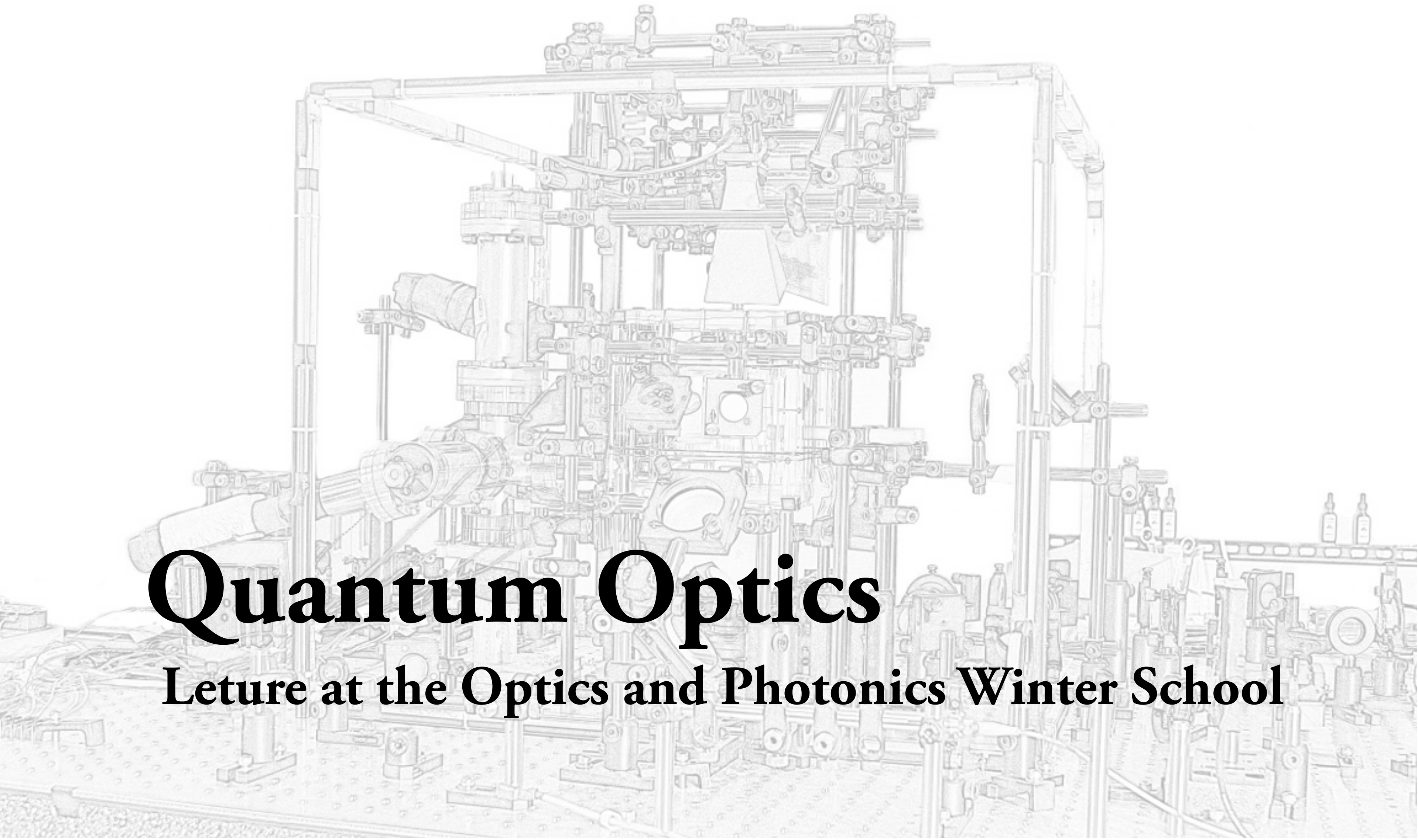


Quantum Optics

Leture at the Optics and Photonics Winter School

Poul Jessen

Brian Anderson
Rolf Binder
Jason Jones
Galina Khitrova
Miroslav Kolesik
Masud Mansuripur
Jerome Moloney
Ewan Wright

