Quantum Optics

Lecture at the Optics and Photonics Winter School

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What is Quantum Optics?

– Not classical optics, but something more and different
– Science of non-classical light
– Any science combining light and quantum mechanics

What is Light?

– Electromagnetic waves?
– Photons?
What is Quantum Optics?
Not so fast!

The Nobel Prize in Physics 1955

Willis Eugene Lamb
Prize share: 1/2

Polykarp Kusch
Prize share: 1/2

The Nobel Prize in Physics 1955 was divided equally between Willis Eugene Lamb "for his discoveries concerning the fine structure of the hydrogen spectrum" and Polykarp Kusch "for his precision"
Not so fast!

Anti-photon

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Abstract. It should be apparent from the title of this article that the author does not like the use of the word “photon”, which dates from 1926. In his view, there is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity among physicists and optical scientists. I admit that the word is short and convenient. Its use is also habit forming. Similarly, one might find it convenient to speak of the “aether” or “vacuum” to stand for empty space, even if no such thing existed. There are very good substitute words for “photon”, (e.g., “radiation” or “light”), and for “photons” (e.g., “optics” or “quantum optics”). Similar objections are possible to use of the word “phonon”, which dates from 1932. Objects like electrons, neutrinos of finite rest mass, or helium atoms can, under suitable conditions, be considered to be particles, since their theories then have viable non-relativistic and non-quantum limits. This paper outlines the main features of the quantum theory of radiation and indicates how they can be used to treat problems in quantum optics.

1 A short history of pre-photonic radiation


A decisive work in 1801 by T. Young, on the two-slit diffraction pattern, showed that the wave version of optics was much to be preferred over the corpuscular form...
With all due respect to W. Lamb, let us try again

What is light?

- a wave?
- a stream of particles (photons)?

Take the question seriously

- test each hypothesis through experimentation!
Key signature of wave behavior? – Interference!

Double-slit experiment

Single-slit diffraction

Double-slit diffraction
Key signature of particle behavior?

Einstein: photo-electric effect

Electrons are released only for light with a frequency $\nu$ such that $h\nu$ is greater than the work function of the metal in question.

But the quantum theory of electron excitation can explain this based on classical electromagnetic fields, so the photo-electric effect only confirms that electrons are particles.

Indivisibility

A particle incident on a barrier is either transmitted or reflected.
Wave behavior

Particle behavior
Evidently a single photon can behave like a wave or a particle, depending on the experiment we do. This is what we know as wave-particle duality.

Does the photon “know” when it hits the first BS if we are doing a wave or particle experiment and then behaves accordingly?

Wigner’s gedanken experiment: Delayed Choice! Decide at random whether to put in the second BS only after the photon has passed the first BS.
In 1988, Greenberger and Yasin noticed that in an unbal-


destruing the interference pattern, which remains

an imperfect—but significant—knowledge of WPI, with-

meter considered as a quantum device allows one to gain

mediate situation in which interaction with the interfer-

of the particle into the interferometer \[\text{accuracy even if the choice, made by a quantum random}

particle entered the interferometer \[\text{two complementary measurements is made long after the}

behavior accordingly \[\text{interaction with the interferometer—offers the}

to two different notions. The

guishability parameter \(V\) and inter-

izations between the two all-or-nothing cases.

inequality (\(V^2 + D^2 \leq 1\)).
Wigner's experiment was done in 2008

FIG. 1 (color online). Delayed-choice complementarity-test experiment. A single-photon pulse is sent into a Mach-Zehnder interferometer, composed of a 50/50 input beam splitter (BS) and a variable output beam splitter (VBS). The reflection coefficient is randomly set either to the null value or to an adjustable value \( R \), after the photon has entered the interferometer. The single-photon photodetectors \( P_1 \) and \( P_2 \) allow to record both the interference and the WPI.

FIG. 2 (color online). Variable output beam splitter (VBS) implementation. The optical axis of the polarization beam splitter (PBS) and the polarization eigenstates of the Wollaston prism (WP) are aligned, and make an angle \( \beta \) with the optical axis of the EOM. The voltage \( V_{\text{EOM}} \) applied to the EOM is randomly chosen accordingly to the output of a Quantum Random Number Generator (QRNG), located at the output of the interferometer and synchronized on the 4.2-MHz clock that triggers the single-photon emission.
Wigner's experiment was done in 2008

FIG. 3 (color online). Interference visibility $V$ measured in the delayed-choice regime for different values of $V_{EOM}$. (a)–(c) correspond to $V_{EOM} \approx 150\, V$ ($R = 0.43$ and $V = 93 \pm 2\%$), $V_{EOM} \approx 40\, V$ ($R = 0.05$ and $V = 42 \pm 2\%$), and $V_{EOM} = 0$ ($R = 0$ and $V = 0$). Each point is recorded with 1.9 s acquisition time. Detectors dark counts, corresponding to a rate of 60 s$^{-1}$ for each, have been substracted to the data.

