Introduction to Optical Engineering

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Optics is the field of science and engineering encompassing the physical phenomena associated with the generation, transmission, manipulation, detection and utilization of light.

From National Research Council Report: "Harnessing Light"

Euclid

Raphael's *School of Athens*

Euclid ~300BC

Greek mathematician

His book *Elements* used to teach geometry for over 2000 years.

Also wrote about optics of the eye, but thought rays exit the eye and we "see" whatever these rays fall on.

Archimedes ~250BC

- Attributed with a “heat ray” which would focus sunlight onto enemy ships and set them on fire.
- Mythbusters “busted” this theory in 2006.

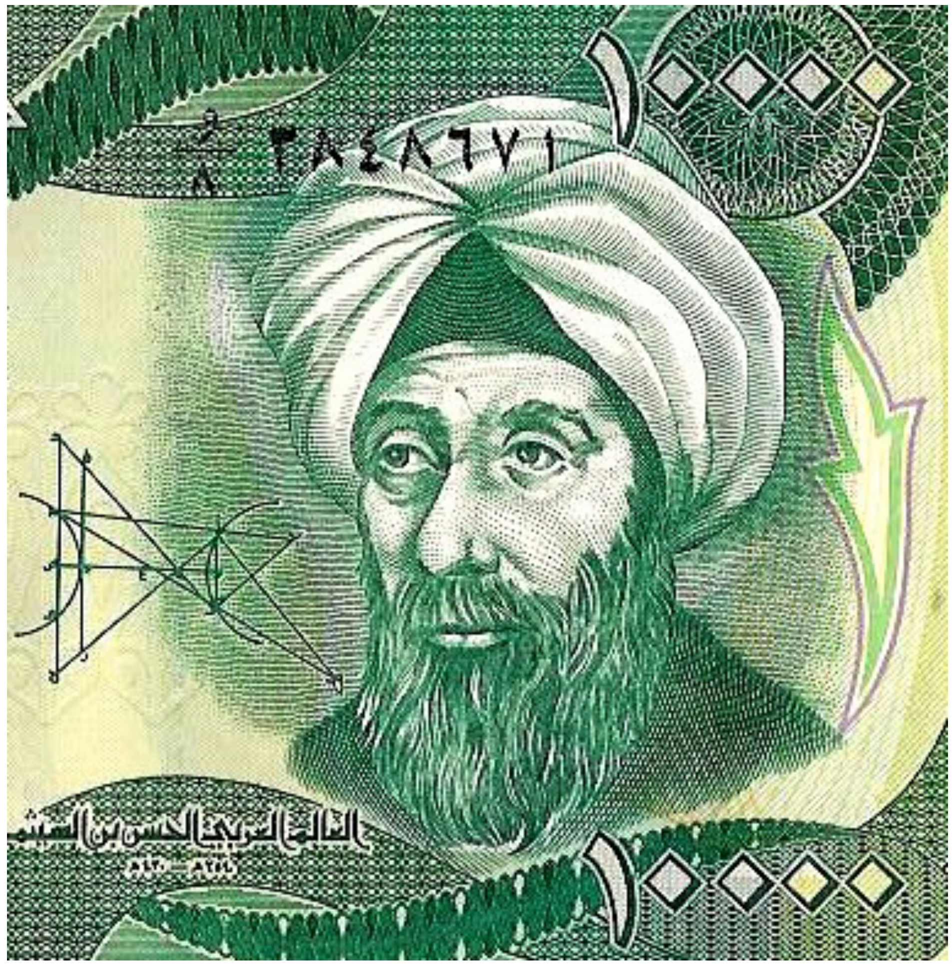
Alhazen ~1000AD

Born in modern day Iraq and called the father of modern optics.

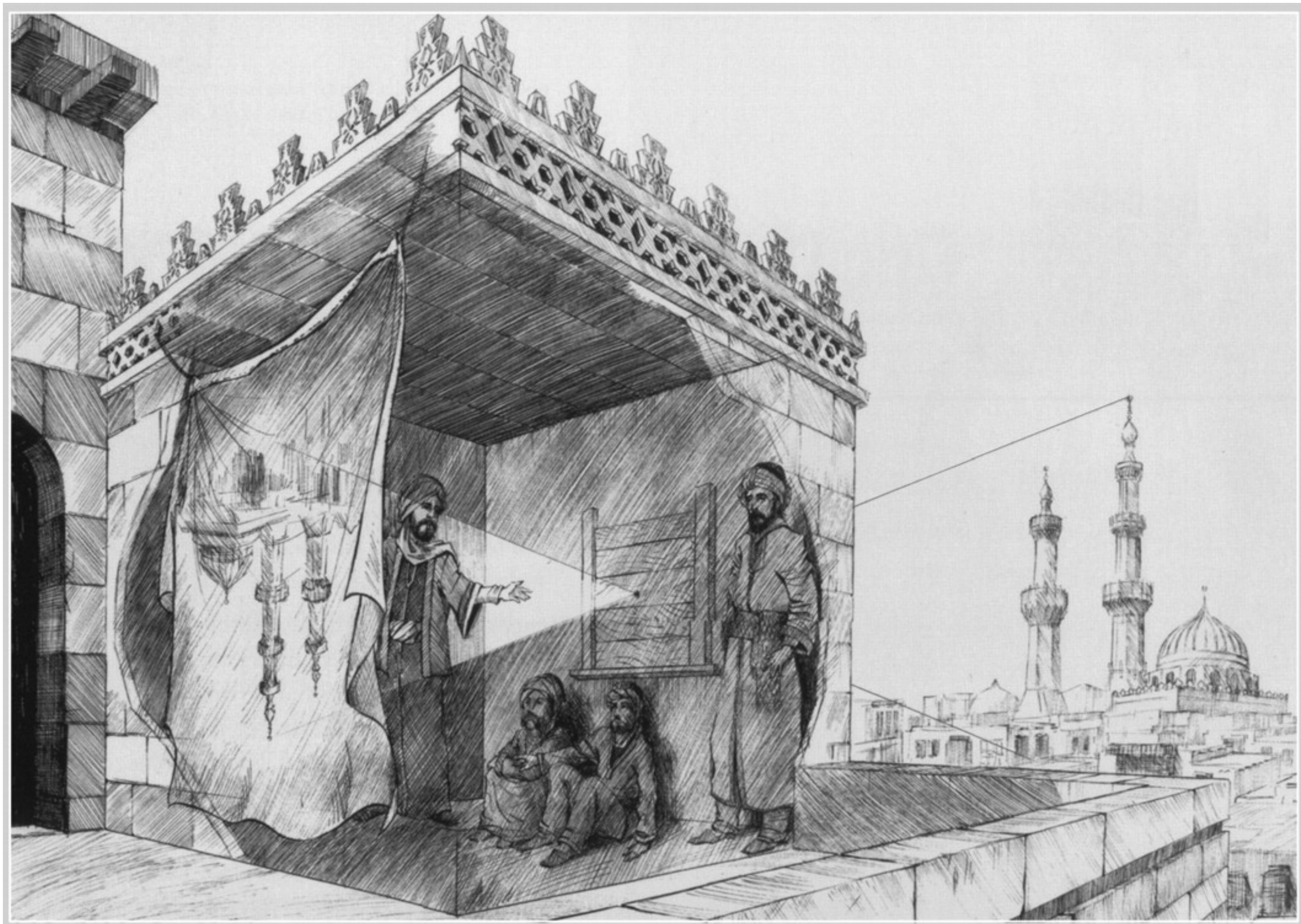
His *Book of Optics* was translated into Latin and later influenced European scholars.

Described magnification in lenses, as well as spherical and parabolic mirrors.

Rays from objects enter the eye.

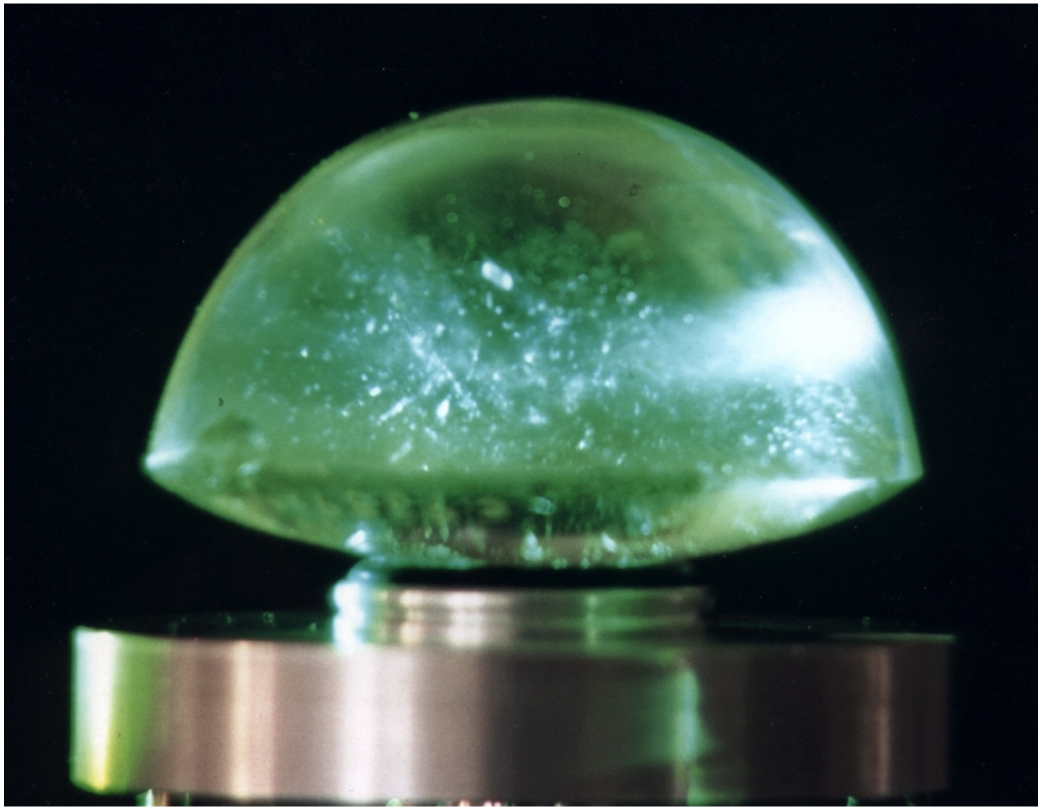


Alhazen - Camera Obscura



Visby Lenses ~11-12th Century

- Found in modern Sweden.
- Possibly used as magnifiers or to start fires.



Zacharias Janssen ~1590AD

- Dutch Spectacle maker
- First compound microscope attributed to him in 1590.
- First telescope sometimes attributed to him.
- Caught counterfeiting coins in 1618.

Hans Lipperhay ~1608AD

- Dutch Spectacle maker
- Next door neighbor of Zacharias Janssen.
- First telescope is attributed to him in 1608.
- Telescope has positive and negative lens and upright image.

HANS LIPPERHEY.

Secundus Conspiciliorum inventor.

Galileo 1609AD

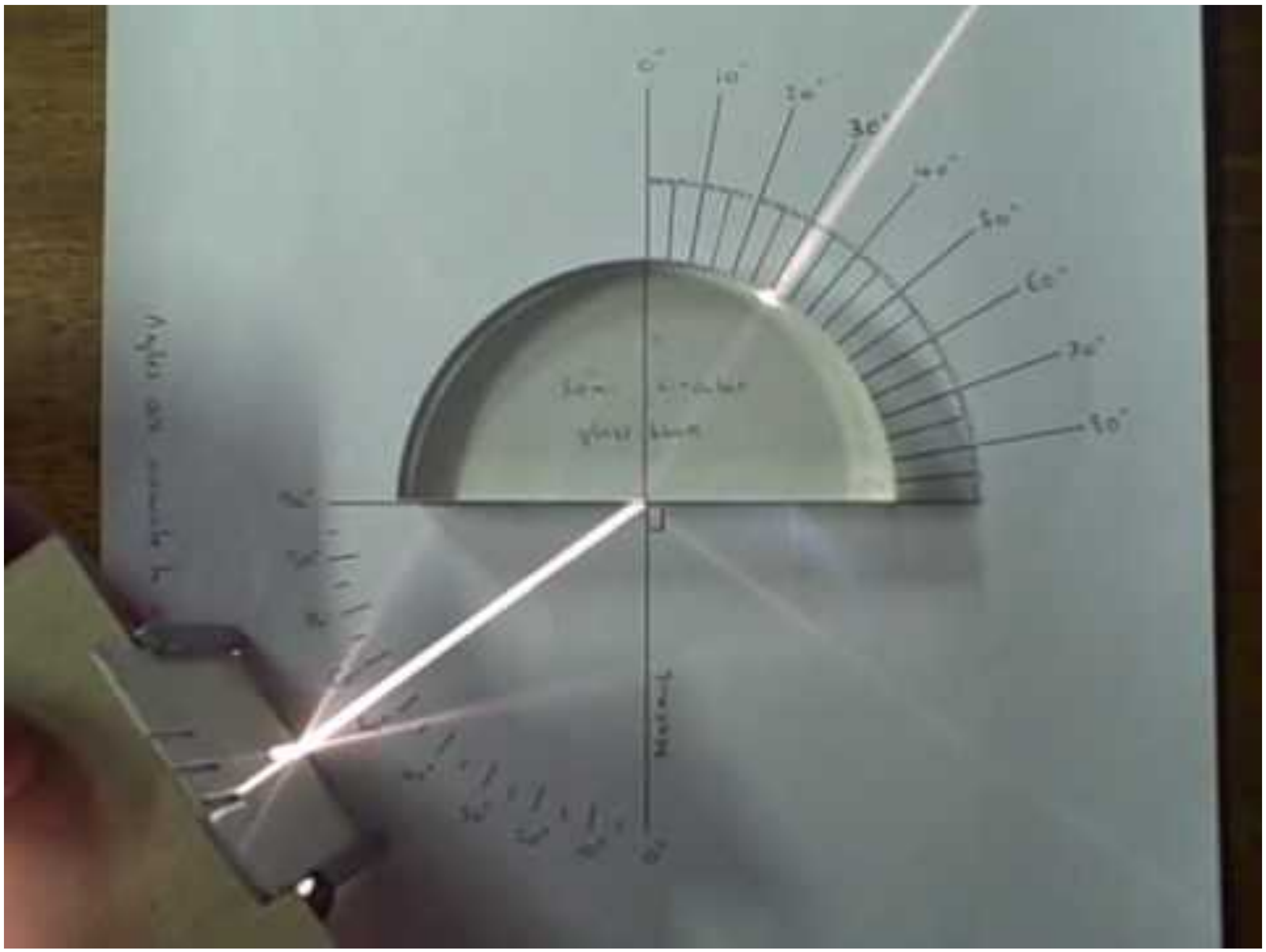
- Constructs in 1609 one of Lipperhay's telescopes.
- Father of observational astromnomy.
- Observed phases of Venus, sunspots and moons of Jupiter.
- Got in trouble for heliocentric views.

Johannes Kepler 1611AD

- German astronomer
- Describes telescopes and microscopes in his book *Dioptrics*.
- Proposes telescope with two positive lenses and upside down image.

Willebrord Snellius 1621

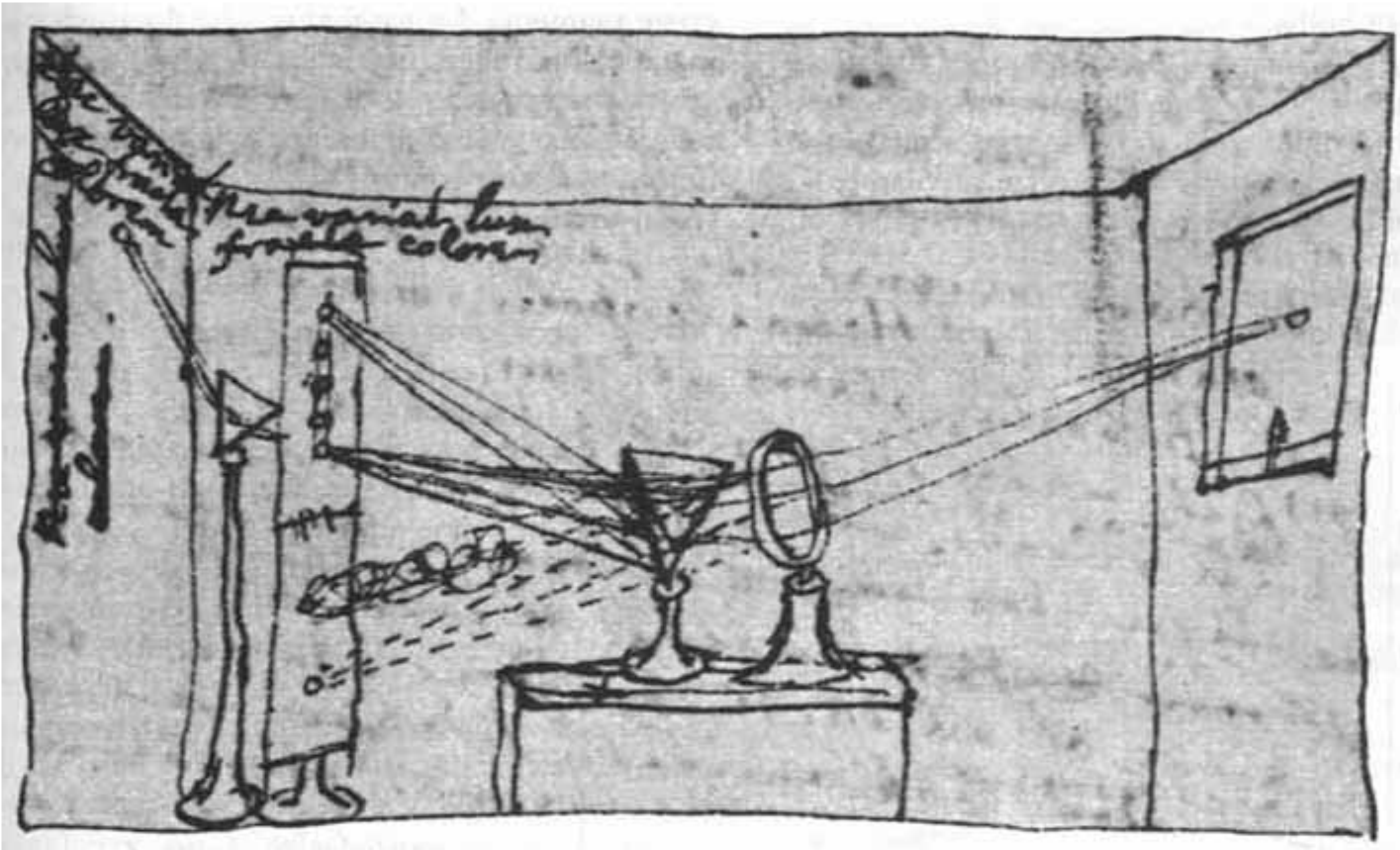
Developed "Snell's Law" of refraction $n_1\sin\theta_1 = n_2\sin\theta_2$, although Ibn Sahl describes this effect in 984 and by Harriot in 1602 who corresponded with Kepler about it.



Isaac Newton

1666 showed white light is composed of all the colors of the spectrum.

1668 suggested all reflective telescope to avoid chromatic aberration. Didn't think lenses could be made to correct chromatic aberration.