College of Optical Sciences
The University of Arizona

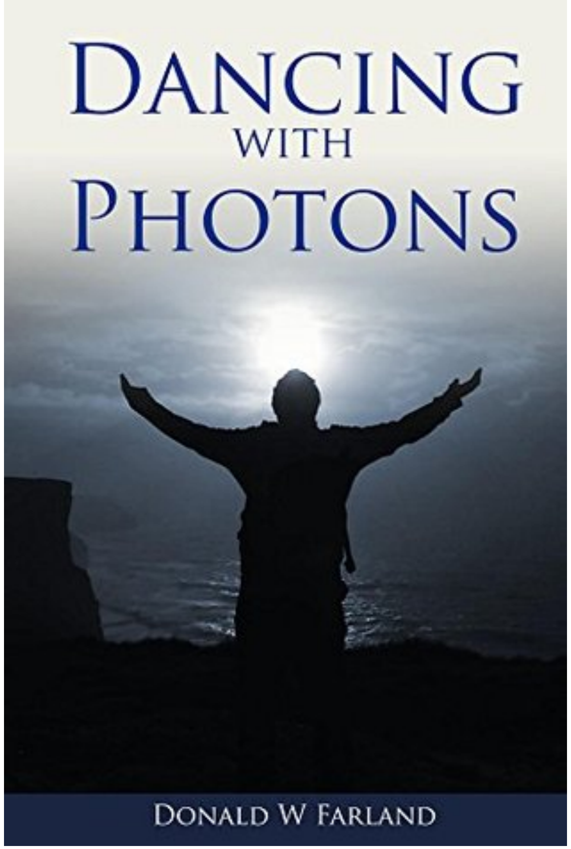
What is Quantum Optics?

- Not classical optics, but something more and different
- Science of non-classical light
- Any science combining light and quantum mechanics

What is Light?

- Electromagnetic waves?
- Photons?

What is Quantum Optics?



Not so fast!

The Nobel Prize in Physics 1955

Willis Eugene Lamb
Prize share: 1/2

Polykarp Kusch
Prize share: 1/2

The Nobel Prize in Physics 1955 was divided equally between Willis Eugene Lamb *"for his discoveries concerning the fine structure of the hydrogen spectrum"* and Polykarp Kusch *"for his precision*

Not so fast!

Anti-photon

W.E. Lamb, Jr.

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Received: 23 July 1994 / Accepted: 18 September 1994

Abstract. It should be apparent from the title of this article that the author does not like the use of the word "photon", which dates from 1926. In his view, there is no such thing as a photon. Only a comedy of errors and historical accidents led to its popularity among physicists and optical scientists. I admit that the word is short and convenient. Its use is also habit forming. Similarly, one might find it convenient to speak of the "aether" or "vacuum" to stand for empty space, even if no such thing existed. There are very good substitute words for "photon", (e.g., "radiation" or "light"), and for "photonics" (e.g., "optics" or "quantum optics"). Similar objections are possible to use of the word "phonon", which dates from 1932. Objects like electrons, neutrinos of finite rest mass, or helium atoms can, under suitable conditions, be considered to be particles, since their theories then have viable non-relativistic and non-quantum limits. This paper outlines the main features of the quantum theory of radiation and indicates how they can be used to treat problems in quantum optics.

afterward, there was a population explosion of people engaged in fundamental research and in very useful technical and commercial developments of lasers. QTR was available, but not in a form convenient for the problems at hand. The photon concepts as used by a high percentage of the laser community have no scientific justification. It is now about thirty-five years after the making of the first laser. The sooner an appropriate reformulation of our educational processes can be made, the better.

1 A short history of pre-photonic radiation

Modern optical theory [2] began with the works of Ch. Huyghens and I. Newton near the end of the seventeenth century. Huyghen's treatise on wave optics was published in 1690. Newton's "Optiks", which appeared in 1704, dealt with his corpuscular theory of light.

A decisive work in 1801 by T. Young, on the two-slit diffraction pattern, showed that the wave version of optics

With all due resepect to W. Lamb, let us try again

What is light?