Light is both a particle and a wave at the same time.

What property we see depends on what property we decide to measure.

This is totally in line with our general quantum theory.
BTW, it works for ultracold atoms too!
We need a quantum theory of light where behaviors such as wave-particle duality is built in

- A formal procedure exists for obtaining a quantum theory from a known classical theory based on Lagrange – Hamilton formalism.

- One cannot “prove” that this procedure is correct. It is justified only by repeated observation that quantum theories obtained in this fashion “work”, in the sense that their predictions agree with experiment.

- This should not surprise us. Quantum Mechanics contains new physics that is absent from Classical Mechanics and cannot be derived from it.

- An unfortunate consequence is that we often seem to pull ideas out of thin air when we teach quantum mechanics. The problem gets worse when we try to do things quickly, e. g., a 50 minute lecture on the quantum theory of light.
Quantization of the Electromagnetic Field

Starting point: Maxwell’s equations from classical Electromagnetism

\[
\begin{align*}
\nabla \cdot \mathbf{E}(\mathbf{r}, t) &= \frac{1}{\varepsilon_0} \rho(\mathbf{r}, t) \\
\nabla \cdot \mathbf{B}(\mathbf{r}, t) &= 0 \\
\nabla \times \mathbf{E}(\mathbf{r}, t) &= -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} \\
\nabla \times \mathbf{B}(\mathbf{r}, t) &= \frac{1}{c^2} \frac{\partial}{\partial t} \mathbf{E}(\mathbf{r}, t) + \frac{1}{\varepsilon_0 c^2} \mathbf{j}(\mathbf{r}, t)
\end{align*}
\]

In Quantum Electrodynamics (QED) the ME’s are still valid, but the fields \( \mathbf{E} \) and \( \mathbf{B} \) become operators that depend on space and time just like the classical electric and magnetic fields.

For simplicity we consider empty space without charges or currents, and use the two last Maxwell equations to derive a wave equation

\[
\left[ \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \mathbf{E}(\mathbf{r}, t) = 0
\]

\[
\left[ \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right] \mathbf{E}(z, t) = 0 \quad \text{paraxial waves propagating along the } z\text{-axis}
\]
Electromagnetic Field in a 1-Dimensional Cavity

Cavity of length $L$, cross section $A$, and volume $V = LA$, field $E = x E_x$

Any field inside the cavity can be expressed as a superposition of the cavity normal modes, which are standing wave solutions to the wave equation with nodes on the mirror surfaces.

$$E_x(z,t) = \sum_{j=1}^{\infty} A_j q_j(t) \sin(k_j z), \quad k_j = \frac{\omega_j}{c} = \frac{j \pi}{L}, \quad A_j = \sqrt{\frac{2\omega_j^2 m_j}{\varepsilon_0 V}}$$

Here $q_j(t)$ are the time varying amplitudes for the different modes, and $A_j$ is chosen so $E_x$ has units of electric field ($V/m$) and $q_j(t)$ has units of length ($m$).
Each normal mode is an independent degree of freedom that can be quantized by itself.

For mode number $j$ we have electric and magnetic fields (the latter can be found from $E_x$ using Maxwells equations)

$$E_x^{(j)}(z,t) = A_j q_j(t) \sin(k_j z), \quad B_y^{(j)}(z,t) = \frac{A_j}{k_j c^2} \dot{q}_j(t) \cos(k_j z), \quad \dot{q}_j(t) = \frac{d}{dt} q_j(t)$$

Using classical E & M we can show that the energy of the field in mode $j$ is

$$H = \varepsilon_0 \int_V dr^3 \left( |E|^2 + c^2 |B|^2 \right)$$

$$= \varepsilon_0 A \int_0^L dz \left[ A_j^2 q_j(t)^2 \sin^2(k_j z) + \frac{A_j^2}{k_j^2} \dot{q}_j(t)^2 \cos^2(k_j z) \right]$$

$$= \frac{1}{2} m_j \omega_j^2 q_j^2 + \frac{1}{2} m_j \dot{q}_j^2$$
Writing the field energy in this form is *highly suggestive*.