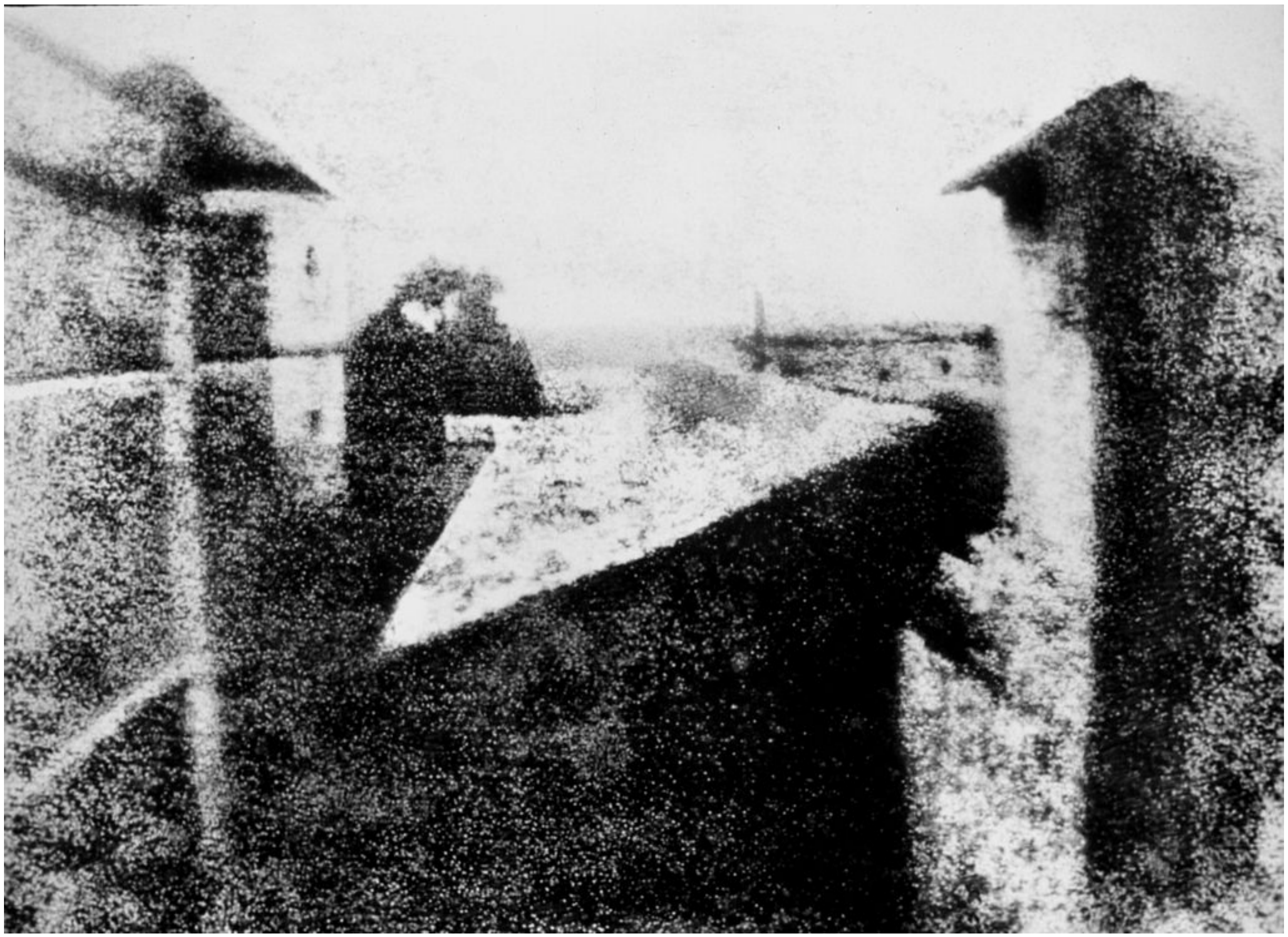


Augustin-Jean Fresnel

- ~1818 Developed the concept for a Fresnel lens for lighthouses which have large aperture and short focal length.
- Major impact in areas of wave optics including polarization and diffraction theory.

Nicéphore Niépce ~1826

- Developed heliography, earliest known photographic process for recording images in a camera obscura.
- Required several days to expose.



Louis Daguerre ~1839

- First practical and commercial demonstration of photography.
- Worked with Niepce until Niepce died in 1933.
- Required several minutes to expose.

<u>Carl Friedrich Gauss 1840</u>

German mathematician

Published *Dioptrische Untersuchungen* which describes the paraxial or Gaussian theory of optics.

Imaging properties of lenses can be determined by their cardinal points.

Death in 1855

Carl Zeiss

German Instrument Maker
Developed high quality microscopes
and later camera lenses.

George Eastman ~1888

Licensed patents from Peter and David Houston covering various aspects of using rolls of film in a camera.

Started Eastman Kodak company which popularized photography.

Original box camera came pre-loaded with film and the customer returned camera for Kodak to make prints and reload.

Thomas Edison 1891

Developed the Kinetoscope which enable one person at a time to view motion pictures.

Louis Lumiere 1895

First portable motion picture camera, film processing and projection system.

Oskar Barnack ~1912

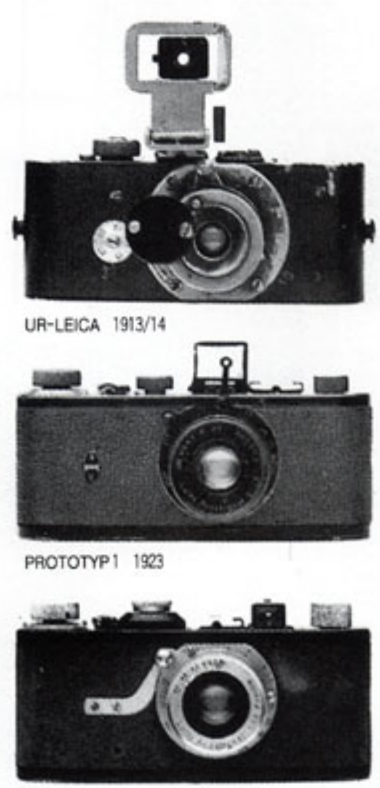
Worked for Leica

Developed portable camera with rolls of film based on Kc
35 mm wide film.

24 x 36 mm images "35 mm format"

Roll of film as long as he could stretch his arms.

Enlarger & Dark room required.



UR-LEICA 1913/14

PROTOTYP 1 1923

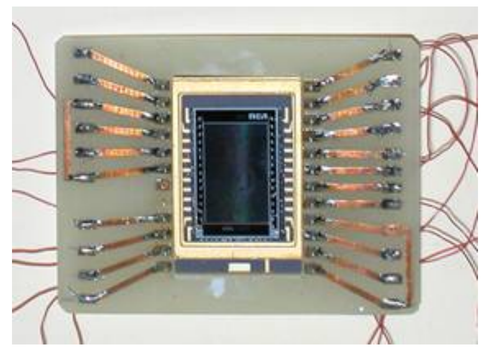
LEICA1 Anastigmat 1 : 3,5 1925

Boyle & Smith 1969

Demonstrated a charged-couple device where charge could be shifted along the surface of a semiconductor to storage capacitors.

Fairchild semiconductor developed the technology into commercial devices.

Kodak developed a digital camera in 1975 based on the Fairchild sensor. (0.01 Mpix).



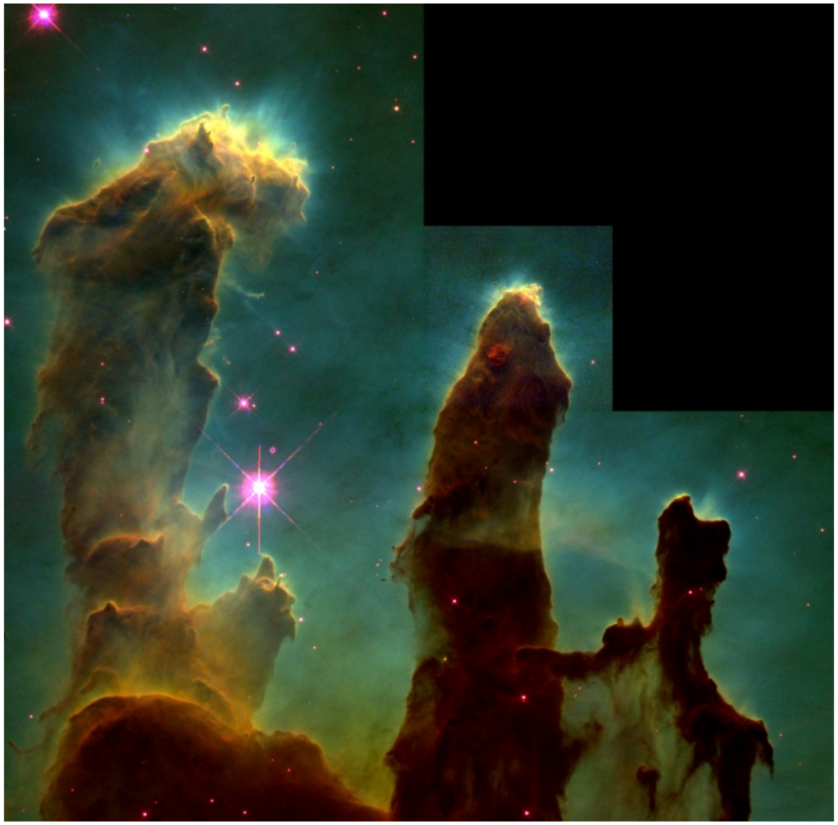
Hubble Telescope

First launched in 1990.

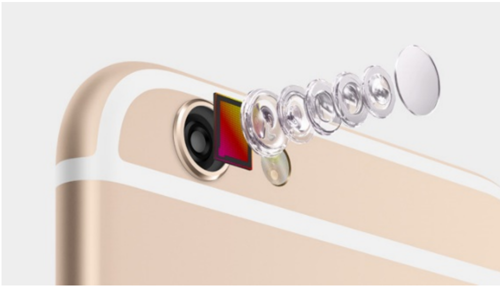
Primary mirror was made incorrectly and had spherical aberration.

Repair mission performed in 1993.

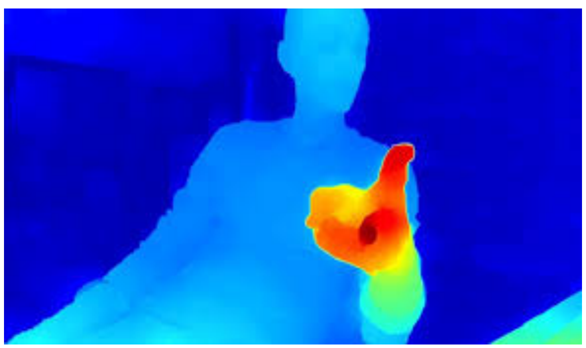
Corrected system now provides some of the most iconic astro-photographs ever captured.

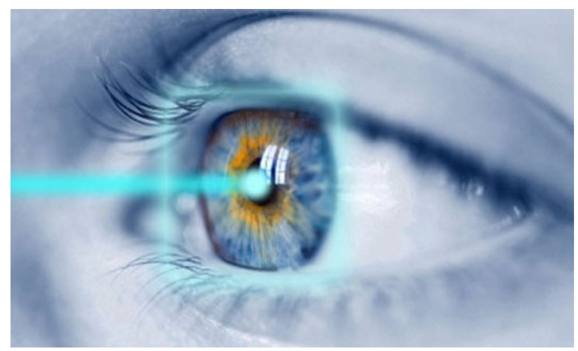


Evolution



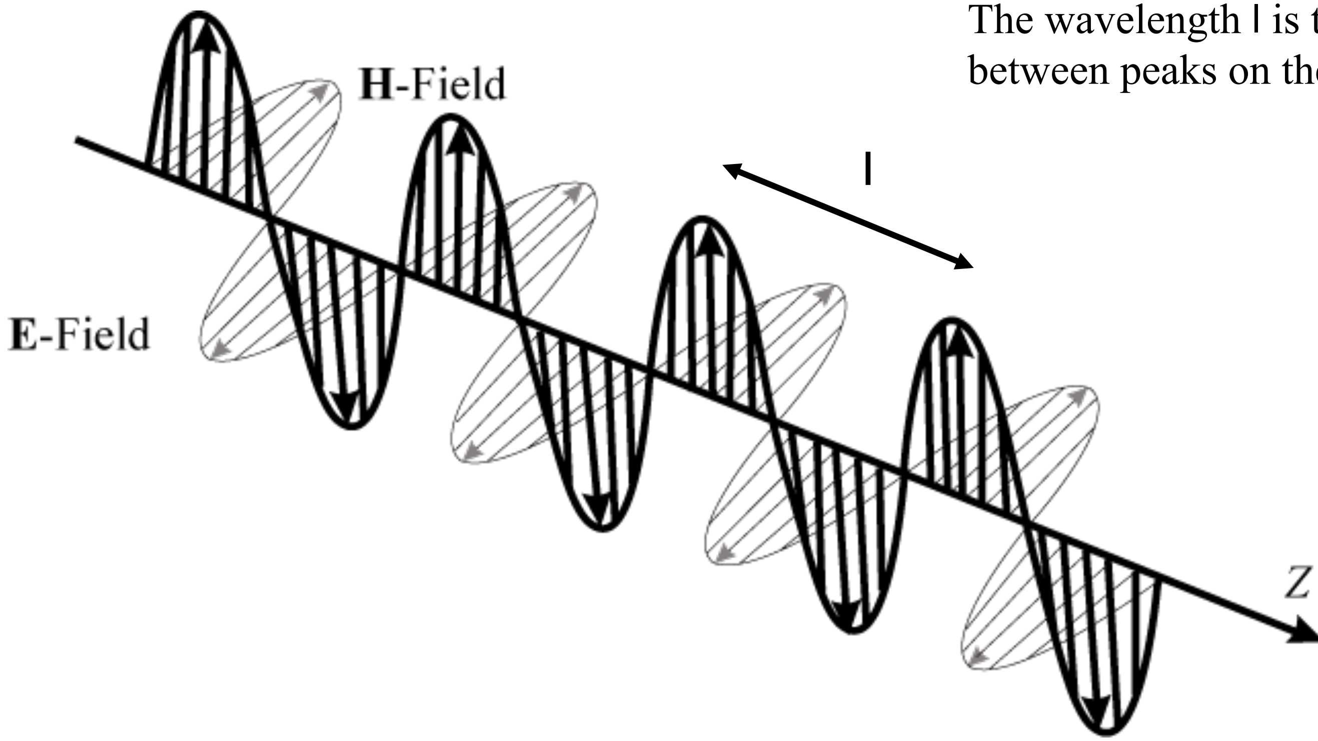




What is Light??

Light is a self-propagating electro-magnetic wave. The electric and magnetic fields are perpendicular or transverse to the direction of propagation.

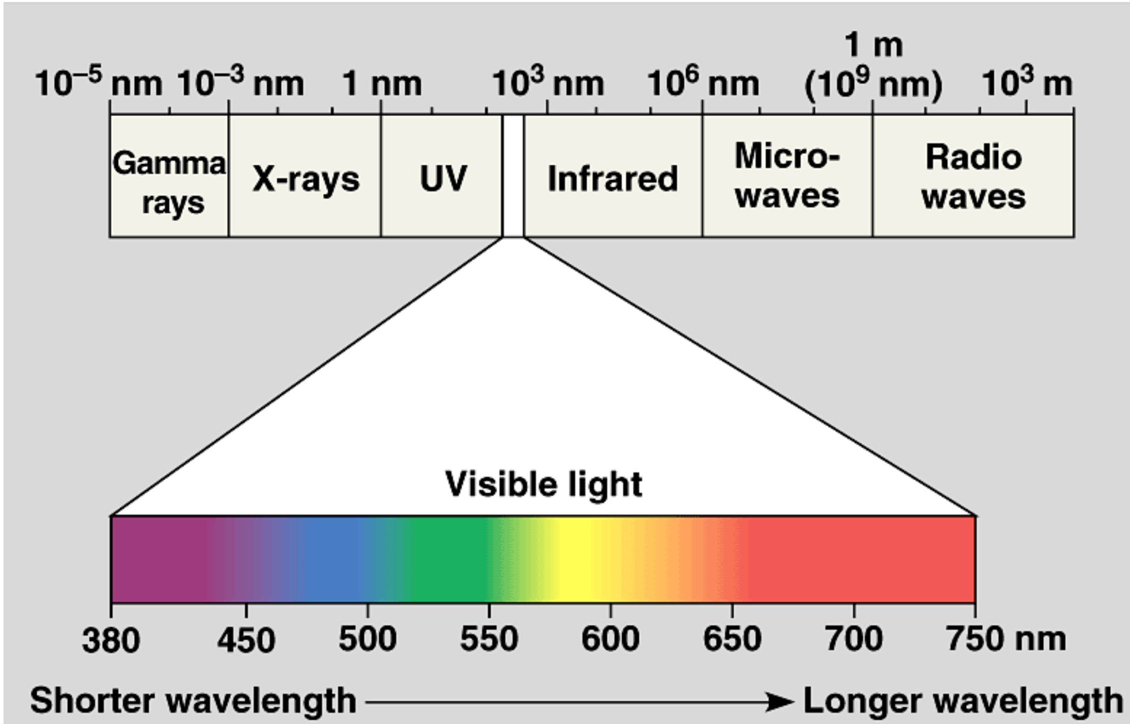


The wavelength l is the distance between peaks on the wave.

In vacuum, the EM wave propagates a the speed of light:

$$c = 2.99792458 \times 10^{8} \text{ m/s}$$

Electro-Magnetic Spectrum



~ 350 nm < l < ~ 750 nm for Visible Light

~ 0.35 µm < l < ~ 0.75 µm for Visible Light

The field of optics often implies only the visible portion of the spectrum, but more generally optics includes both the ultraviolet and infrared portions of the spectrum.

Example:

$$\lambda = 550\ nm = 0.55\ \mu m \quad \text{(Green)}$$

$$c = 2.99792458 \times 10^{8}\ \text{m/s}$$

$$\nu = 5.45 \times 10^{14}\ Hz$$

Wavelength and Frequency

If the wave is propagating with a velocity or speed V, the frequency n of the wave is the number of cycles or wavelengths that pass a point in one second.

The time for one wavelength to pass is $$T = \frac{\lambda}{V}$$

The frequency is then $$\frac{1}{T} = \nu = \frac{V}{\lambda}$$

$$\lambda = \frac{V}{\nu} \qquad \text{in vacuum: } \lambda = \frac{c}{\nu}$$

Units:	l	m
	V	m/sec
	n	1/sec or Hz

Index of Refraction

Speed of light in vacuum

$$c = 2.99792458 \times 10^{8} \text{ m/s}$$

Index of Refraction n

$$n \equiv \frac{\text{Speed of Light in Vacuum}}{\text{Speed of Light in Medium}} = \frac{c}{V} \qquad V = \frac{c}{n}$$

The index of refraction tells how much light slows down in a medium compared to its speed in a vacuum.

Note that in a medium, the frequency does not change as this is determined by the source oscillation. The wavelength changes with the index.

$$\lambda_M = \frac{\lambda_0}{n}$$

Example:

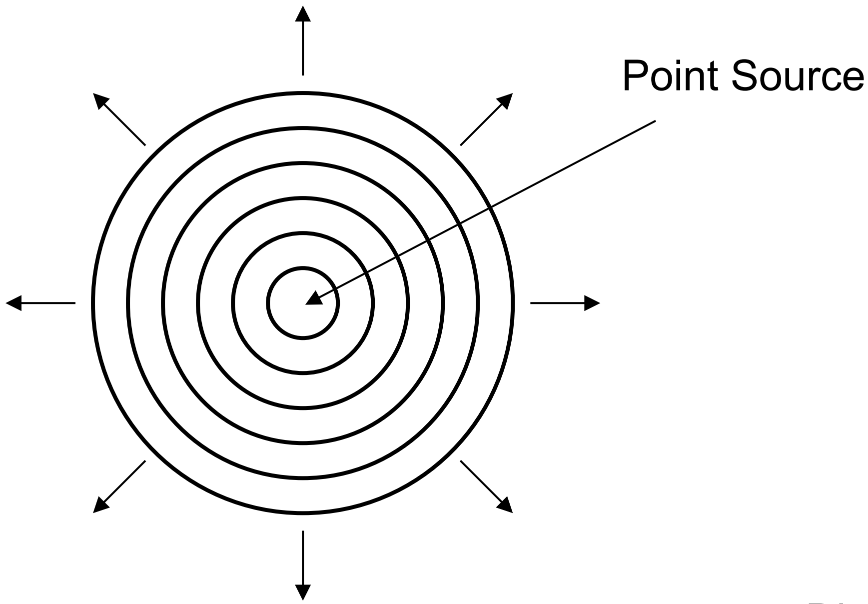
$$\lambda_0 = 550\ nm = 0.55\ \mu m \quad \text{(Green)}$$
$$\nu = 5.45 \times 10^{14}\ Hz$$
$$n = 1.5$$
$$V = 1.993 \times 10^{8} \text{ m/s}$$
$$\lambda_M = 367\ nm = 0.367\ \mu m$$

Typical Indices of Refraction

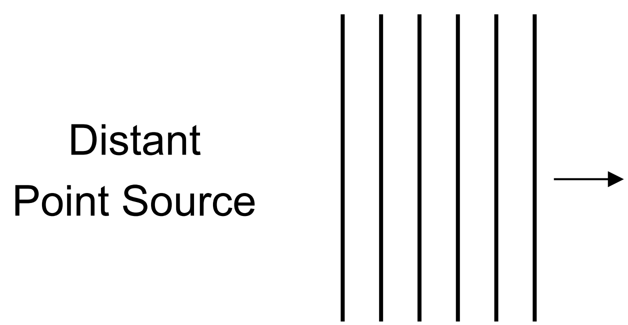
vacuum		1.0
helium		1.000036
hydrogen		1.000132
air		1.000293
water		1.33
fused silica		1.46
plastics		1.48-1.6
borosilicate crown glass		1.51
crown glass		1.52
light flint glass		1.57
dense barium crown glass		1.62
dense flint glass		1.72
diamond		2.4
Silicon	@ 10 mm	3.4
Germanium	@ 13 mm	4.0

Propagation and Wavefronts

A point source in a homogeneous median will produce an expanding spherical wave. A wavefront is a surface of constant propagation time from the source.



At a great distance from the source, a portion of the spherical wave will appear to be a plane wave.



There is a direct analogy to water waves, which are also transverse waves. The water moves up and down – the water does not propagate, but the wave does.

Geometrical Optics

Any object is comprised of a collection of independently radiating point sources. Each source is infinitesimally small; there is no interference. Each point source is independently imaged through the system, and the image is the superposition of intensity patterns from all of the point images.

First-order optics is the study of perfect optical systems, or optical systems without aberrations. Analysis methods include Gaussian optics and paraxial optics. Small angle approximations are used.