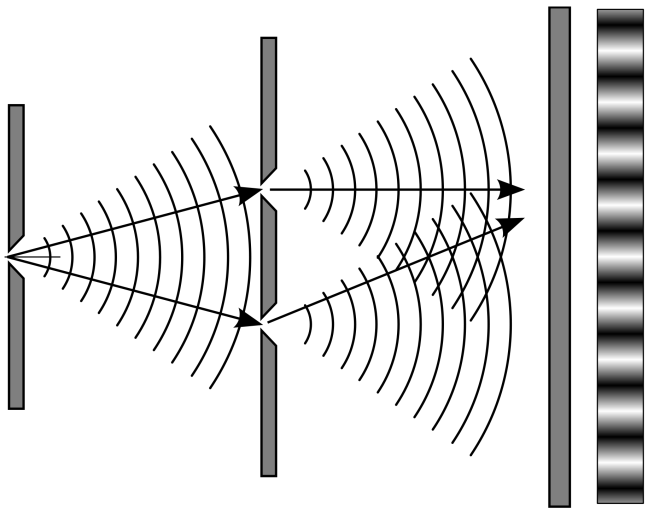
– a wave?
– a stream of particles (photons)?

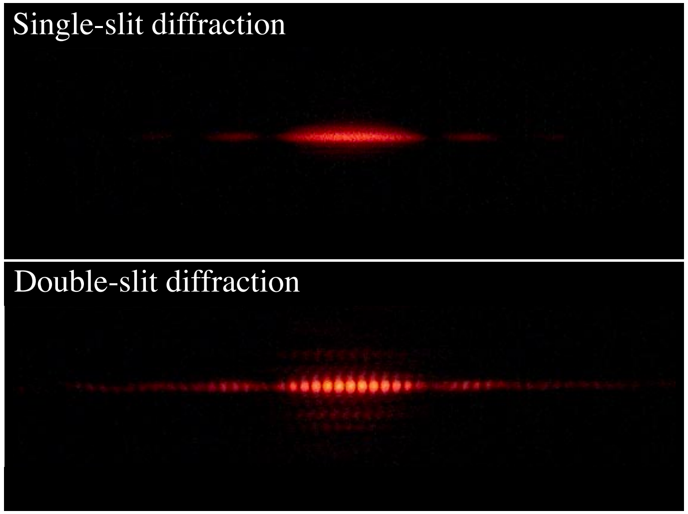
Take the question seriously

– test each hypothesis through experimentation!

Key signature of wave behavior? – Interference!

Double-slit experiment

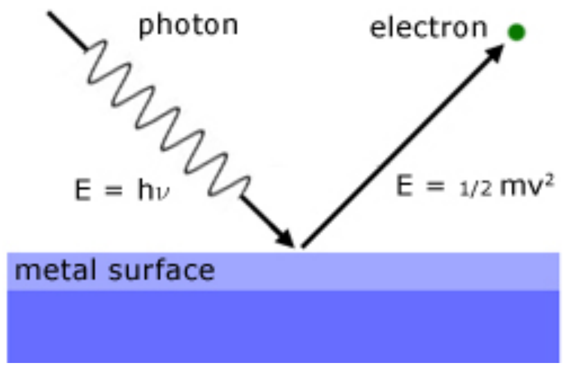




Key signature of particle behavior?

Einstein: photo-electric effect

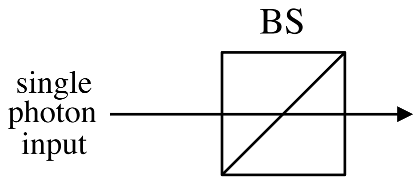
Electrons are released only for light with a frequency ν such that $h\nu$ is greater than the *work function* of the metal in question



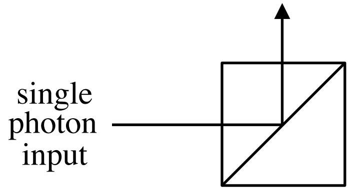
But the quantum theory of electron excitation can explain this based on classical electromagnetic fields, so the photo-electric effect only confirms that *electrons* are particles.