Harmonic Oscillator

\[ H = \frac{1}{2} m \omega^2 x^2 + \frac{1}{2} m v^2 \]
\[ = \frac{1}{2} m \omega^2 x^2 + \frac{1}{2m} p^2 \]

Quantum Theory

\[ x \rightarrow \hat{x} = x \]
\[ p \rightarrow \hat{p} = -i\hbar \frac{d}{dx} \]
\[ [\hat{x}, \hat{p}] = i\hbar \]

EM field in mode \( j \)

\[ H = \frac{1}{2} m_j \omega_j^2 q_j^2 + \frac{1}{2} m_j \dot{q}_j^2 \]

Vs.

Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator
Quick review of the Quantum Harmonic Oscillator

Standard paradigm: mass on a spring

\[ \omega = \sqrt{\frac{K}{m}} \]

Observables: \( \hat{x}, \hat{p} \)  

Hamiltonian:

\[ \hat{H} = \frac{1}{2} m \omega^2 \hat{x}^2 + \frac{1}{2m} \hat{p}^2 \]

Energy eigenvalues and eigenfunctions:

\[ \hat{H} \psi_n(z) = E_n \psi_n(z) \Rightarrow \]

\[ E_n = \hbar \omega (n + 1/2), \quad n \geq 0 \]

\[ \psi_n(z) \propto e^{-\beta z^2/2} H_n(\beta z) \]

\( H_n \): Hermite polynomial

\[ V(x) \]
Energy eigenfunctions (stationary states): look at probability density $|\psi_n(x)|^2$

Unique features:

Mean position in state $\psi_n(x)$

$\langle \hat{x} \rangle = 0$

Quantum fluctuations

$\Delta x_n = \sqrt{\frac{\hbar}{m\omega}} \sqrt{n + 1 / 2}$

Heisenberg uncertainty relation

$\Delta x \Delta p = \hbar(n + 1 / 2)$
Creation and annihilation operators

Introduce dimensionless variables \( \hat{X} = \hat{x} \sqrt{m\omega/2\hbar} \), \( \hat{P} = \hat{p}/\sqrt{2m\hbar\omega} \)

Define \( \hat{a} = \hat{X} + i\hat{P} \) annihilation operator
\( \hat{a}^\dagger = \hat{X} - i\hat{P} \) creation operator

Can show that
\[
\psi_{n-1} \propto \hat{a} \psi_n
\]
\[
\psi_{n+1} \propto \hat{a}^\dagger \psi_n
\]
annihilation/creation of an excitation

\( \hat{a}^\dagger \hat{a} \psi_n = n \psi_n \)
(#) of excitations

What is a “phonon”? A quantum of excitation in a Harmonic Oscillator
Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator.
QED paradigm: normal mode $j$ in cavity

Observables: $\hat{q} (\propto \hat{E}_x)$, $\hat{p} (\propto \hat{B}_y)$

Hamiltonian: $\hat{H} = \frac{1}{2} m \omega^2 \hat{q}^2 + \frac{1}{2m} \hat{p}^2$

Energy eigenvalues and eigenfunctions:

$E_n = \hbar \omega (n + 1/2), \ n \geq 0$

Number states $\psi_n$

No “wavefunction”, use

Dirac notation $\psi_n \rightarrow |\psi_n\rangle$

$V(x)$

$\Delta E = \hbar \omega$

$E_0 = \hbar \omega(1/2)$

$E_1 = \hbar \omega(3/2)$

$E_n = \hbar \omega(n + 1/2)$
Creation and annihilation operators

Introduce dimensionless variables

\[ \hat{Q} = \hat{q} \sqrt{m\omega / 2\hbar}, \quad \hat{P} = \hat{p} / \sqrt{2m\hbar\omega} \]

Define

\[ \hat{a} = \hat{Q} + i\hat{P} \quad \text{annihilation operator} \]
\[ \hat{a}^\dagger = \hat{Q} - i\hat{P} \quad \text{creation operator} \]

Can show that

\[ |\psi_{n-1}\rangle \propto \hat{a} |\psi_n\rangle \]
\[ |\psi_{n+1}\rangle \propto \hat{a}^\dagger |\psi_n\rangle \]

annihilation/creation of an excitation

\[ \hat{a}^\dagger \hat{a} |\psi_n\rangle = n |\psi_n\rangle \]

number states

What is a “photon”? A quantum of excitation in a Normal Mode of the EM field
Photons as particles

Standing wave normal modes $\Rightarrow$ photons are \textit{delocalized} in space