Imaging properties (image location and magnification)
Radiometric properties
Preliminary system design and layout

Aberrations are the deviations from perfection of the optical system. These aberrations are inherent to the design of the optical system, even when perfectly manufactured. Additional aberrations can result from manufacturing errors.

Third-order optics (and higher-order optics) includes the effects of aberrations on the system performance. The effects of diffraction are sometimes included in the analysis.

Image quality
Optical design

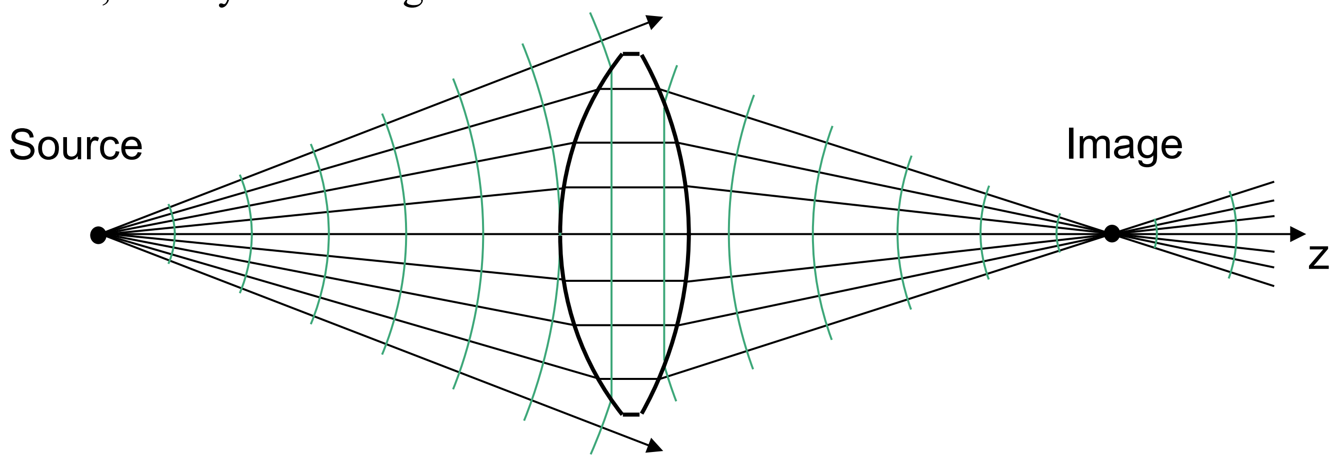
Geometrical optics principles can be applied to radiation of any wavelength.

<u>Wavefronts and Rays</u>

Optical Path Length is the product of the local refractive index and the distance travelled.

Wavefronts are surfaces of constant OPL from a source point.

Rays indicate the direction of energy propagation and are normal to the wavefront surfaces.
If n is constant, the rays are straight lines.



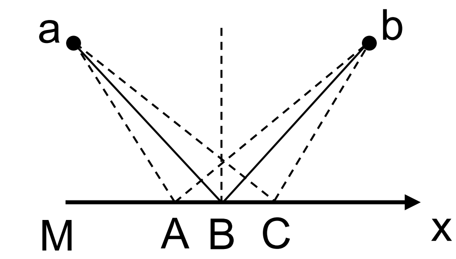
In a perfect optical system or a first-order optical system:

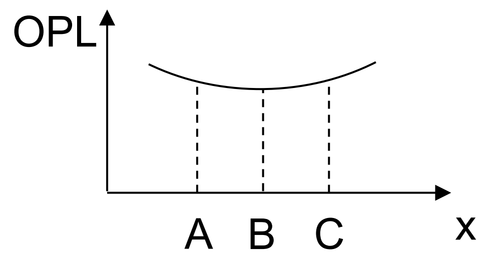
- All wavefronts are spherical or planar.
- The OPL along each ray from the object point to the image point is constant.

If n is not constant, the rays may be bent. An example is GRIN or Gradient Index materials.

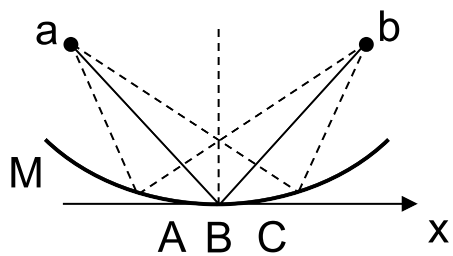
<u>Fermat Examples</u>

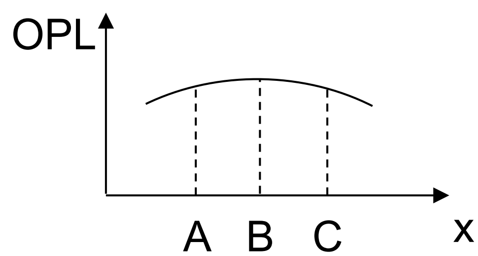
Minimum OPL – a plane mirror



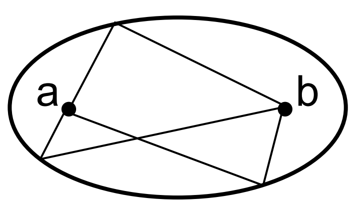


Maximum OPL – a concave mirror





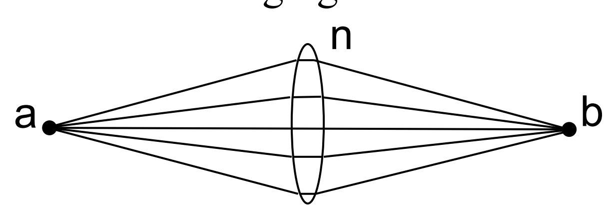
Equal OPL – an elliptical reflector



(a and b are the foci of the ellipse)

An ellipse can be drawn with a piece of string of fixed length with its ends fixed at the foci.

Equal OPL – an imaging lens



(a and b are object and image)

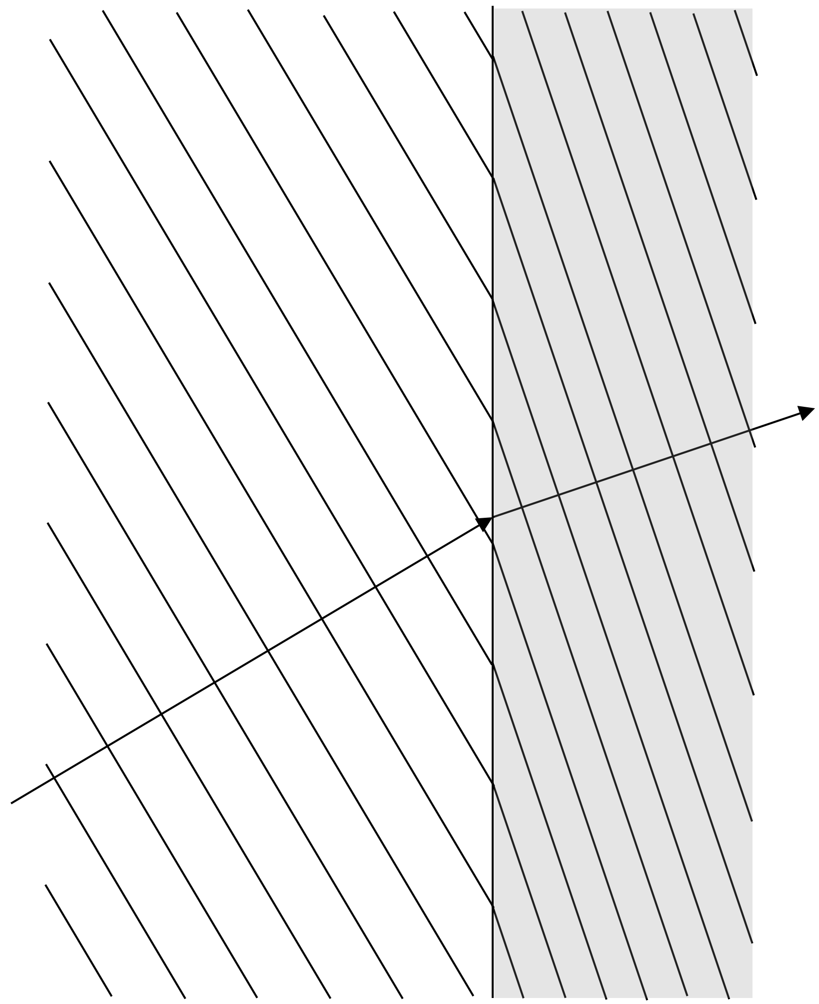
The extra OPL through the center of the lens (nt) compensates for the extra distance to the edge of the lens.

Refraction and Wavefronts

The wavefront spacing is different across the boundary, and the wavefronts must meet at the interface.

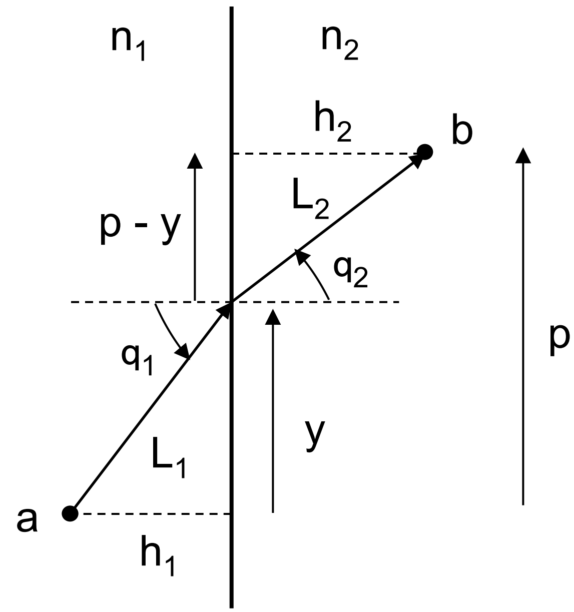
The Law of Refraction or Snell's Law can be easily derived from this condition.

The same arguments and analysis lead to the Law of Reflection.



Refraction at a Surface

Use Fermat's Principle to determine the valid ray path across a refractive boundary. The ray goes from point a to point b. The variable y defines the ray intersection at the interface:



$$OPL = n_1 L_1 + n_2 L_2$$

$$L_1 = \sqrt{h_1^2 + y^2}$$

$$L_2 = \sqrt{h_2^2 + (p-y)^2}$$

$$\frac{dOPL}{dy} = n_1 \frac{dL_1}{dy} + n_2 \frac{dL_2}{dy} = 0 \quad \text{for a valid ray path}$$

$$\frac{dL_1}{dy} = \frac{y}{\sqrt{h_1^2 + y^2}} = \frac{y}{L_1} = \sin\theta_1$$

$$\frac{dL_2}{dy} = \frac{-(p-y)}{\sqrt{h_2^2 + (p-y)^2}} = -\frac{(p-y)}{L_2} = -\sin\theta_2$$

Then

$$n_1 \sin\theta_1 - n_2 \sin\theta_2 = 0$$

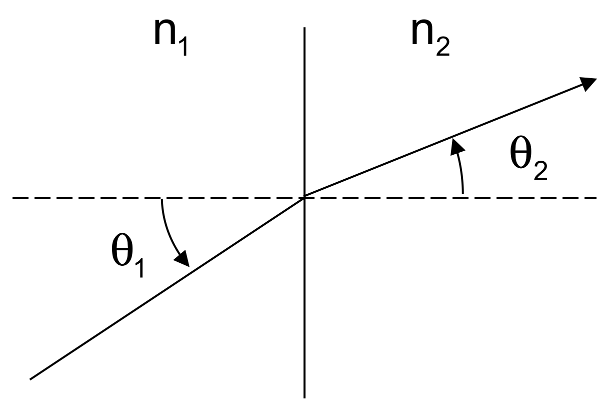
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This is, of course, Snell's Law

Snell's Law of Refraction

For refraction at a surface,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$



where n_1 and n_2 are the indices of refraction in the two media and q_1 and q_2 are the angles between the rays and the surface normal.

An important second part of this law is that the incident ray, the refracted ray and the surface normal must be coplanar.

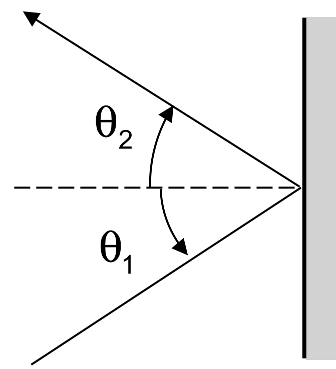
For refraction at a curved surface, the surface slope and angles at the ray intercept are used.

When propagating through a series of parallel interfaces, the quantity n sinq is conserved.

Law of Reflection

For reflection at a surface, the angle of incidence equals the angle of reflection:

$$\theta_2 = -\theta_1$$



Both angles are measured relative to the surface normal, and the minus sign is due to sign conventions.

Once again, the two rays and the surface normal must be coplanar.

The law of reflection is often considered a special case of Snell's law where $n_2 = - n_1$. The sign of the index of refraction is now a function of propagation direction.

Following a reflection, light propagates from right to left, and its velocity can be considered to be negative. Using velocity instead of speed in the definition of *n*, the index of refraction is now also negative.

$$n \equiv \frac{\text{Velocity of Light in Vacuum}}{\text{Velocity of Light in Medium}} = \frac{c}{V}$$

Critical Angle and Total Internal Reflection TIR

When a ray propagates from a high index medium to a low index medium, the angle of refraction is larger than the angle of incidence. For a sufficiently large incident angle, there is no solution to Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{and} \quad n_1 > n_2$$

There is no solution when

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 > 1$$

Under this condition, the light ray is reflected at the surface by *total internal reflection* (TIR). There is no refracted beam and the beam is completely reflected. This process achieves 100% reflection, and is often used in prisms to achieve high reflectivity.

The incident angle that just satisfies this inequality is the *critical angle*:

$$\sin\theta_1 = \sin\theta_C = \frac{n_2}{n_1}$$

Incident rays above the critical angle undergo TIR; rays at angles less than the critical angle are refracted.

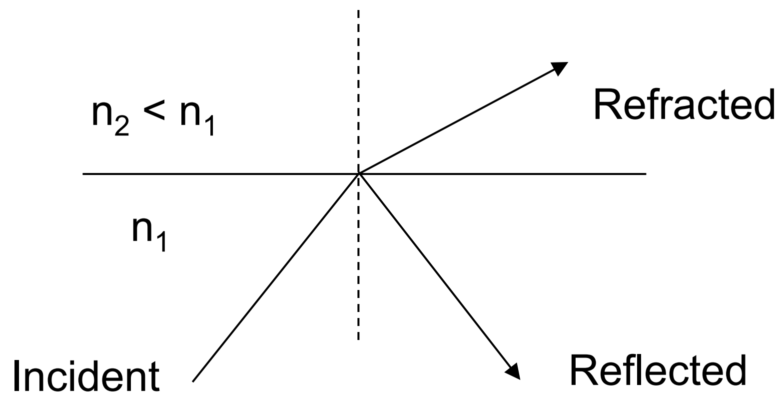
Critical angles for $n_2 = 1.0$

1.3	50.3°
1.4	45.6°
1.5	41.8°
1.6	38.7°
1.7	36.0°
1.8	33.7°
1.9	31.8°
2.0	30.0°

Partial Reflection

For common glasses (n = 1.5), the critical angle is about 42°. Even at angles below the critical angle there is a significant portion of the ray that is reflected. The ratio of reflection to transmission is governed by the Fresnel reflection coefficients, and the reflectance increases rapidly as the critical angle is approached.

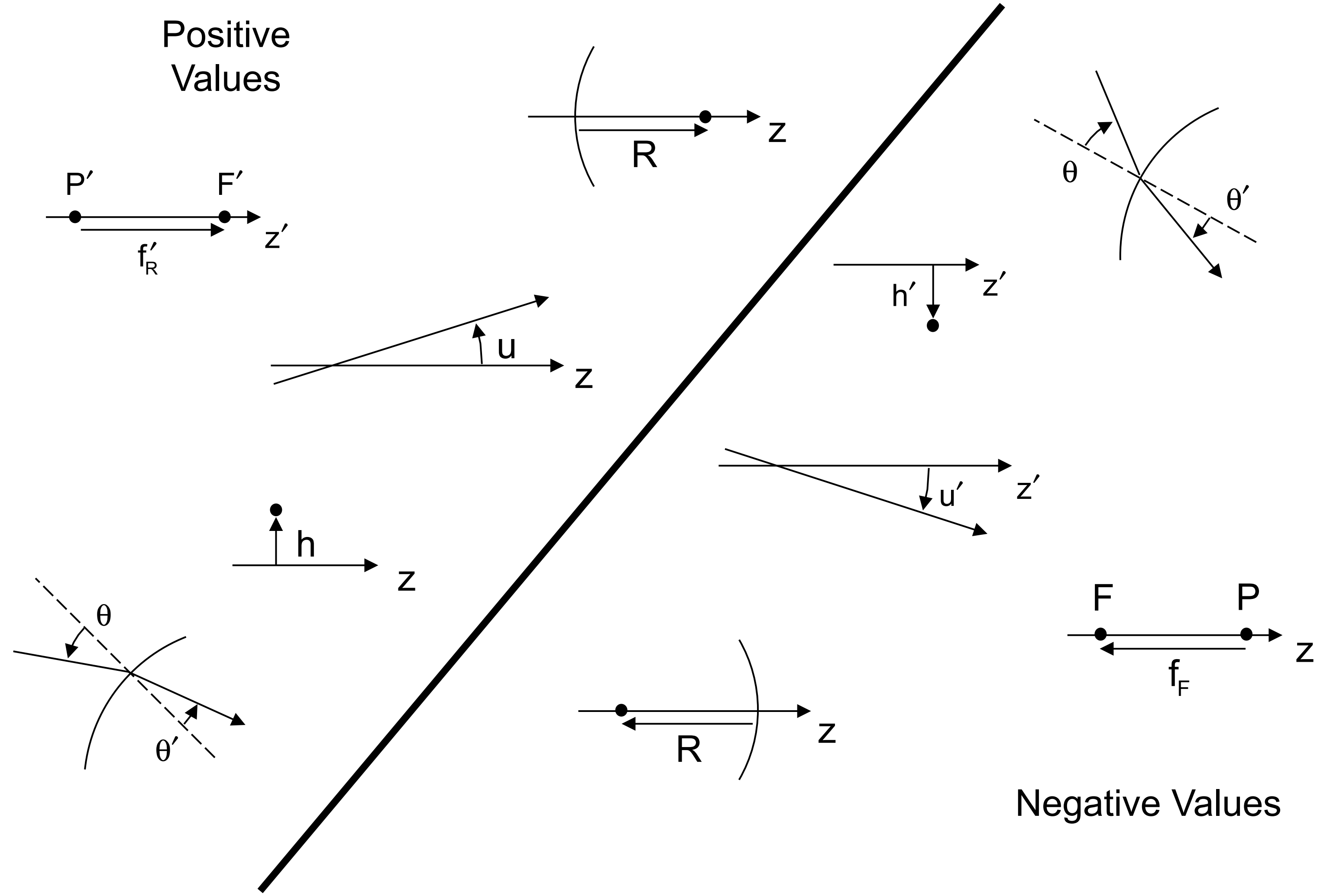
Care must be taken in the design of optical systems with steep surface slopes; TIR instead of refraction may occur at glass/air interfaces.



At normal incidence, with no absorption, the reflectance of an interface is

$$\rho = \left(\frac{n_2 - n_1}{n_2 + n_1} \right)^2$$

Sign Conventions



The value and sign for an angle or position depends on the reference. For example, a ray angle can be measured relative to either the optical axis or a surface normal.

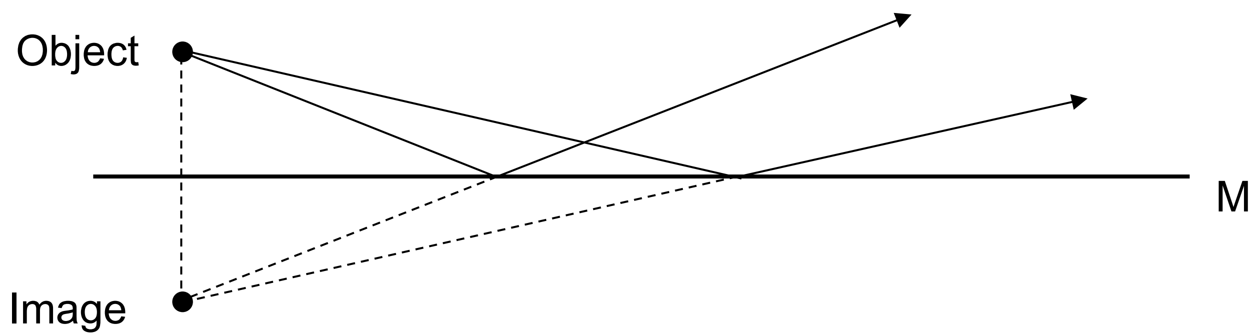
MIRRORS & PRISMS

Plane Mirrors

Plane mirrors are used to:

- Produce a deviation
- Fold the optical path
- Change the image parity

Each ray from the object point obeys the law of reflection at the mirror surface. A virtual image of the object point is produced.