Indivisibility

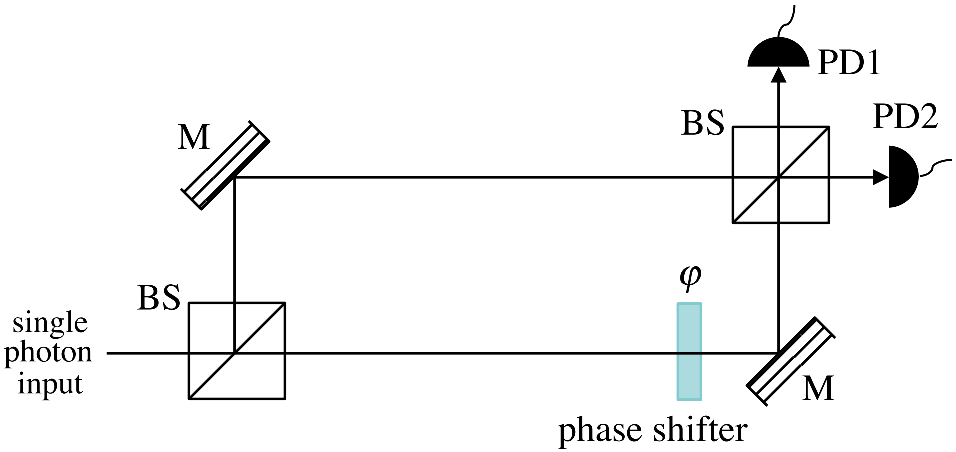
A particle incident on a barrier is either transmitted or reflected.