We can make superpositions of standing waves that correspond to \textit{wavepackets} & use these as our normal modes $\Rightarrow$ photons become localized in space

It is in this sense than we can talk about, e. g, a photon traveling along a specific path in an interferometer, as in the first part of the lecture.
More about number states (Foch states):

Mean field in state $|\psi_n\rangle$

$$\langle \hat{E}_x \rangle \propto \langle \hat{q} \rangle = 0$$

Quantum fluctuations

$$\Delta \hat{q}_n = q_0 \sqrt{n + 1/2} \quad \Rightarrow$$

$$\langle \Delta \hat{E}_x \rangle_n = E_0 \sqrt{n + 1/2}$$

Vacuum fluctuations

$$\Delta \hat{q}_{n=0} = q_0 / 2 \quad \Rightarrow$$

$$\langle \Delta \hat{E}_x \rangle_{n=0} = E_0 / 2$$

Does a laser emit light in a number state with a well defined number of photons?
Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator
Number states are highly non-classical – look at the probability density $|\psi_n(x)|^2$

A quasi-classical state is a minimum-uncertainty oscillating wavepacket
We can make a quasi-classical state $\psi_\alpha(t)$ as a superposition of number states

$$\psi_\alpha(t) = e^{-|\alpha|^2/2} \sum_n \frac{(\alpha e^{i\omega t})^n}{\sqrt{n!}} \psi_n \Rightarrow \langle \hat{X} \rangle \propto \cos(\omega t), \quad \langle \hat{P} \rangle \propto \sin(\omega t)$$

A coherent state is the equivalent superposition of photon number states

$$|\psi_\alpha(t)\rangle = e^{-|\alpha|^2/2} \sum_n \frac{(\alpha e^{i\omega t})^n}{\sqrt{n!}} |\psi_n\rangle \Rightarrow$$

$$\langle \hat{E}_x \rangle \propto \langle \hat{Q} \rangle \propto \cos(\omega t), \quad \langle \hat{B}_y \rangle \propto \langle \hat{P} \rangle \propto \sin(\omega t)$$

Probability of detecting $n$ photons

$$P(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \quad \text{(shot noise)}$$
An ideal laser comes very close to emitting a coherent state.

This is the closest we can come to a classical, monochromatic light field.
Other Interesting Topics in Quantum Optics

– The quantum beam splitter (photons are bosons)

– Quantum theory of interferometers

– How to make number states, squeezed states, coherent states, etc.

– Two-level atoms in single-mode cavities (Jaynes-Cummings model)

– Generalizations of the Jaynes-Cummings model

– Excited atoms interacting with the vacuum, decoherence & decay

– Quantum theory of photodetection

– Quasi-probability distributions and non-classical light
Atoms and Photons: Confessions of a Self-Admitted Control Freak

Poul Jessen
Daniel Hemmer
Nathan Lysne
David Melchior

Enrique Montano
Hector Sosa-Martinez
Kyle Taylor
Light, Atoms and Quantum Control

- Quantum Mechanics is our reigning theory of everything
  - behavior of light and matter on scales from micro- to macroscopic ✓

- Quantum Computing, Quantum Information Science
  - quantum mechanics is a resource that allows us to do different things ✓

- Central Challenge: Quantum Control
  - how to make quantum systems do what we want, not what comes naturally ?
Experimental Setup – “NMR” w/Cold Atoms
The butterfly effect gets entangled

Cold-atom experiments show chaotic fingerprints in the quantum world.

Zeeya Merali

A hidden partnership between two of the hottest topics in physics — quantum entanglement and chaos theory — may have been uncovered by a series of ingenious experiments with caesium atoms. The relationship could provide clues about where the quantum realm ends and the classical world begins.

Chaos theory describes how the slightest change in the starting conditions of a system can have dramatic effects on how it develops. It's usually explained using the 'butterfly effect', in which the atmospheric changes caused by the beating of a butterfly's wings are amplified, leading to completely different regimes.

The 'butterfly effect' has now been seen at the quantum level.

Millard H. Sharp / Science Photo Library

kicking this 'quantum top' caused the spin to change chaotically.

“They've brought together two sexy concepts in physics that are usually thought to operate in completely different regimes.”

Nir Davidson
Weizmann Institute of Science

According to the team, quantum top behaviour is akin to the butterfly effect. Kicking it should produce outcomes depending on its spin. If the top’s initial conditions are chosen from a subset of three subdivisions, dubbed islands of stability, a single kick would knock the top from one island to another. If, however, the top’s conditions fall outside these islands, the spin should jump unpredictably.

When they did the experiment, they found aln