The rules of plane mirrors:

- The line connecting an object point and its image is perpendicular to the mirror and is bisected by the mirror.
- Any point on the mirror surface is equidistant from a given object point and its image point.
- The image parity is changed on reflection.

Image Parity and Orientation

Terminology:

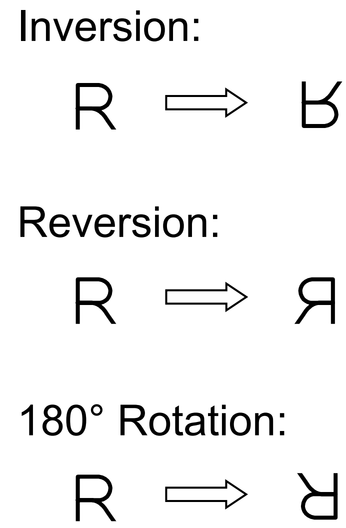
Image Rotation – the image is rotated about a line normal to the image (optical axis). An image rotation does not produce a parity change.

Image Parity Changes

Invert – Parity change about horizontal line; vertical flip.

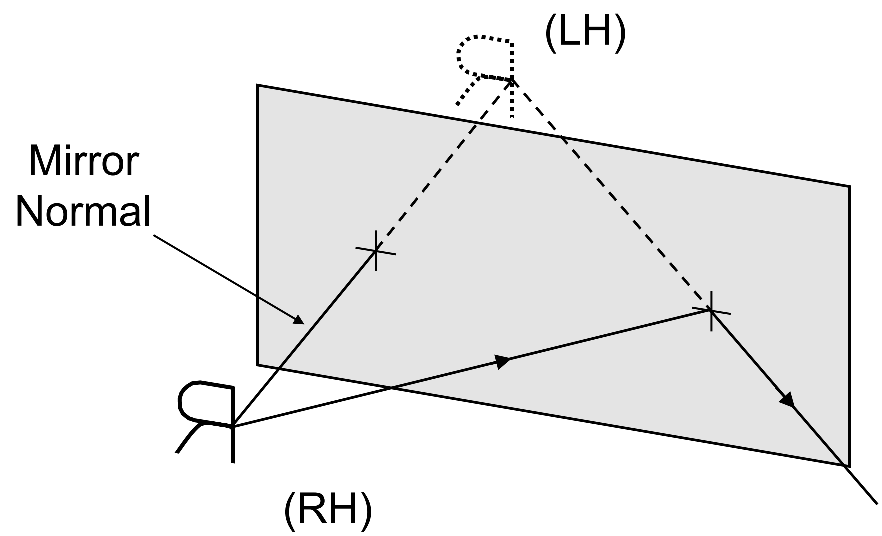
Revert – Parity change about a vertical line; horizontal flip.

An inversion and a reversion is equivalent to a 180° rotation.



An image seen by an even number of reflections maintains its parity. An odd number of reflections changes the parity.

Parity is determined by looking back against the propagation direction towards the object or image in that optical space; let the light from the object or image come to you.

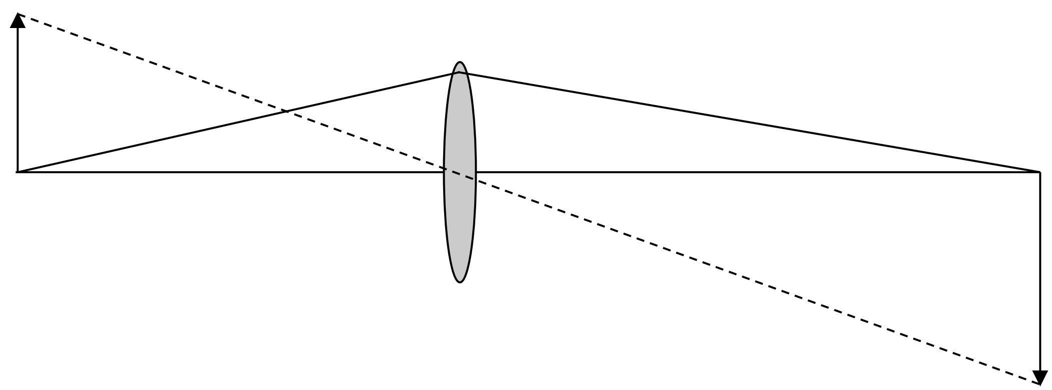


Parity and Imaging

The object and the image formed by a lens are

flipped top and bottom

flipped left and right



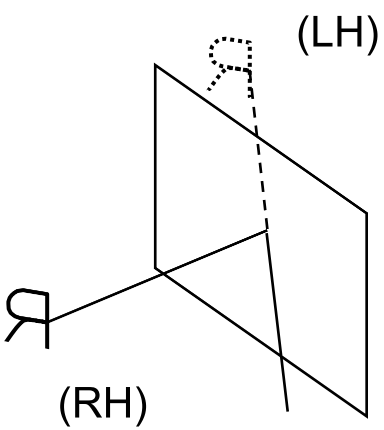
This constitutes an inversion and a reversion.

The image is rotated 180° about the optical axis.

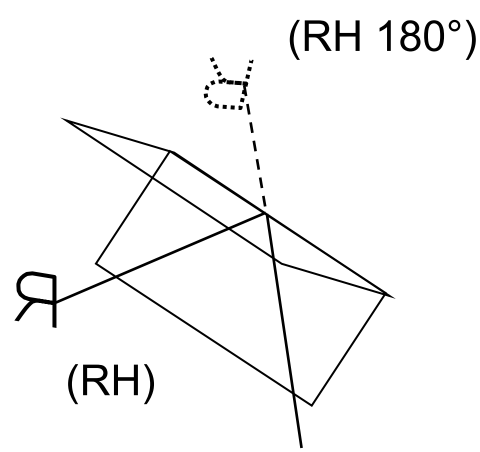
There is no parity change.

Roof Mirror

A roof mirror is two plane mirrors with a dihedral angle of 90°, and the input and output rays are anti-parallel. This roof mirror can replace any flat mirror to insert an additional reflection or parity change. It preserves parity on reflection.

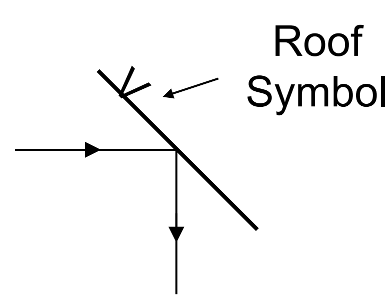


Plane Mirror



Roof Mirror

The dihedral line is often in the plane of the drawing, and the presence of a roof mirror is indicated by a "V" at the equivalent mirror or dihedral line.



90° Deviation Prisms

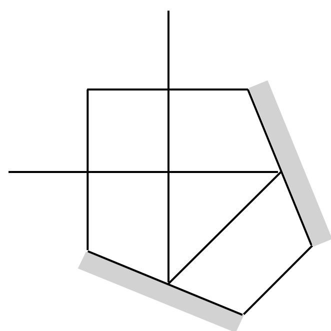
Right angle prism (1 R) – the actual deviation depends on the input angle and prism orientation. Image is inverted or reverted depending on prism orientation.

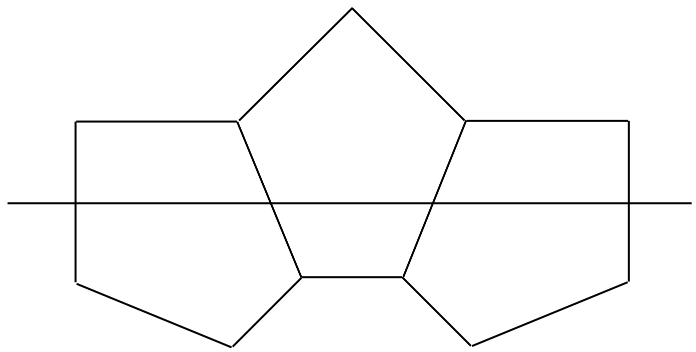
Amici or Roof prism (2 R) – a right angle prism with a roof mirror. The image is rotated 180°. No parity change.

The tunnel diagram is identical to the right angle prism.

More 90° Deviation Prisms

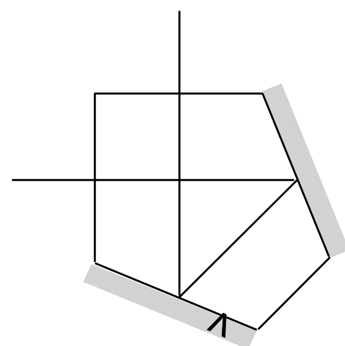
Pentaprism (2 R) – two surfaces at 45° produce a 90° deviation independent of the input angle. It is the standard optical metrology tool for defining a right angle. The two reflecting surfaces must be coated. No parity change.



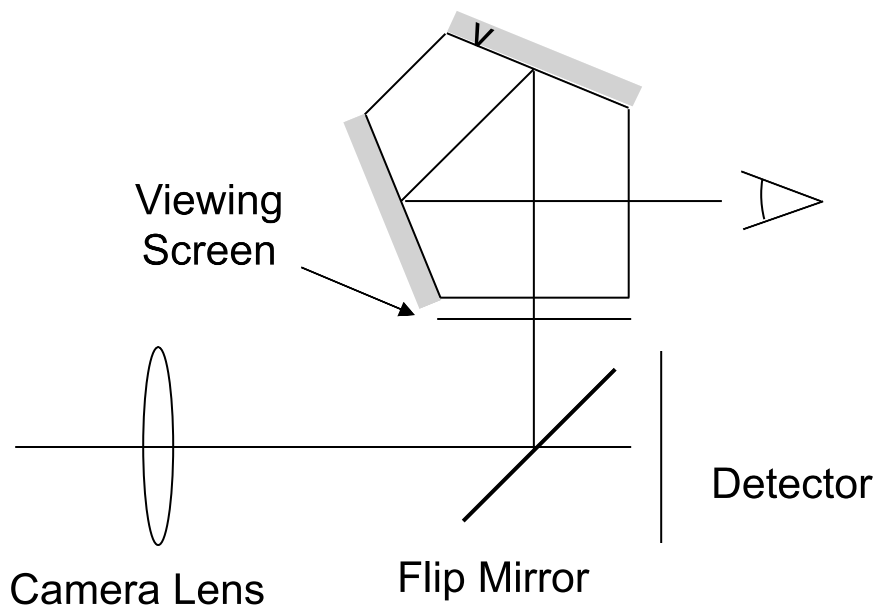


More 90° Deviation Prisms – Reflex Prism

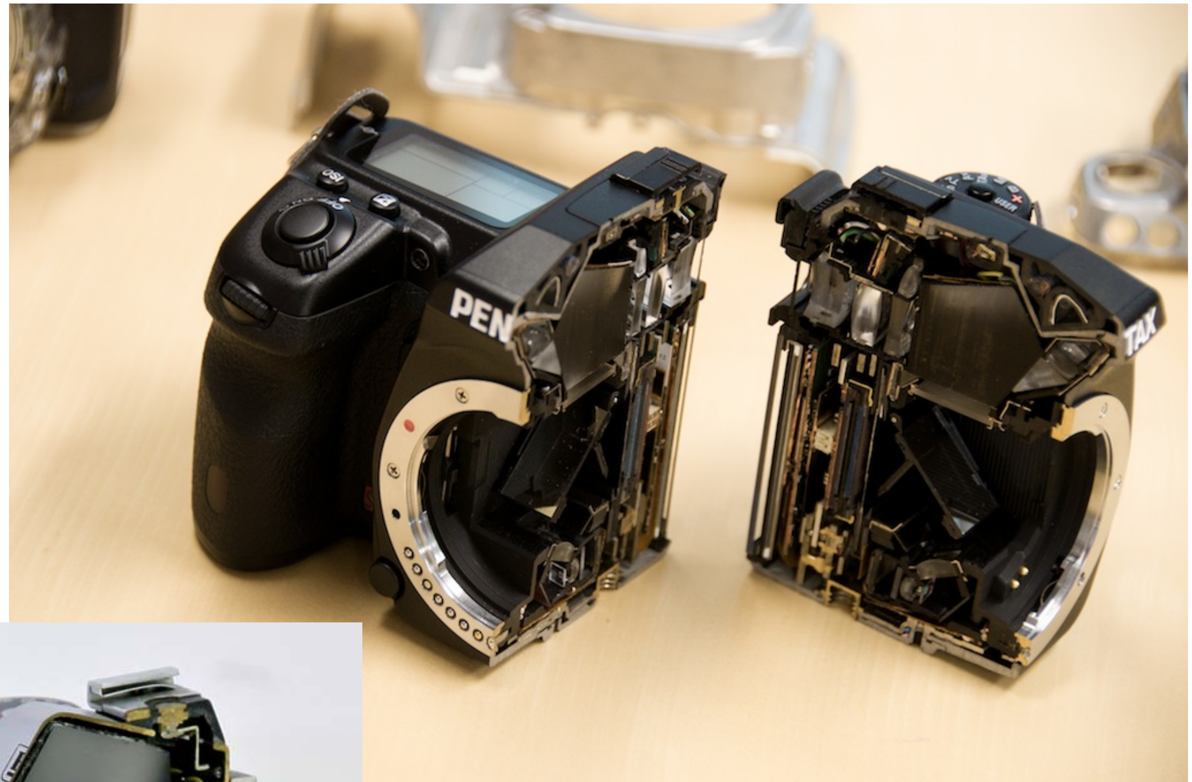
Reflex prism (3 R) – a pentaprism with an added roof mirror. Used in single lens reflex (SLR) camera viewfinders to provide an erect image of the proper parity. The roof surfaces must also be coated.



SLR Camera – the flip mirror rotates up during the exposure. The film and viewing screen planes are optically coincident.



SLR Camera

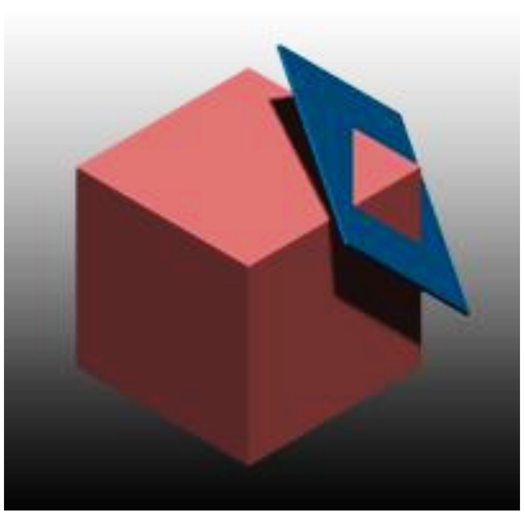


Wikipedia

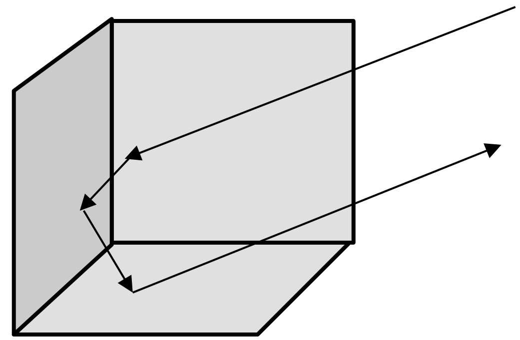
LoneTreeImages

College of Optical Sciences
The University of Arizona

<u>Retroreflectors</u>



Three reflections are made and the light returns to the source.



Taillights and Bicycle Reflectors:

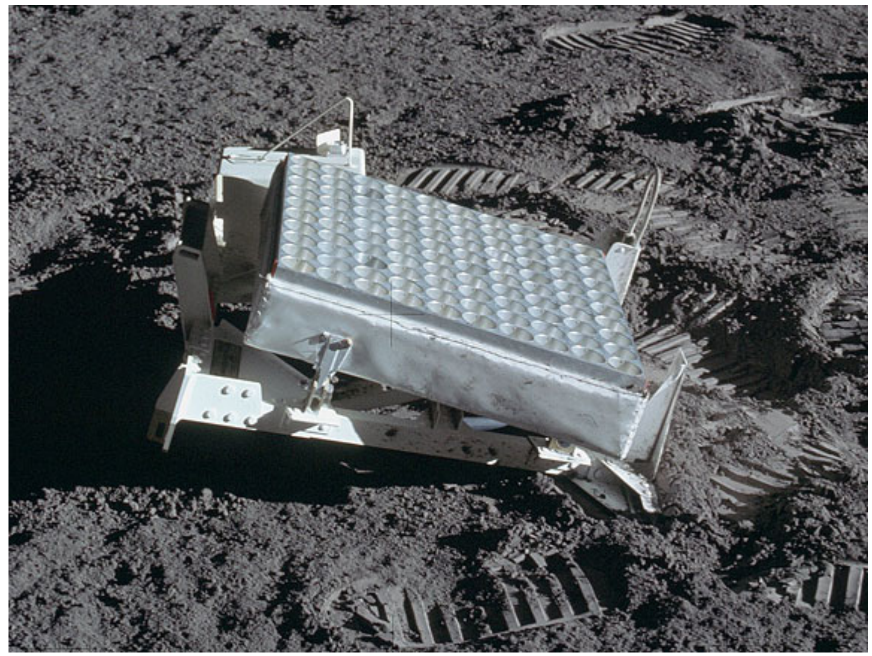
bannerengineering.com

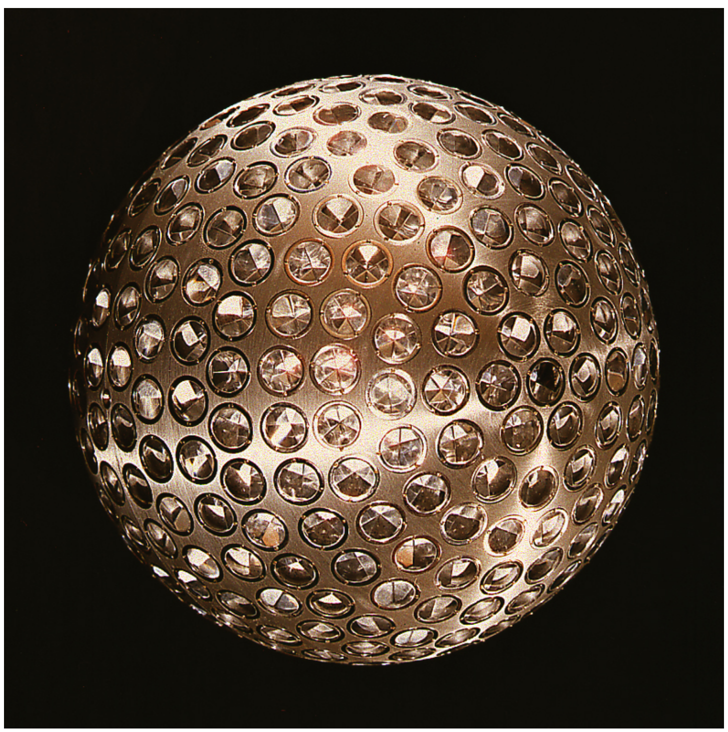
Applications of Corner Cubes – Time-of-Flight Measurements

Surveying Prism

Lunar Laser Ranging Retroreflector Arrays

Apollo 11 and 14: 100 prisms Apollo 15: 300 prisms





Laser Geodynamics Satellite (Orbit at 5,900 km):

The LAGEOS mission goals:

- Provide an accurate measurement of the satellite's position with respect to Earth,
- Determine the planet's shape (geoid), and
- Determine tectonic plate movements associated with continental drift.

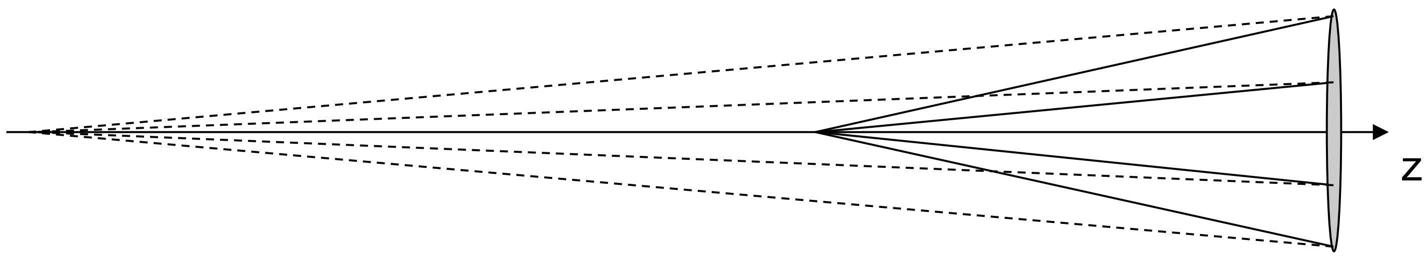
THIN LENSES

Object at Infinity

An object at infinity produces a set of collimated set of rays entering the optical system.