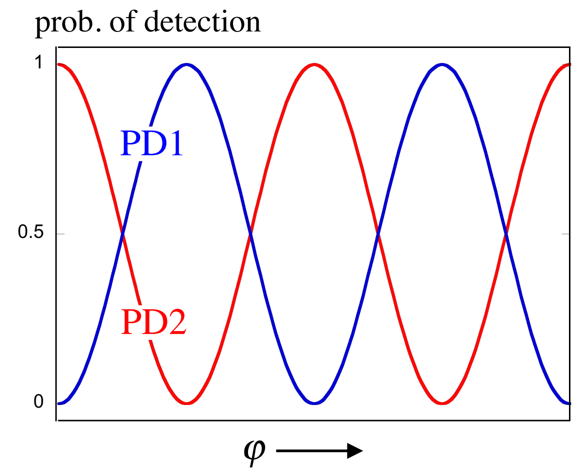


or

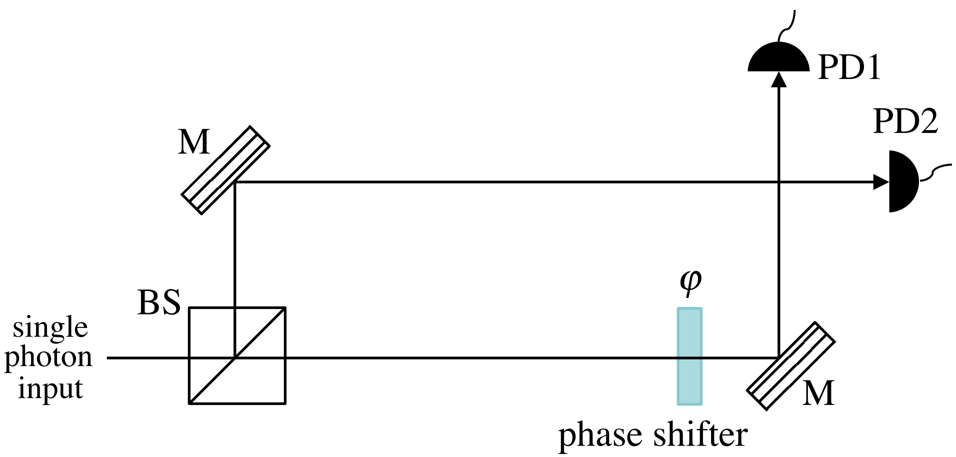


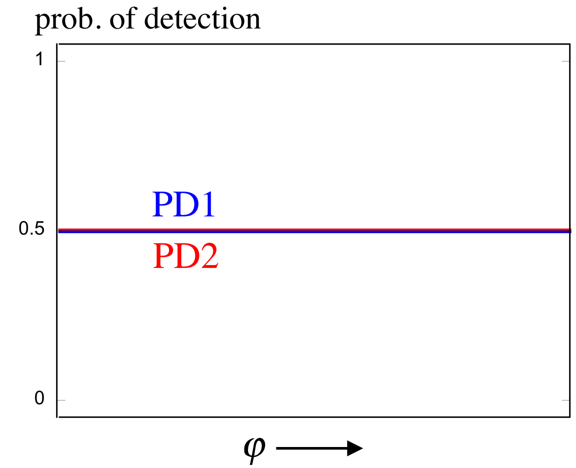
Wave behavior





Particle behavior





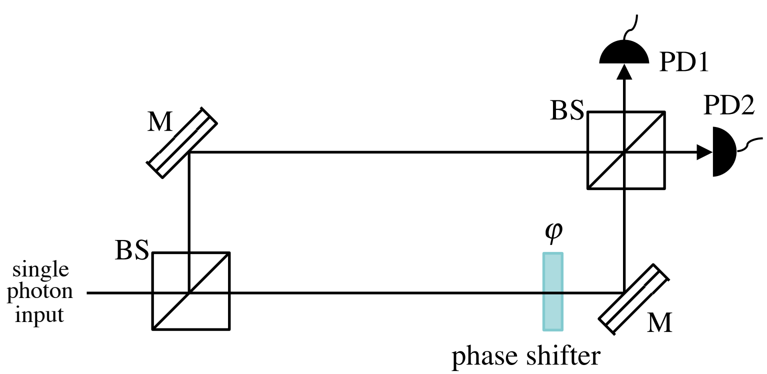
– Evidently a single photon can behave like a wave or a particle, depending on the experiment we do. This is what we know as *wave-particle duality*.

– Does the photon "know" when it hits the first BS if we are doing a wave or particle experiment and then behaves accordingly?

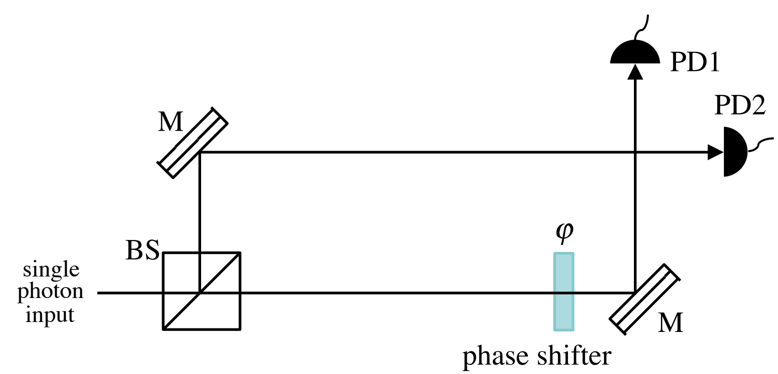
– Wigner's gedanken experiment: Delayed Choice!

Decide *at random* whether to put in the second BS only *after* the photon has passed the first BS

Wave behavior



Particle behavior



Wigners experiment was done in 2008

PRL **100,** 220402 (2008) PHYSICAL REVIEW LETTERS week ending 6 JUNE 2008

Delayed-Choice Test of Quantum Complementarity with Interfering Single Photons

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(Received 12 February 2008; published 3 June 2008)

We report an experimental test of quantum complementarity with single-photon pulses sent into a Mach-Zehnder interferometer with an output beam splitter of adjustable reflection coefficient R. In addition, the experiment is realized in Wheeler's delayed-choice regime. Each randomly set value of R allows us to observe interference with visibility V and to obtain incomplete which-path information characterized by the distinguishability parameter D. Measured values of V and D are found to fulfill the complementarity relation $V^2 + D^2 \leq 1$.

Wigners experiment was done in 2008

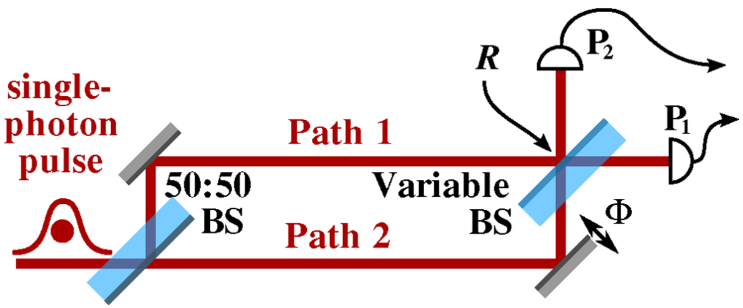

FIG. 1 (color online). Delayed-choice complementarity-test experiment. A single-photon pulse is sent into a Mach-Zehnder interferometer, composed of a 50/50 input beam splitter (BS) and a variable output beam splitter (VBS). The reflection coefficient is randomly set either to the null value or to an adjustable value R, after the photon has entered the interferometer. The single-photon photodetectors P_1 and P_2 allow to record both the interference and the WPI.

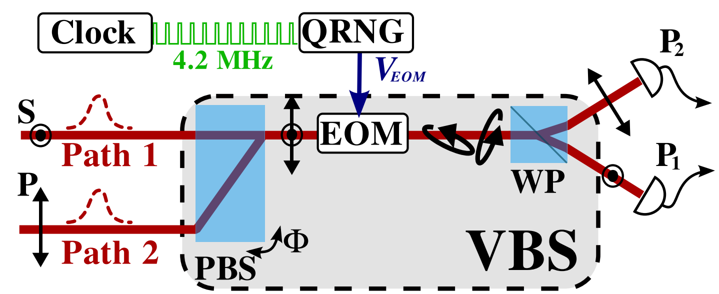

FIG. 2 (color online). Variable output beam splitter (VBS) implementation. The optical axis of the polarization beam splitter (PBS) and the polarization eigenstates of the Wollaston prism (WP) are aligned, and make an angle β with the optical axis of the EOM. The voltage V_{EOM} applied to the EOM is randomly chosen accordingly to the output of a Quantum Random Number Generator (QRNG), located at the output of the interferometer and synchronized on the 4.2-MHz clock that triggers the single-photon emission.