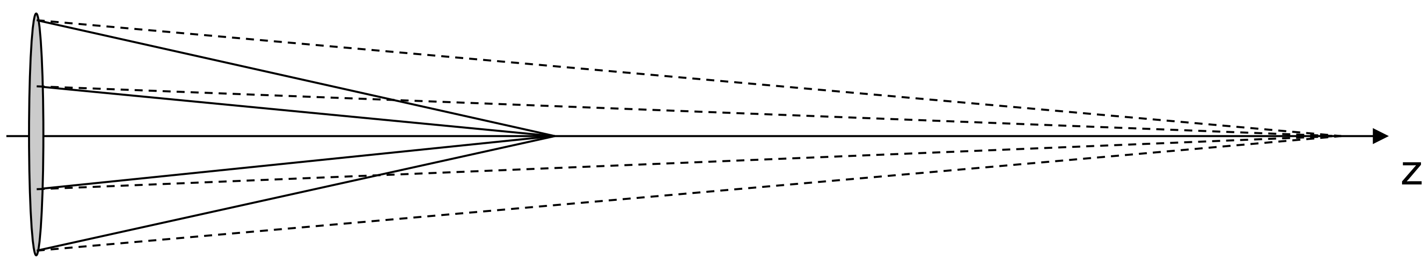
Consider the rays from a finite object located on the axis.

When the object becomes more distant the rays become more parallel.



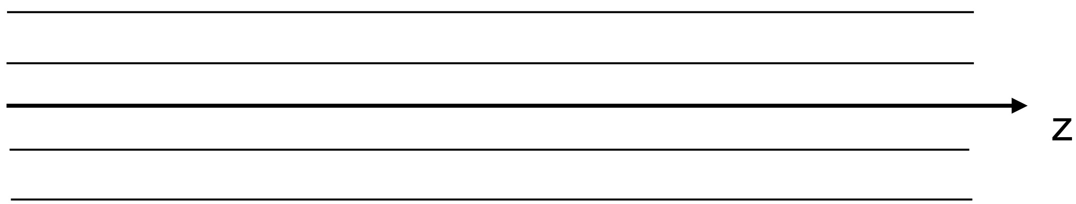
When the object goes to infinity, the rays become parallel or collimated.

An image at infinity is also represented by collimated rays.



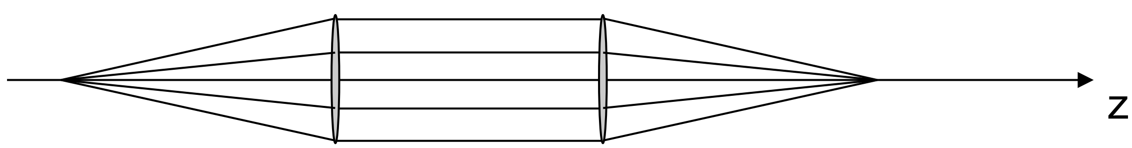
Parallel Rays

Parallel rays are used to represent either an object at infinity or an image at infinity.



One way to think of this is that parallel lines intersect at a point at infinity. Without a lens, these rays could represent either an object at negative infinity or an image at positive infinity. With a lens, it is easy to tell an object from an image:

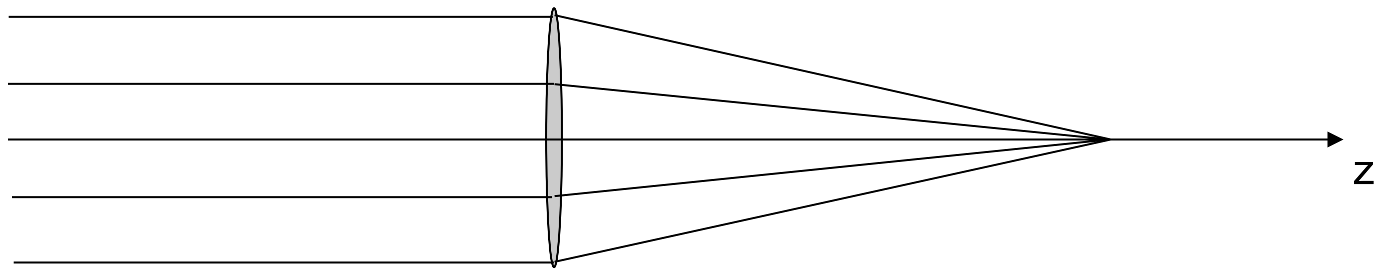
But an image at positive infinity can serve as an object at negative infinity:



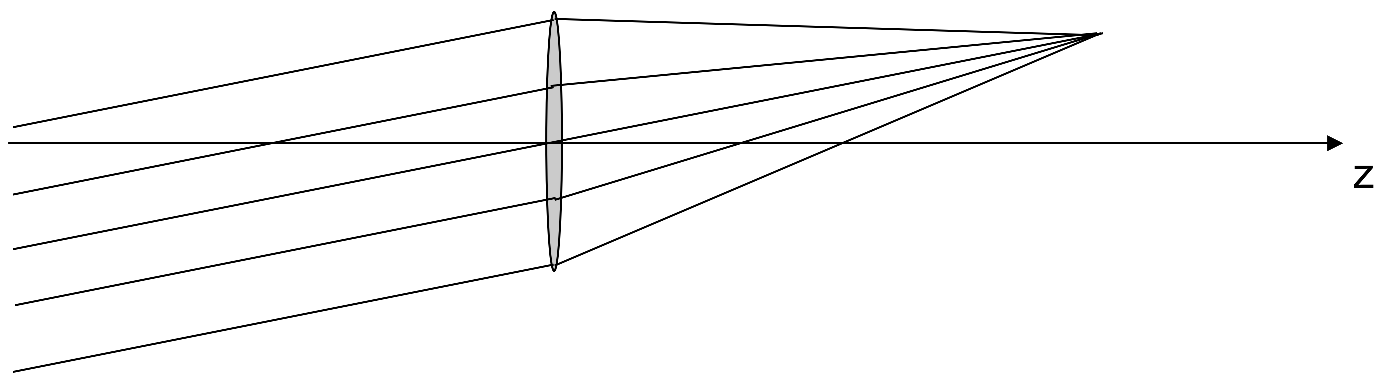
Infinity is infinity and it does not matter if we are discussing positive or negative infinity.

<u>On Axis and Off Axis – Object at Infinity</u>

An on-axis object is aligned with the optical axis and the image will be on the optical axis:



For an off-axis object, collimated light still enters the optical system, but at an angle with respect to the optical axis. The image is formed off the optical axis.



In practice, no object is ever really at infinity – just at a great distance. The rays from that distant object point are "approximately" collimated, but the approximation is incredibly good!

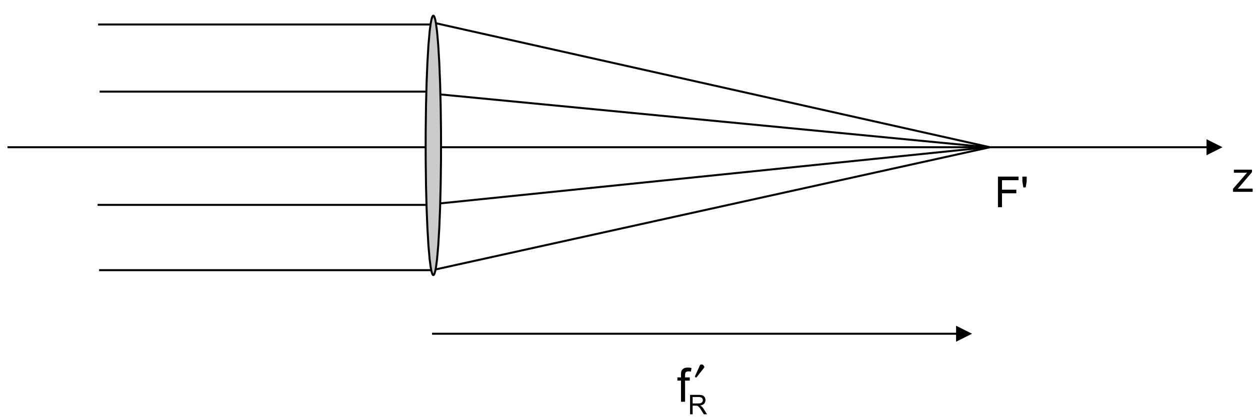
Imaging with a Thin Lens in Air

Many optical systems are first modeled as a thin lens*. A thin lens is an optical element with zero thickness that has refracting power. It is almost always used in air (n = n' = 1.0) and is characterized by its focal length f.

An object at infinity is imaged to the *Rear Focal Point* of the lens F'.

- All rays parallel to the axis produce rays that cross at the rear focal point.
- The distance from the lens to the rear focal point is the *rear focal length* of the lens.

$$f'_R = f$$



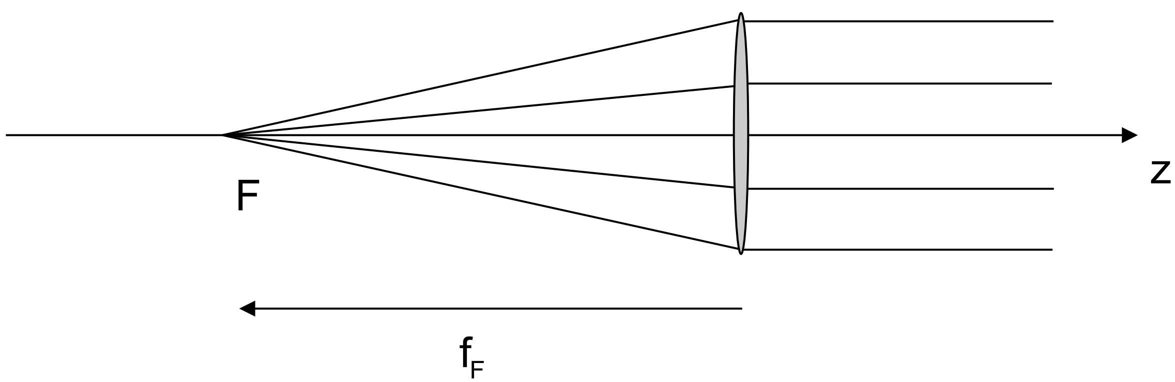
* This discussion should actually refer to "paraxial lenses" which are perfect first-order lenses of zero thickness. Thin lenses, which also have zero thickness, may have aberrations. However, the term "thin lens" is commonly used to describe both of these idealized elements.

Thin Lens – Front Focal Point

An object at the *Front Focal Point* of the lens F is imaged to infinity.

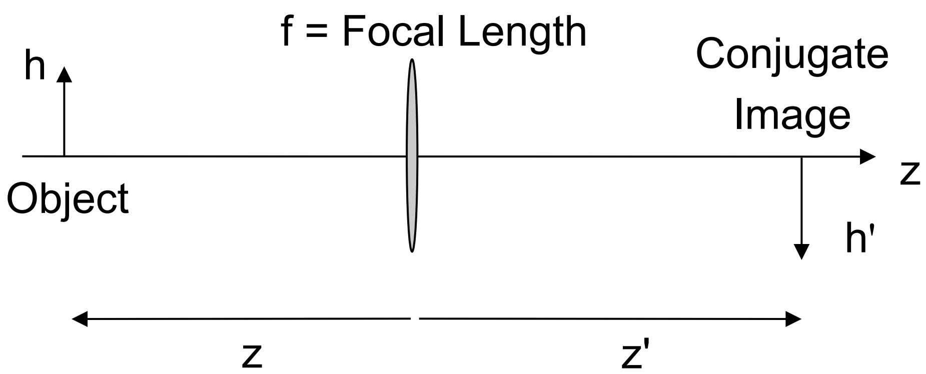
- All rays crossing at the front focal point emerge from the lens parallel to the axis.
- The distance from the lens to the front focal point is the *front focal length* of the lens which equals minus the focal length.

$$f_F = -f$$



Thin Lens - Conjugates

An object and its image are conjugate, and the respective distances from the lens are called conjugate distances (z and z') or object and image distances.



Because of sign conventions, an object to the left of the lens has a negative object distance.

The magnification of this object and image is defined as the ratio of the image height to the object height:

$$m \equiv \frac{h'}{h}$$

In this figure, h' is negative so that the magnification is also negative.

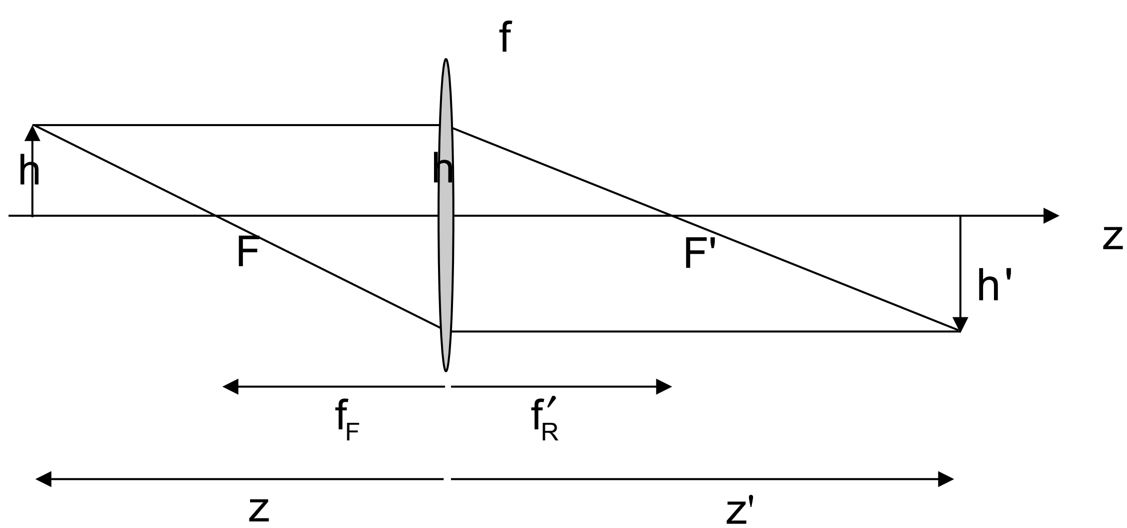
A pair of conjugate planes (the object and its image) is related by their magnification.

- A given magnification defines a unique pair of conjugate planes.
- For every object position z, there is a single image location z'.
- Each object position (or image position) has a unique, associated magnification.

Thin Lens – Imaging Relationships

The relationships between the object position, the image position, the magnification, and the focal length can be determined from the properties of the focal points.

- A point is defined by the intersection of two rays.
- Construct two rays intersecting at the object point.
- Determine the two conjugate rays.
- The image point is found at the intersection of these two rays.



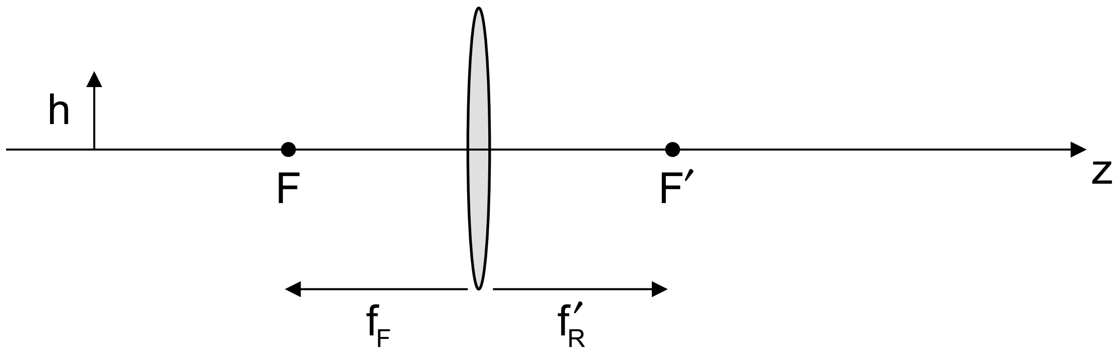
$$f_F = -f$$
$$f'_R = f$$

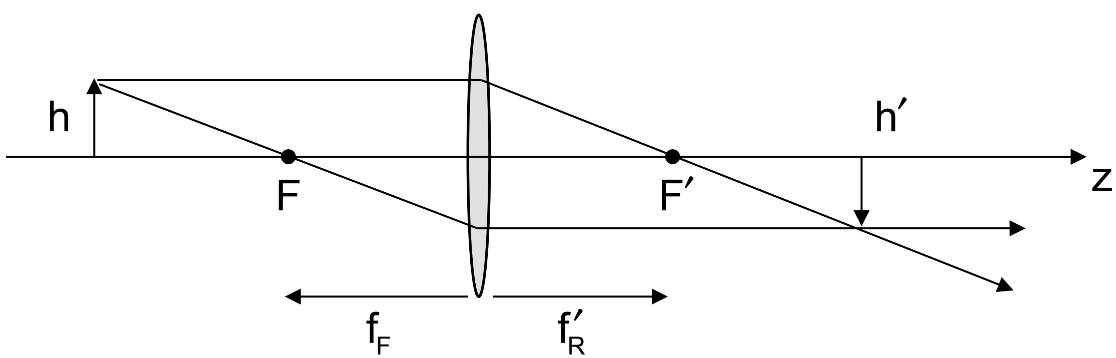
The two rays that are normally used are:

- A ray parallel to the optical axis emerges through the rear focal point.
- A ray through the front focal point emerges parallel to the optical axis.

Locating an Image with the Focal Points – Example 1

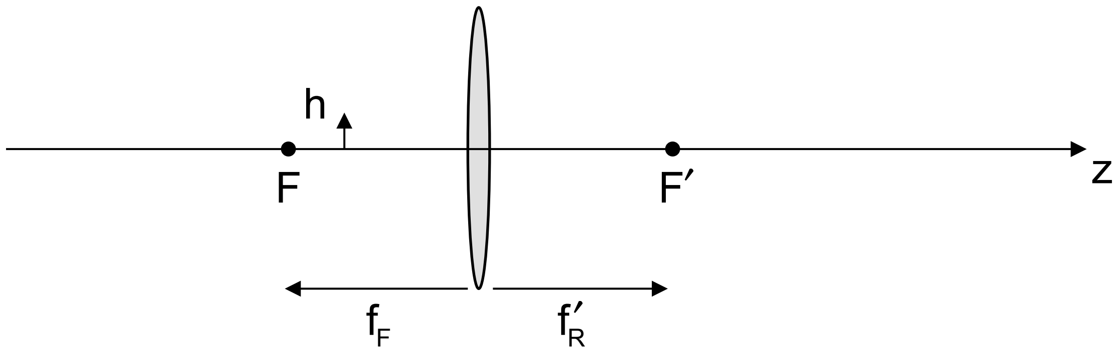
Positive Lens– Real object to the left of the front focal point F

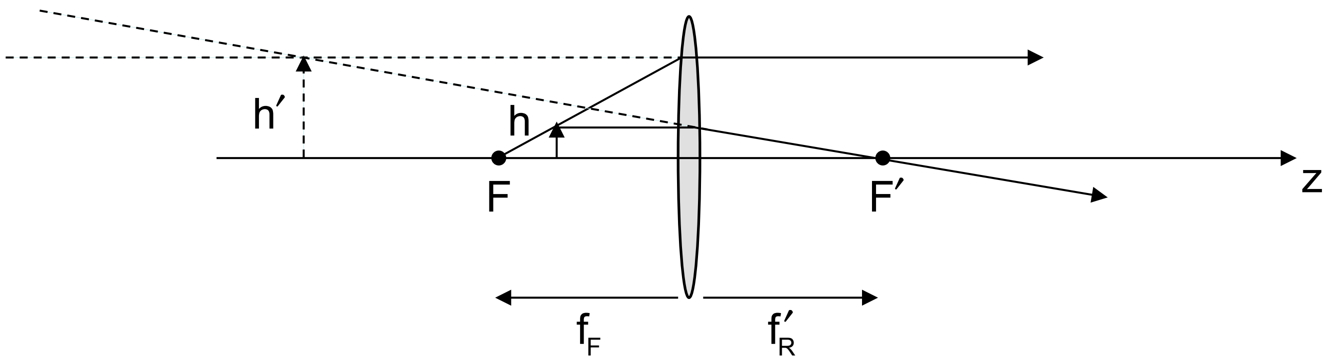




Locating an Image with the Focal Points – Example 2

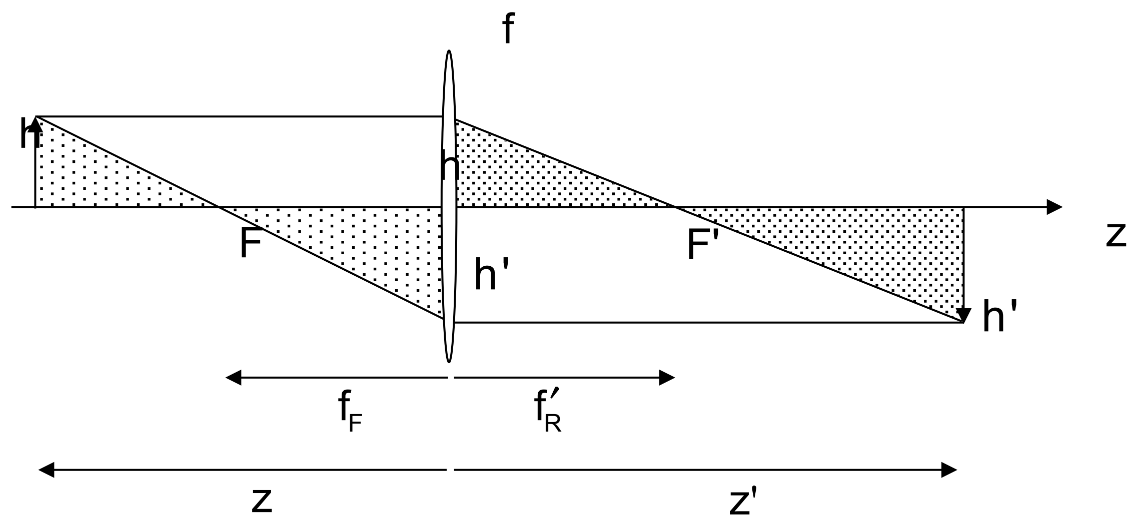
Positive Lens– Real object to the right of the front focal point F





The two image rays diverge and have a virtual crossing. An enlarged, erect virtual image is produced. The image is in image space.