Wigners experiment was done in 2008

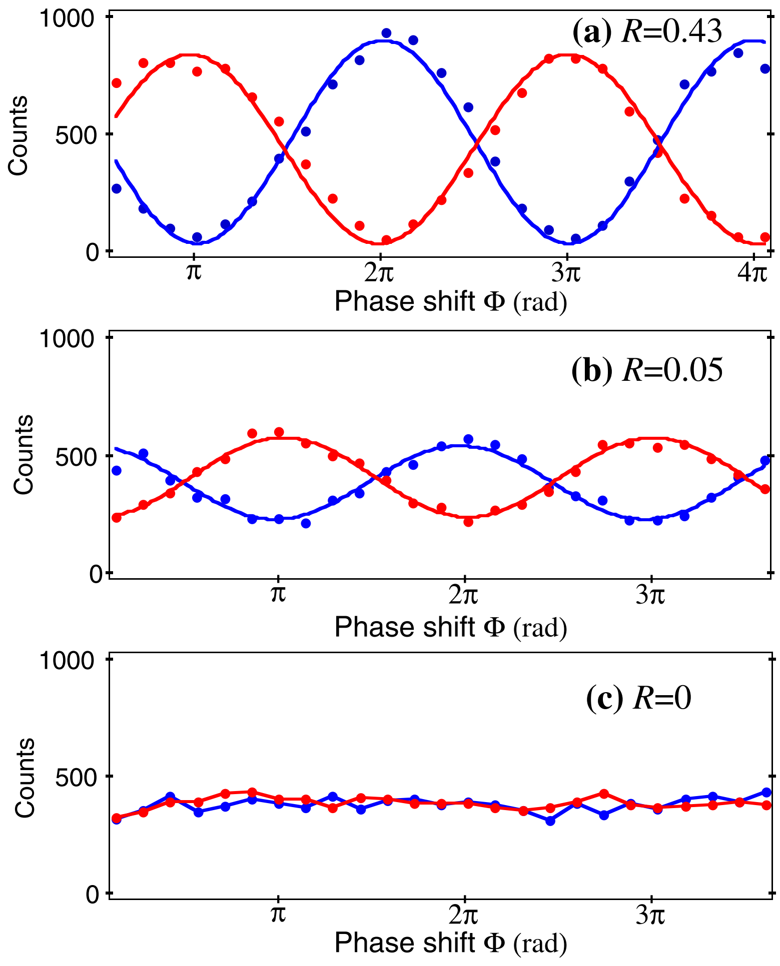

FIG. 3 (color online). Interference visibility V measured in the delayed-choice regime for different values of V_{EOM}. (a)–(c) correspond to $V_{\mathrm{EOM}} \approx 150$ V ($R = 0.43$ and $V = 93 \pm 2\%$), $V_{\mathrm{EOM}} \approx 40$ V ($R = 0.05$ and $V = 42 \pm 2\%$), and $V_{\mathrm{EOM}} = 0$ ($R = 0$ and $V = 0$). Each point is recorded with 1.9 s acquisition time. Detectors dark counts, corresponding to a rate of 60 s^{-1} for each, have been substracted to the data.

Light is *both* a particle and a wave *at the same time*.

What property we see depends on what property we decide to measure.

This is totally in line with our general quantum theory.

BTW, it works for ultracold atoms too!

nature physics

LETTERS

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Wheeler's delayed-choice gedanken experiment with a single atom

A. G. Manning, R. I. Khakimov, R. G. Dall and A. G. Truscott*

The wave–particle dual nature of light and matter and the fact that the choice of measurement determines which one of these two seemingly incompatible behaviours we observe are examples of the counterintuitive features of quantum mechanics. They are illustrated by Wheeler's famous 'delayed-choice' experiment[1], recently demonstrated in a single-photon experiment[2]. Here, we use a single ultracold metastable helium atom in a Mach–Zehnder interferometer to create an atomic analogue of Wheeler's original proposal. Our experiment confirms Bohr's view that it does not make sense to ascribe the wave or particle behaviour to a massive particle before the measurement takes place[1]. This result is encouraging for current work towards entanglement and Bell's theorem tests in macroscopic systems of massive particles[3].

The question of whether light behaves like a particle or wave had a long and strongly contested history until the advent of quantum mechanics, where it was accepted that it could indeed exhibit either behaviour. Conversely, it was de Broglie's hypothesis of matter waves[4] that deviated from the preceding view of massive bodies exclusively as particles, which was confirmed by the electron diffraction experiments of Davisson and Germer[5]. Even more

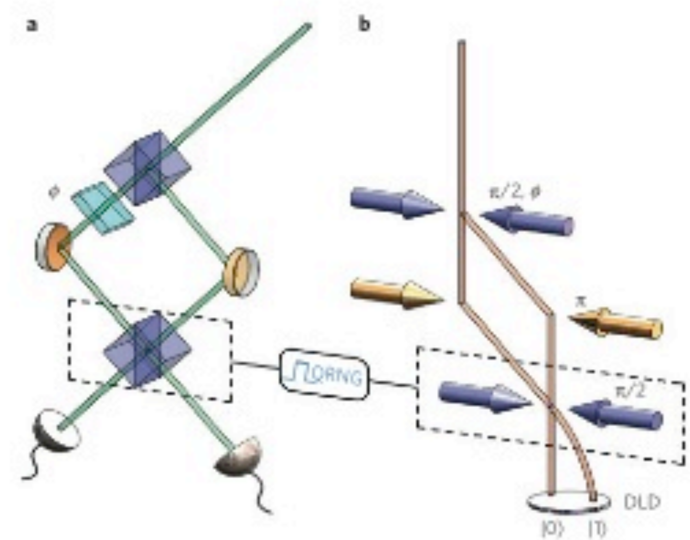


Figure 1 | Schematics of Wheeler's delayed-choice experiments. a, Optical version of Wheeler's delayed-choice experiment. **b**, Atomic version of

We need a quantum theory of light where behaviors such as wave-particle duality is built in

- A formal procedure exists for obtaining a quantum theory from a known classical theory based on Lagrange – Hamilton formalism.
- One cannot "prove" that this procedure is correct. It is justified only by repeated observation that quantum theories obtained in this fashion "work", in the sense that their predictions agree with experiment.
- This should not surprise us. Quantum Mechanics contains new physics that is absent from Classical Mechanics and cannot be derived from it.
- An unfortunate consequence is that we often seem to pull ideas out of thin air when we teach quantum mechanics. The problem gets worse when we try to do things quickly, e. g., a 50 minute lecture on the quantum theory of light.