Thin Lens – Imaging Relationships - Derivation



$$f_F = -f$$
$$f'_R = f$$

Similar triangles:

$$\frac{h}{-(z - f_F)} = \frac{-h'}{-f_F} \qquad \frac{h}{f'_R} = \frac{-h'}{z' - f'_R}$$

$$m \equiv \frac{h'}{h} = \frac{-f_F}{z - f_F} \qquad m \equiv \frac{h'}{h} = \frac{-(z' - f'_R)}{f'_R}$$

$$\frac{1}{m} = 1 - \frac{z}{f_F} = 1 + \frac{z}{f} \qquad m = 1 - \frac{z'}{f'_R} = 1 - \frac{z'}{f}$$

$$\frac{z}{f} = \frac{1}{m} - 1 \qquad \frac{z}{f_F} = 1 - \frac{1}{m} \qquad \frac{z'}{f} = \frac{z'}{f'_R} = 1 - m$$

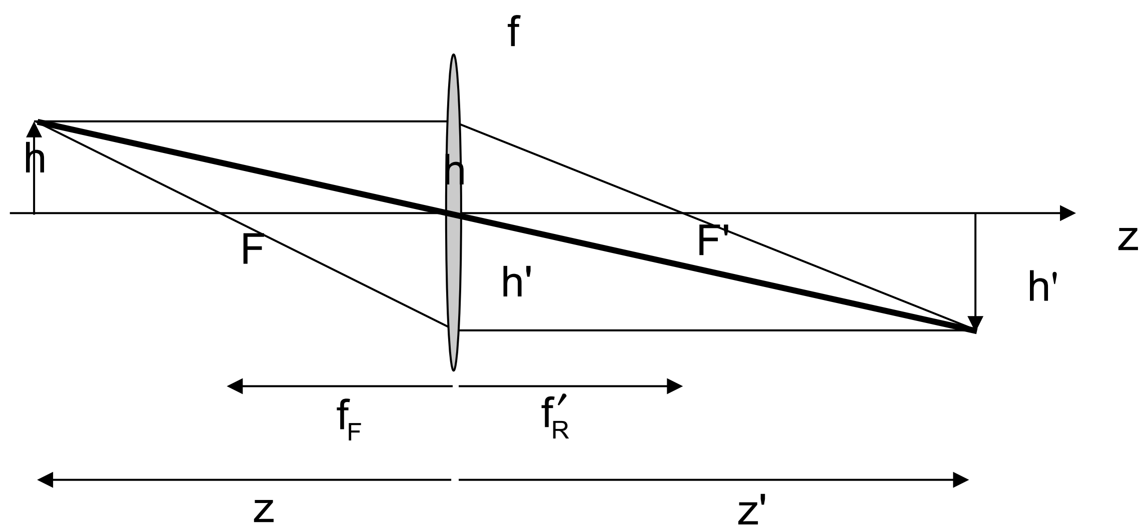
These equations give the conjugate distances for a particular magnification.

Thin Lens - Magnification

To determine the magnification as a function of z and z' (eliminating f), take the ratio of the last two equations:

$$\frac{z'}{z} = \frac{(1-m)f}{\left(\frac{1}{m}-1\right)f} = \frac{1-m}{1-m}m \qquad\qquad m = \frac{z'}{z}$$

This last equation implies that a ray drawn from the object point to the image point must pass through the center of the lens.



The center of the lens can be considered to be a very thin plane parallel plate. The ray is not deviated, and it displacement is zero since the plate thickness is zero for an ideal thin lens.

Thin Lens – Imaging Equation

One final result is to relate the object and image conjugate distances directly with the focal length (eliminate the magnification).

Return to the equation for z' in terms of m:

$$\frac{z'}{f} = 1 - m$$

$$\frac{z'}{f} = 1 - \frac{z'}{z}$$

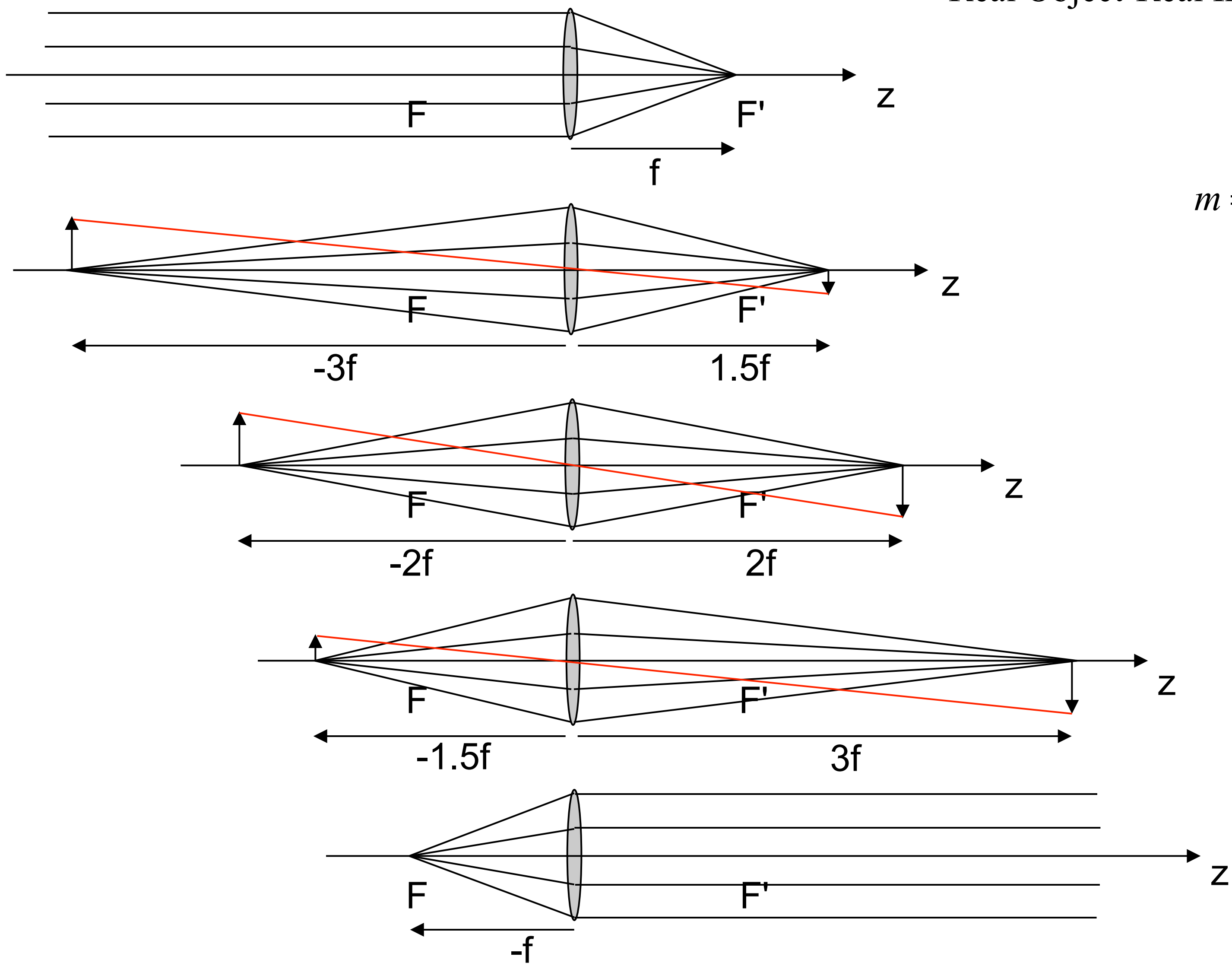
$$\frac{1}{f} = \frac{1}{z'} - \frac{1}{z}$$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

This form of the imaging equation differs from what is usually stated due to sign conventions. We use an object distance measured from the lens (and therefore usually negative.)

Imaging with a Positive Lens

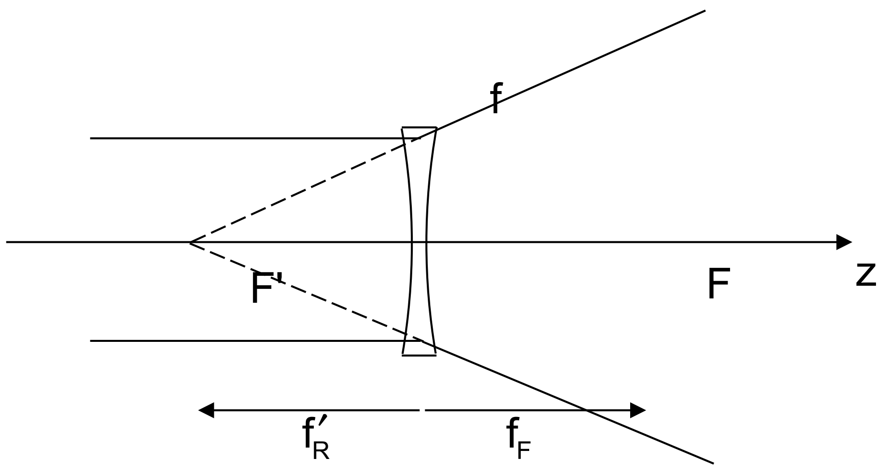
Real Object-Real Image



$$m = \frac{z'}{z}$$

<u>Thin Lens – Negative Lens</u>

A negative lens has a negative focal length and will diverge light. The front and rear focal points are reversed and are virtual.



Negative Lens:

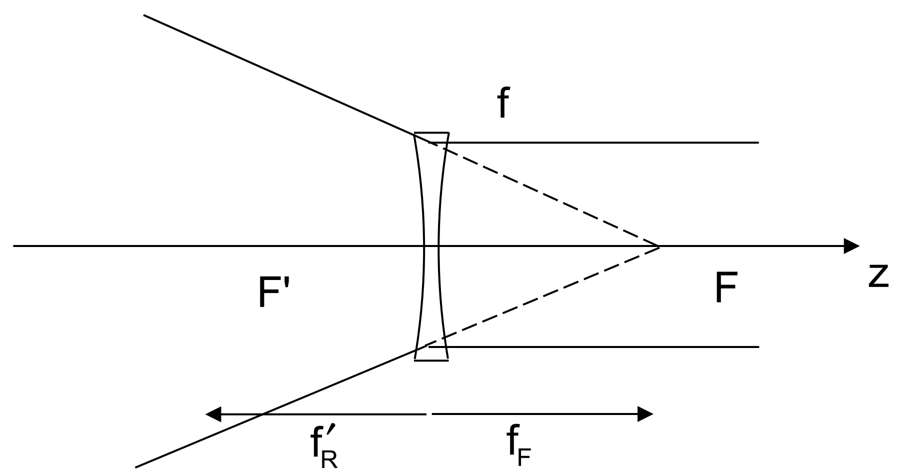
$f < 0$

$f'_R < 0$

$f_F > 0$

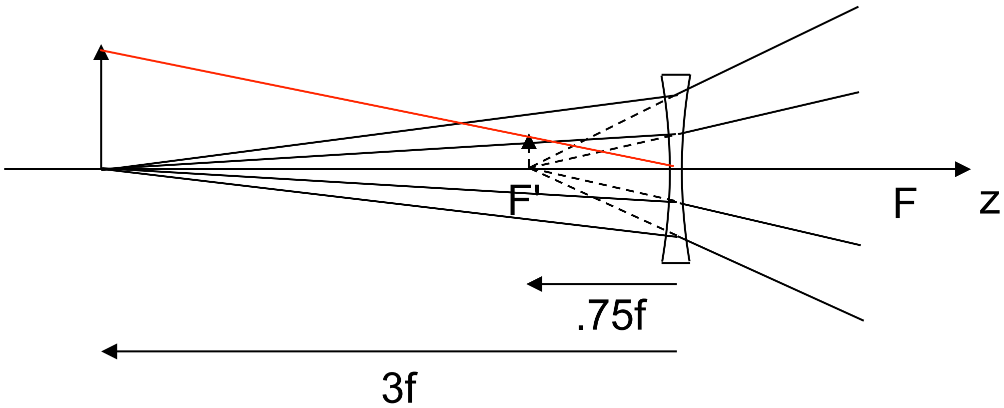
$f = f'_R = -f_F$

The imaging equations also hold for a negative lens, and a virtual image is produced for a real object (z < 0).



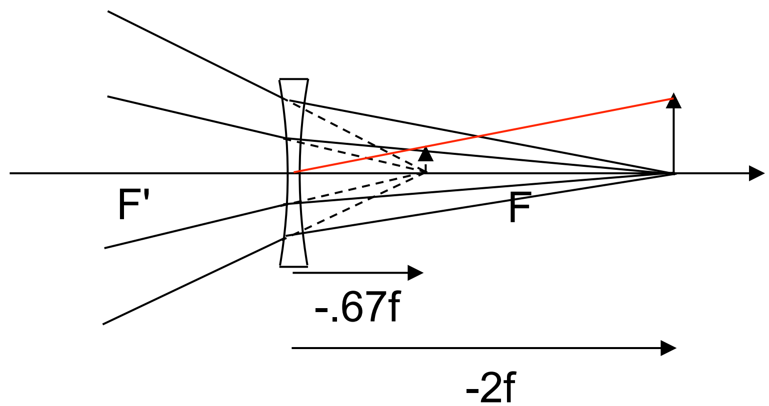
Imaging with a Negative Lens

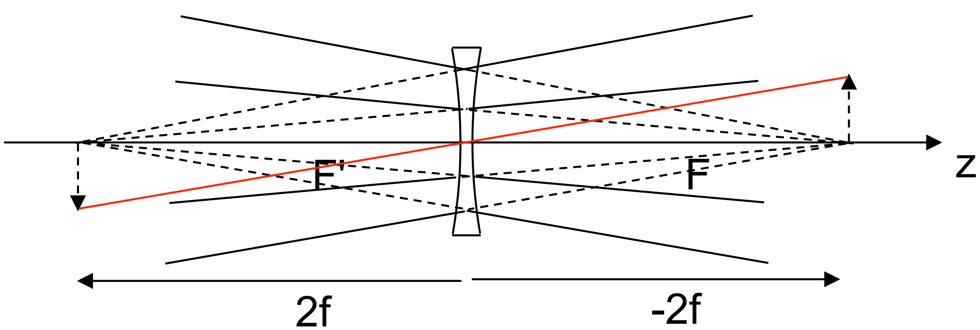
$f < 0$ $m = \frac{z'}{z}$



Real Object-Virtual Image

Virtual Object-Real Image





Virtual Object-Virtual Image

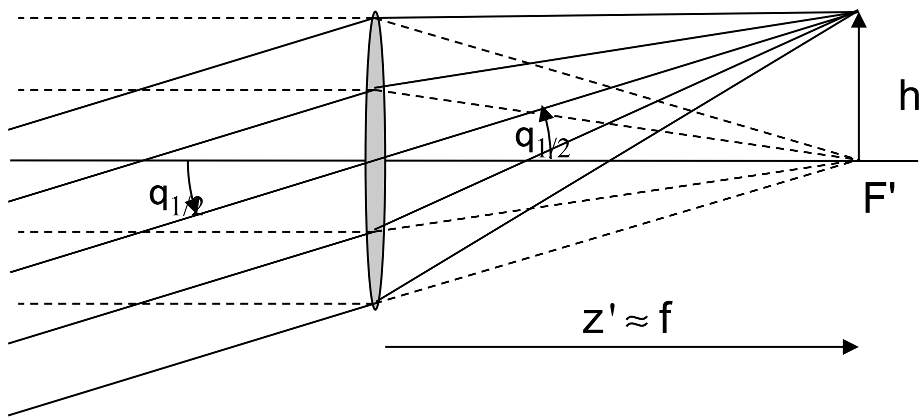
Field of View and Focal Length for a Thin Lens

For distant scenes, the object height is usually expressed as an angular size:

FOV – Field of View – the angular "diameter" of the object

HFOV – Half Field of View – the angular "radius" of the object

The image is formed at (or near) the rear focal point of the system:



$$HFOV = \theta_{1/2}$$

$$h' = f \tan(\theta_{1/2})$$

$$\vec{u}' = \tan(\theta_{1/2}) = \frac{h'}{f}$$

These relationships determine the image height for the entire scene or for an element in the scene:

- the FOV of the camera is 30 degrees
- two stars are separated by 10 arc seconds

1 degree = 60 arc minutes
= 3600 arc seconds

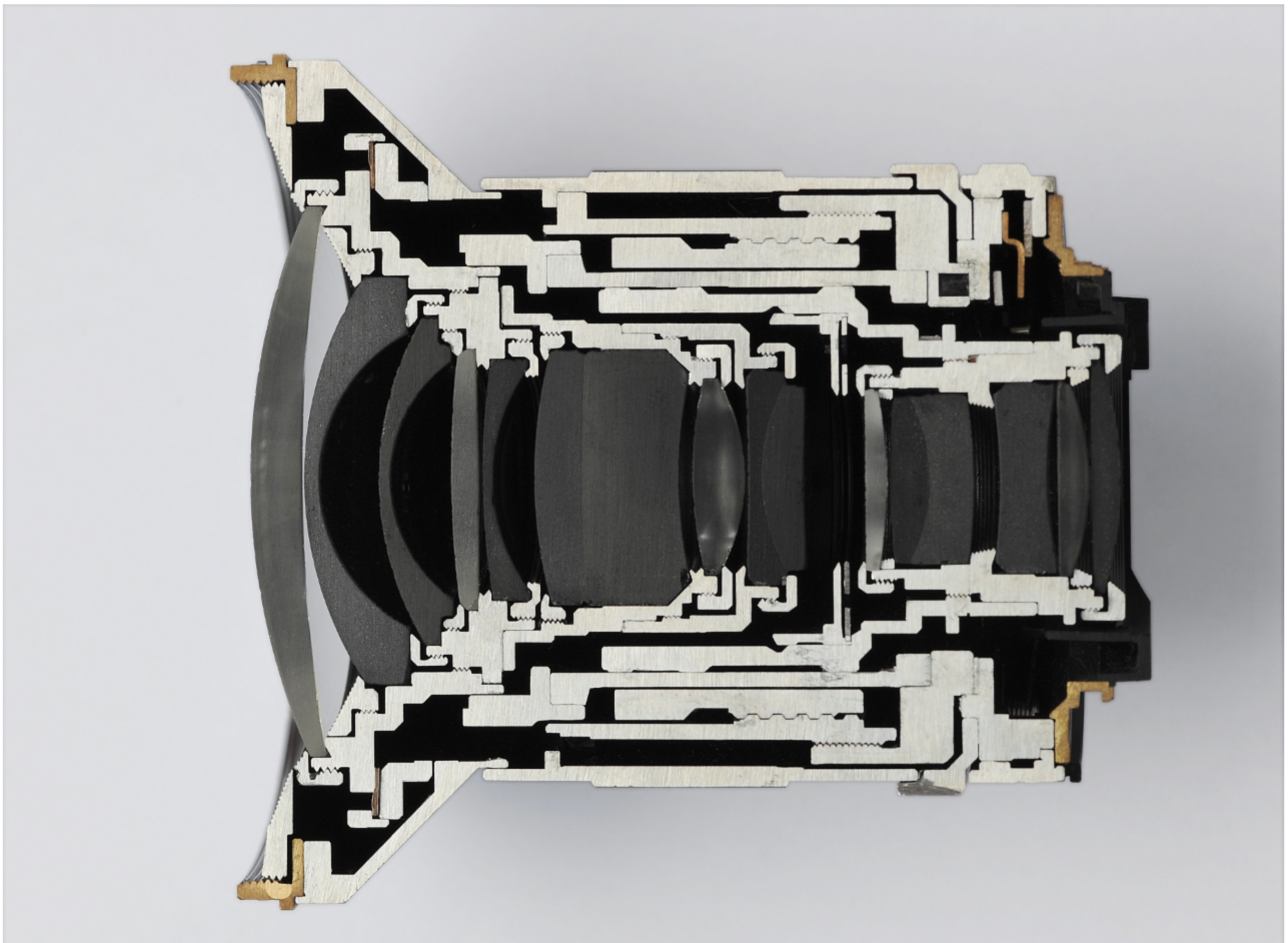
If the detector limits the extent of the FOV, these relationships can be used to determine the angular object FOV.

In the context of stops and pupils, the central ray drawn in the diagram is called the *Chief Ray*, and $\vec{u}'$ is the *Chief Ray Angle* (the slope of the ray).

OPTICAL SYSTEMS

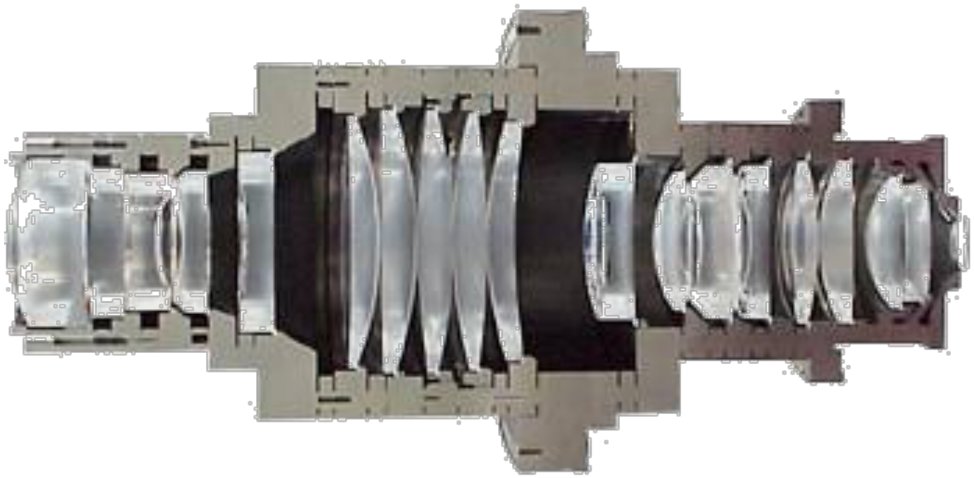
<u>Optical Systems</u>

An optical system is a collection of optical elements (lenses and mirrors). While the optical system can contain multiple optical elements, the first order properties of the system are characterized by a single focal length or magnification.

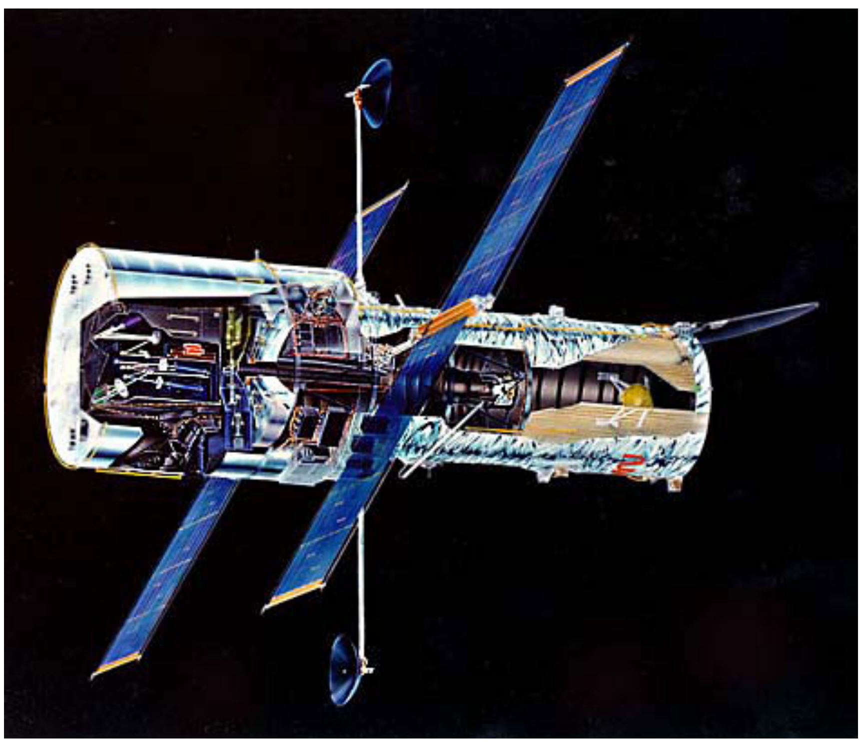


lenses.zeiss.com

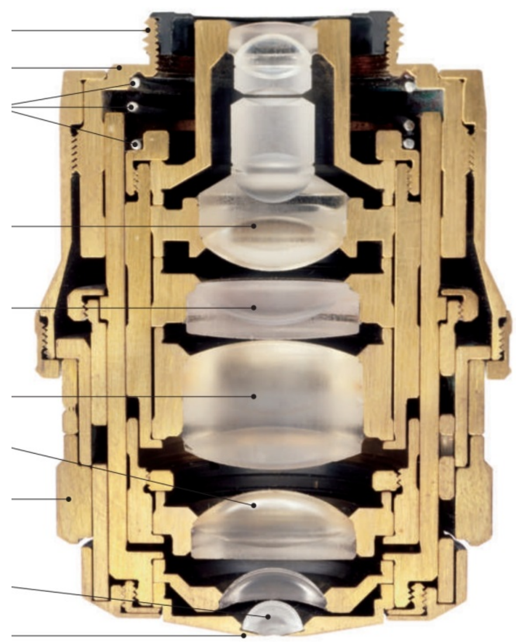
Optical Systems



tf.uni-kiel.de



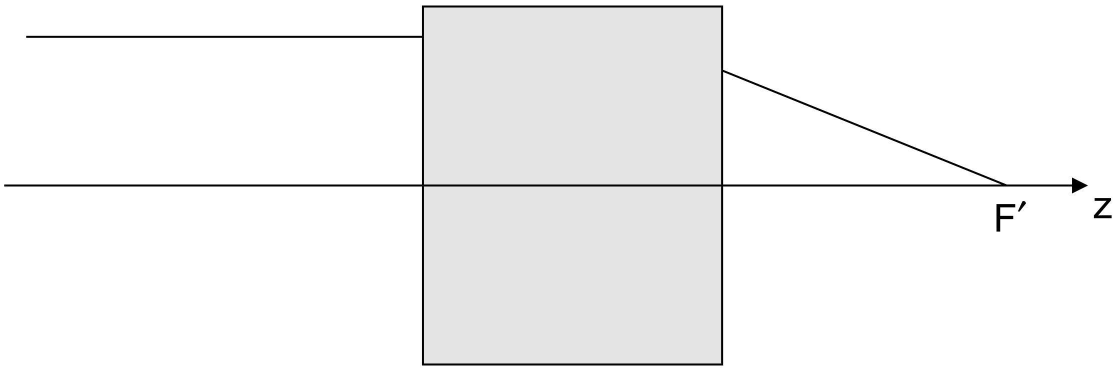
base24.com



oddstuffmagazine.com

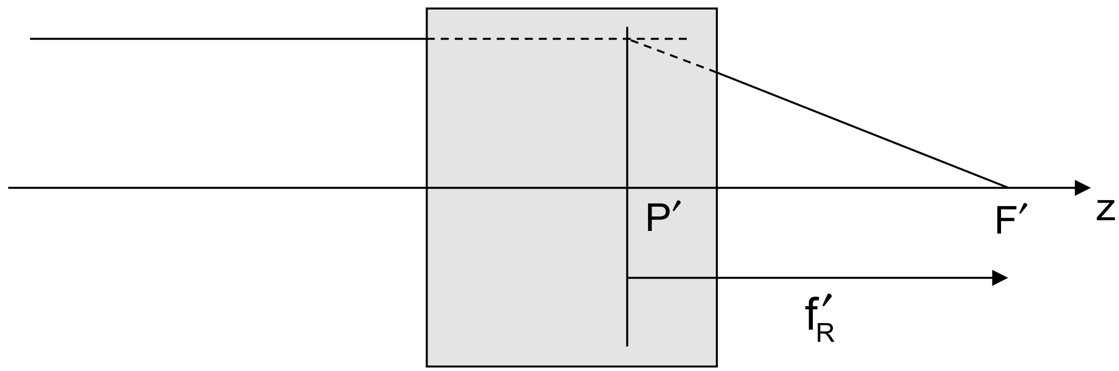
General System

Consider a "black-box" model of an optical system. An optical system is any collection of optical elements (lenses, mirrors, etc.) comprising a rotationally symmetric optical system. A ray from an object at infinity will emerge from the system and go through the Rear Focal Point of the system F':



<u>Planes of Effective Refraction</u>

By extending the image space ray back to the height of the object space ray, the plane of effective refraction into image space for the system is found. All of the refractions at the individual elements within the system are combined into a single effective refraction.

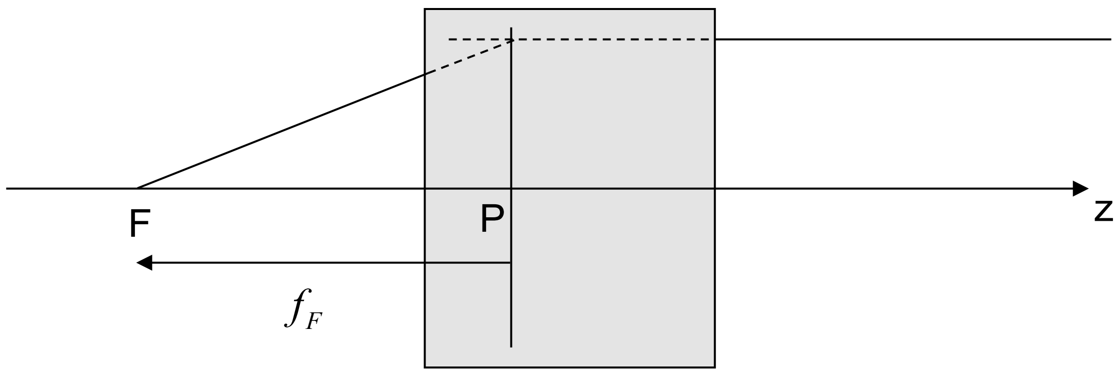


This plane of effective refraction into image space is called the Rear Principal Plane P'.

The distance from the Rear Principal Plane to the Rear Focal Point is the *Rear Focal Length* of the system f'_R .

Planes of Effective Refraction

In a similar manner, the plane of effective refraction out of object space can be found by using a ray starting at the Front Focal Point of the system F. The image ray is parallel to the axis.

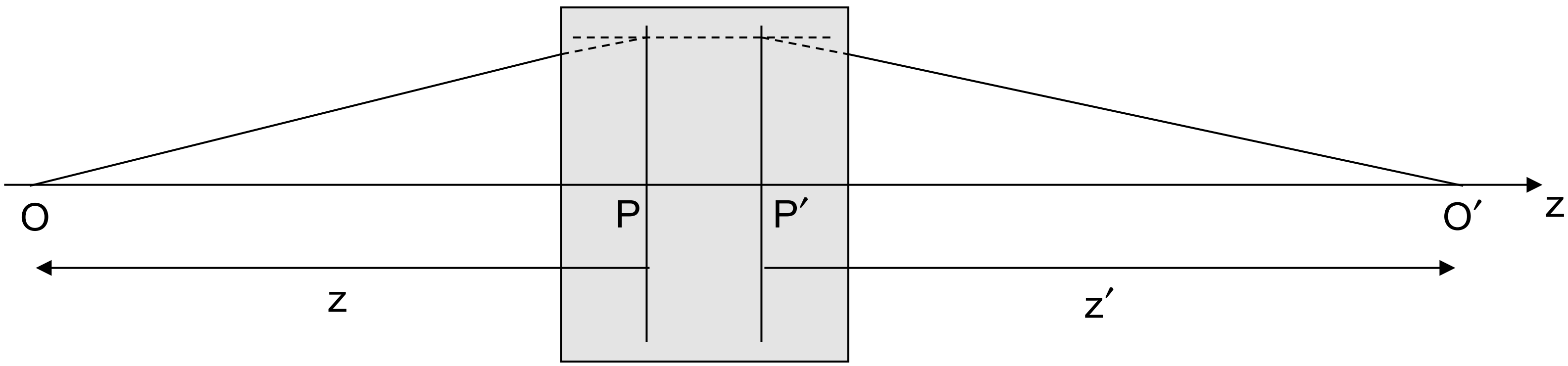


This plane of effective refraction out of object space is called the Front Principal Plane P.

The distance from the Front Principal Plane to the Front Focal Point is the *Front Focal Length* of the system f_F .

Planes of Effective Refraction

For the system used at finite conjugates, the ray from the object point will appear to refract at the Front Principle Plane and emerge from the Rear Principal Plane at the same height. The image point is located where this ray crosses the axis.



The object and image distances z and z' are measured from the respective Principal Planes.

This treatment allows the system to be considered to be just like a thin lens except the refraction occurs at separated planes of effective refraction.

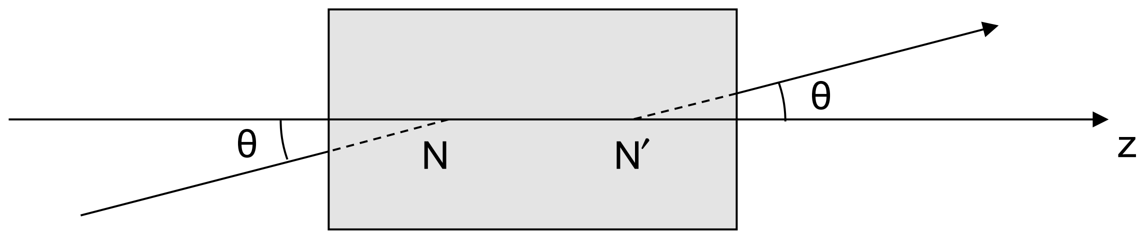
Cardinal Points and Planes

Unprimed variables are in object space.

Primed variables are in image space.

AP	Front antiprincipal plane/point	m = -1
AP'	Rear antiprincipal plane/point	m = -1
N	Front nodal point	Angular Mag = 1
N'	Rear nodal point	Angular Mag = 1

The two nodal points of a system are conjugate points.



For a focal imaging system, an object plane location is related to its conjugate image plane location through the transverse magnification associated with those planes.

Gaussian equations measure object and image distances from the *principal planes*.

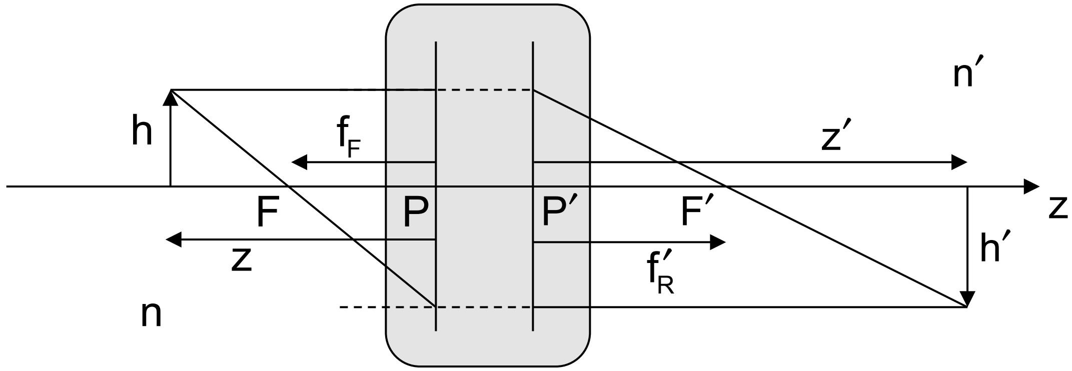
Gaussian Equations Applied to a System

The Gaussian equations describe the focal mapping when the respective *Principal Planes* are the references for measuring the locations of the conjugate object and image planes.

$$f'_R = \frac{n'}{\phi} = n' f_E$$

$$f_F = -\frac{n}{\phi} = -n f_E$$

$$-\frac{f'_R}{f_F} = \frac{n'}{n}$$



$$m \equiv \frac{h'}{h}$$

$$\frac{z}{f_F} = 1 - \frac{1}{m} \qquad \frac{z'}{f'_R} = 1 - m \qquad \frac{z'}{z} = \left(-\frac{f'_R}{f_F}\right) m \qquad \frac{f_F}{z} + \frac{f'_R}{z'} = 1$$

$$\frac{z}{n} = \frac{(1-m)}{m} f_E \qquad \frac{z'}{n'} = (1-m) f_E \qquad m = \frac{z'/n'}{z/n} \qquad \frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E}$$

In air:
$n = n' = 1$

$$z = \frac{(1-m)}{m} f_E \qquad z' = (1-m) f_E \qquad m = \frac{z'}{z} \qquad \frac{1}{z'} = \frac{1}{z} + \frac{1}{f_E}$$

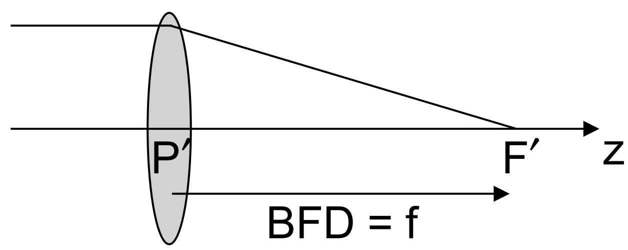
College of Optical Sciences
The University of Arizona

OBJECTIVES & ZOOM LENSES

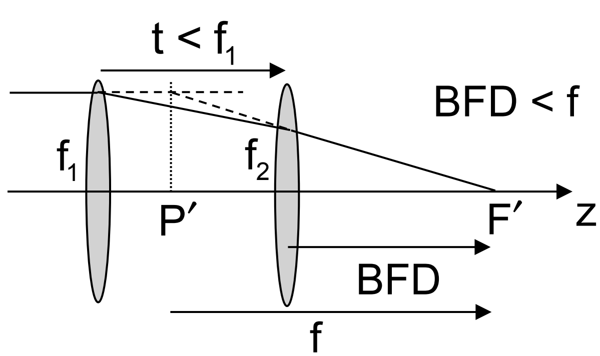
College of Optical Sciences
The University of Arizona

Objectives – Simple and Petzval

Objectives are lens element combinations used to image (usually) distant objects. To classify the objective, separated groups of lens elements are modeled as thin lenses. The simple objective is represented by a positive thin lens.

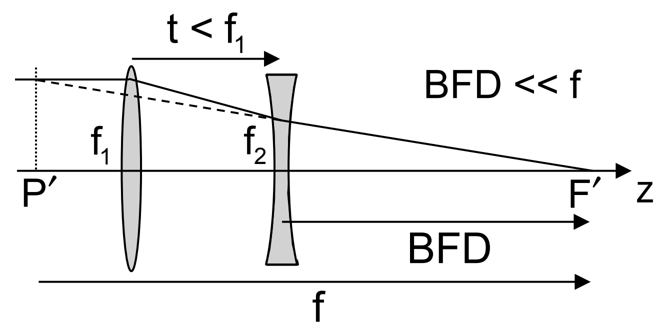


The Petzval objective consists of two separated positive groups of elements. The system rear principal plane is located between the groups.

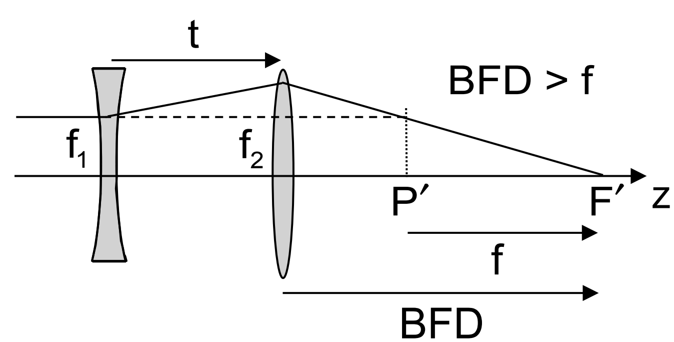


Objectives – Telephoto and Reverse Telephoto

The telephoto objective produces a system focal length longer than the overall system length ($t + BFD$). It consists of a positive group followed by a negative group.

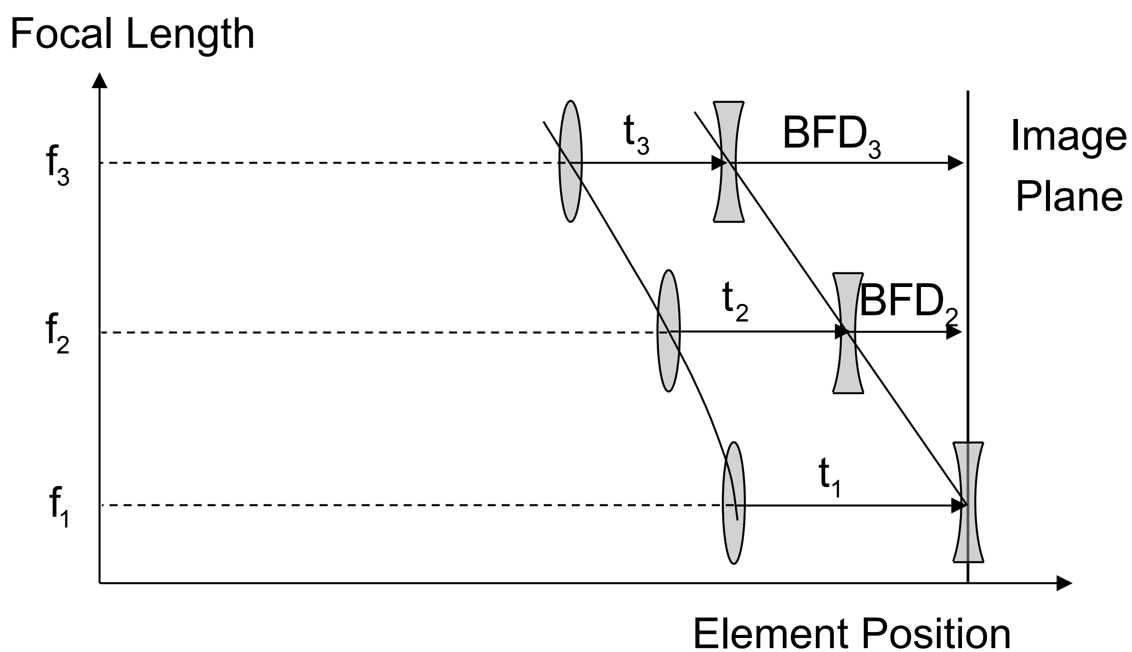


The reverse telephoto objective or retrofocus objective consists of a negative group followed by a positive group. This configuration is used to produce a system with a BFD larger than the system focal length. While this configuration is used for many wide angle objectives, the term reverse telephoto specifically refers to the configuration, not the FOV.



Telephoto Zoom Lens

The element positions and spacing for a telephoto zoom are plotted as a function of system focal length:

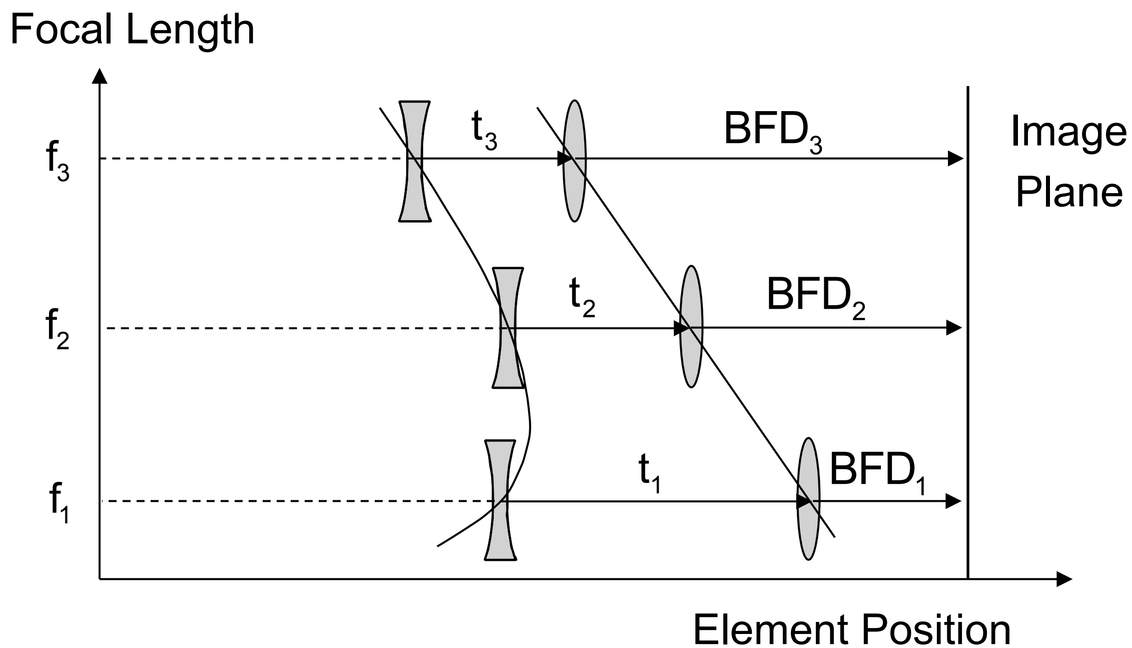


As the element separation approaches the sum of the element focal lengths ($f_1 + f_2$), the system focal length approaches infinity ($f \rightarrow \infty$).

The short end of the zoom range of this configuration is limited by the BFD as the rear element runs into the image plane when the element separation approaches f_1.

Reverse Telephoto Zoom Lens

The element positions and spacing for a reverse telephoto zoom are plotted as a function of system focal length:



As the element separation approaches the sum of the element focal lengths ($f_1 + f_2$), the system focal length approaches infinity ($f \rightarrow \infty$).

This configuration does not have the BFD issue of the telephoto zoom. Of the two configurations, the reverse telephoto zoom is the more commonly used.

MAGNIFIERS, TELESCOPES & MICROSCOPES

Visual Magnification

All optical systems that are used with the eye are characterized by a visual magnification or a visual magnifying power.

While the details of the definitions of this quantity differ from instrument to instrument and for different applications, the underlying principle remains the same:

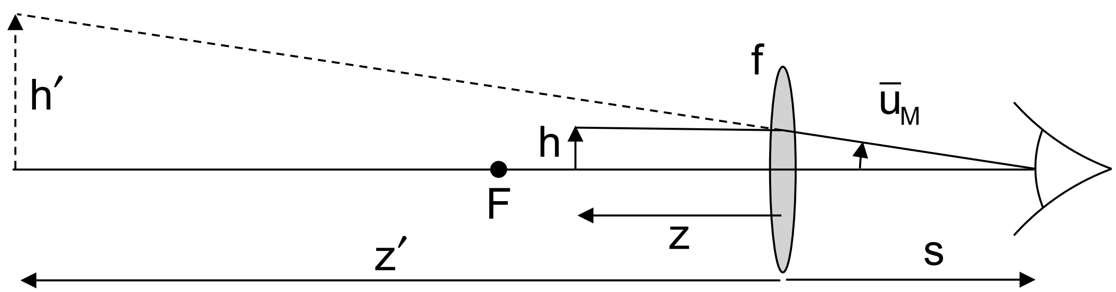
How much bigger does an object appear to be when viewed through the instrument?

The size change is measured as the change in angular subtense of the image produced by the instrument compared to the angular subtense of the object.

The angular subtense of the object is measured when the object is placed at the optimum viewing condition.

Magnifiers

As an object is brought closer to the eye, the size of the image on the retina increases and the object appears larger. The largest image magnification possible with the unaided eye occurs when the object is placed at the near point of the eye, by convention 250 mm or 10 in from the eye. A magnifier is a single lens that provides an enlarged erect virtual image of a nearby object for visual observation. The object must be placed inside the front focal point of the magnifier.



The magnifying power MP is defined as (stop at the eye):

$$MP = \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}}$$

$$MP = \frac{\bar{u}_M}{\bar{u}_U} \qquad d_{NP} = -250 \text{ mm}$$

Magnifiers – Magnifying Power – Continued

$$MP = \frac{d_{NP}}{z'} - \frac{d_{NP}}{f} = \frac{250\,mm}{f} - \frac{250\,mm}{z'}$$

The magnification is a function of both f and the image location. The most common definition of the MP of a magnifier assumes that the lens is close to the eye and that the image is presented to a relaxed eye ($z' = \infty$).

$$MP = -\frac{d_{NP}}{f} = \frac{250\,mm}{f} = \frac{10''}{f}$$

The maximum MP occurs if the image is presented at the near point of the eye:

$$MP = 1 - \frac{d_{NP}}{f} = \frac{250\,mm}{f} + 1 = \frac{10''}{f} + 1 \qquad z' = d_{NP}$$

Telescopes

Telescopes are afocal or nearly afocal systems used to change the apparent angular size of an object. The image through the telescope subtends an angle θ' different from the angle subtended by the object θ. The magnifying power MP of a telescope is

$$MP = \frac{\theta'}{\theta}$$

$$|MP| > 1 \qquad \text{Telescope magnifies}$$

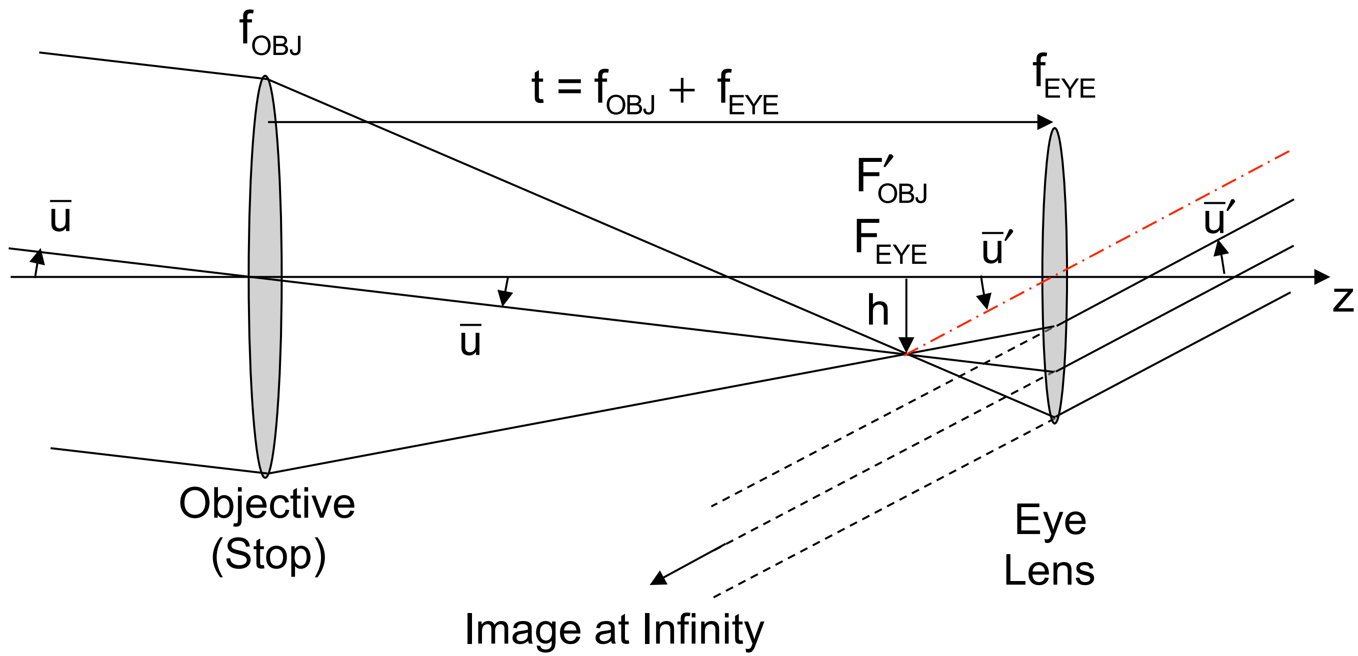
$$|MP| < 1 \qquad \text{Telescope minifies}$$

The angles θ and θ' are often considered to be paraxial angles.

$$MP = \frac{\bar{u}'}{\bar{u}} = \text{ Angular Magnification}$$

Keplerian Telescope

A Keplerian telescope or astronomical telescope consists of two positive lenses separated by the sum of the focal lengths. The system stop is usually at or near the objective lens.



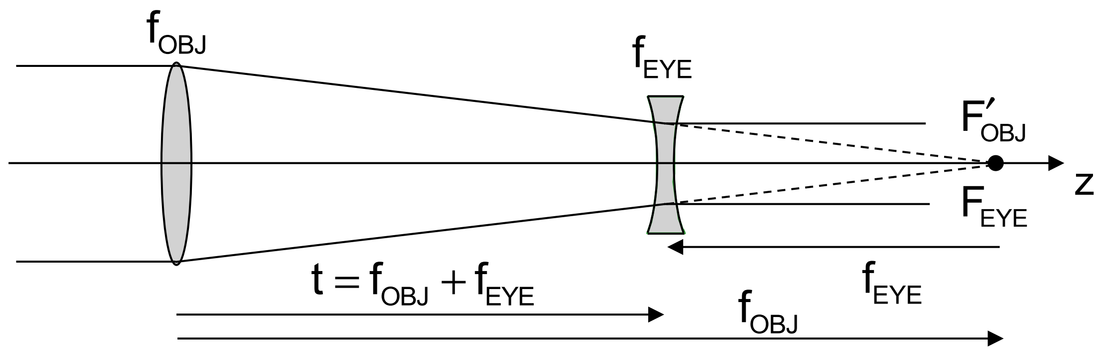
This telescope can be considered to be a combination of an objective plus a magnifier. The objective creates an aerial image (a real image in the air) at the common focal point that is magnified by the eye lens and presented to the relaxed eye at infinity.

$$h = \bar{u} f_{OBJ} \qquad h = -\bar{u}' f_{EYE} \qquad MP = \frac{\bar{u}'}{\bar{u}} = -\frac{f_{OBJ}}{f_{EYE}}$$

The image presented to the eye is inverted and reverted (rotated by 180° or "upside down"). The MP of a Keplerian telescope is negative.

Galilean Telescope

The Galilean telescope uses a positive lens and a negative lens to obtain an erect image and a positive MP (MP > 1).



$$m = -\frac{f_{EYE}}{f_{OBJ}} \qquad 0 < m < 1$$

$$ER = z' = (1 - m) f_{EYE} < 0$$

$$MP = -\frac{f_{OBJ}}{f_{EYE}} \qquad MP > 1$$

The XP is internal or virtual and not accessible to the eye. There is poor coupling between the telescope and the eye, and the FOV of the system is small. There is no intermediate image plane, so it cannot be used with reticles.

The Galilean telescope is used for inexpensive systems such as opera glasses.

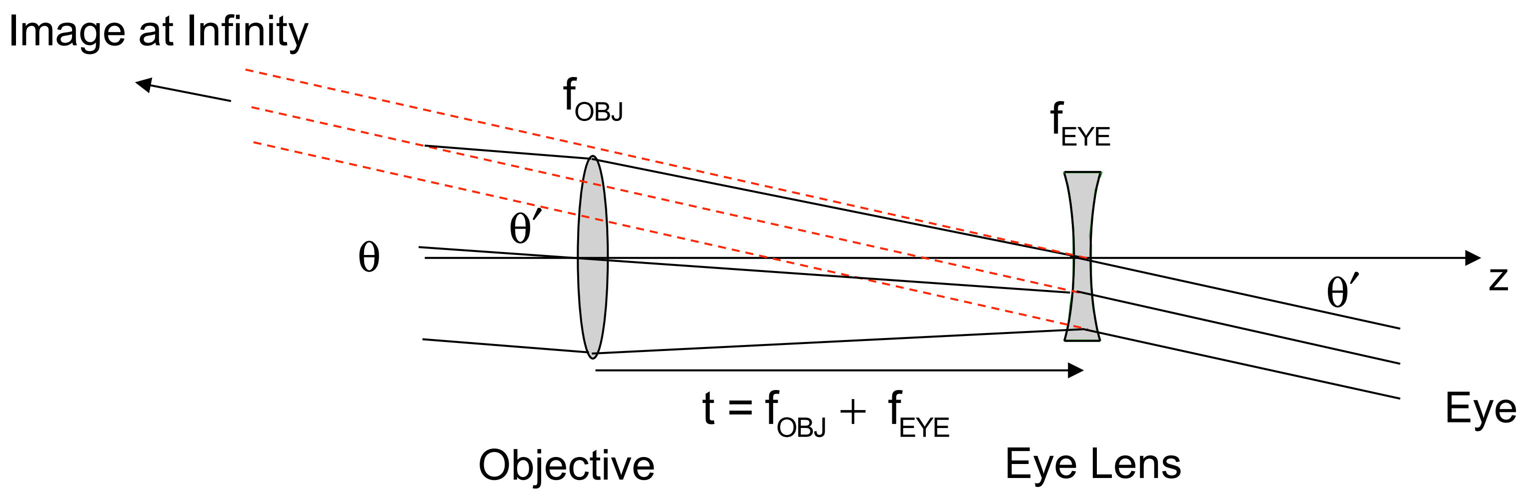
For a Galilean telescope to be constructed, the negative lens must be stronger than the positive lens.

$$|f_{EYE}| < f_{OBJ}$$

Galilean Telescope

Rays from an off-axis object enter the telescope at q and emerge at q′.

$$MP = \frac{\theta'}{\theta} = -\frac{f_{OBJ}}{f_{EYE}}$$

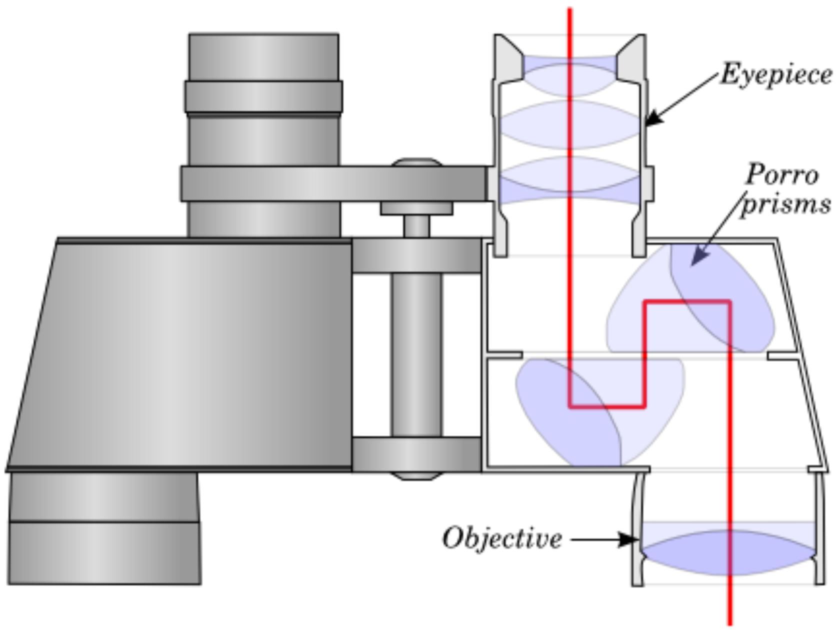


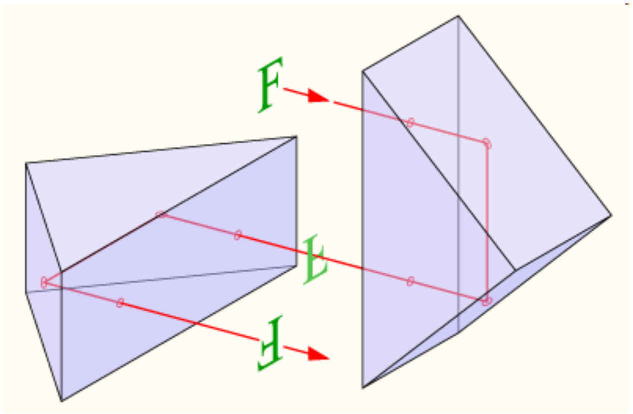
Collimated input light comes out of the telescope at a larger angle. The image appears bigger to the eye.

The image in this configuration is erect (right-side up).

Binoculars

Image erection prisms are inserted in an optical system to provide a fixed 180° image rotation. They are commonly used in telescopes and binoculars to provide an upright image orientation. No parity change.



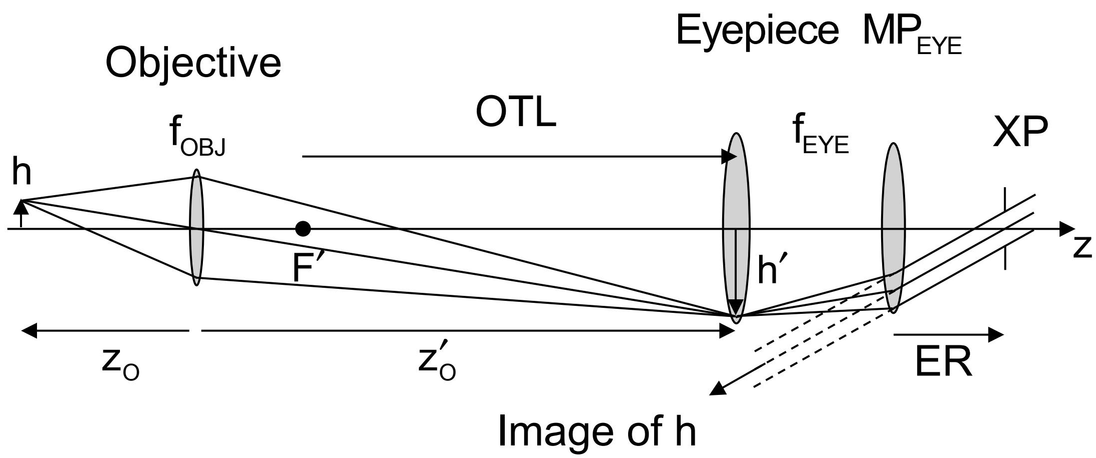


Porro Prism System

Wikipedia

Microscopes

The compound microscope is a sophisticated magnifier which presents an enlarged image of nearby object to the eye. It consists of an objective plus an eyepiece or ocular.



The visual magnification is the product of the lateral magnification of the objective and the MP of the eyepiece.

$$m_{OBJ} = \frac{z'_O}{z_O} \qquad MP_{EYE} = \frac{250\,\text{mm}}{f_{EYE}}$$

$$m_V = m_{OBJ} MP_{EYE} = \frac{z'_O}{z_O}\frac{250\,\text{mm}}{f_{EYE}}$$

Note that the visual magnification is negative (z_O is negative).