Quantization of the Electromagnetic Field

Starting point: Maxwell's equations from classical Electromagnetism

$$\nabla \cdot \mathbf{E}(\mathbf{r},t) = \frac{1}{\varepsilon_0}\rho(\mathbf{r},t)$$

$$\nabla \cdot \mathbf{B}(\mathbf{r},t) = 0$$

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial}{\partial t}\mathbf{B}(\mathbf{r},t)$$

$$\nabla \times \mathbf{B}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{E}(\mathbf{r},t) + \frac{1}{\varepsilon_0 c^2}\mathbf{j}(\mathbf{r},t)$$

In Quantum Electrodynamics (QED) the ME's are still valid, but the fields **E** and **B** become operators that depend on space and time just like the classical electric and magnetic fields.

For simplicity we consider empty space without charges or currents, and use the two last Maxwell equations to derive a *wave equation*

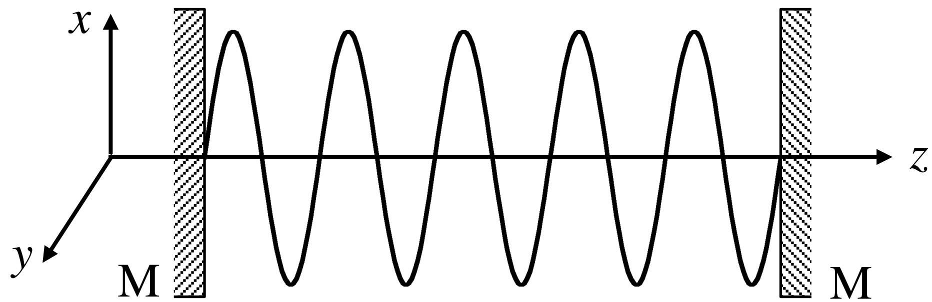
$$\left[\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{E}(\mathbf{r},t) = 0$$

$$\left[\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]\mathbf{E}(z,t) = 0$$

paraxial waves propagating along the z-axis

Electromagnetic Field in a 1-Dimensional Cavity

Cavity of length L, cross section A, and volume $V = LA$, field $\mathbf{E} = \mathbf{x}E_x$



Any field inside the cavity can be expressed as a superposition of the cavity *normal modes*, which are standing wave solutions to the wave equation with nodes on the mirror surfaces.

$$E_x(z,t) = \sum_{j=1}^{\infty} A_j q_j(t)\sin(k_j z), \qquad k_j = \frac{\omega_j}{c} = \frac{j\pi}{L}, \qquad A_j = \sqrt{\frac{2\omega_j^2 m_j}{\varepsilon_0 V}}$$

Here $q_j(t)$ are the time varying amplitudes for the different modes, and A_j is chosen so E_x has units of electric field (V/m) and $q_j(t)$ has units of length (m).

Each normal mode is an independent degree of freedom that can be quantized by itself.

For mode number j we have electric and magnetic fields (the latter can be found from E_x using Maxwells equations)

$$E_x^{(j)}(z,t) = A_j q_j(t)\sin(k_j z), \quad B_y^{(j)}(z,t) = \frac{A_j}{k_j c^2}\dot{q}_j(t)\cos(k_j z), \quad \dot{q}_j(t) = \frac{d}{dt}q_j(t)$$

Using classical E & M we can show that the energy of the field in mode j is

$$\begin{aligned} H &= \varepsilon_0 \int_V dr^3 \left(|\mathbf{E}|^2 + c^2|\mathbf{B}|^2\right) \\ &= \frac{\varepsilon_0 A}{2}\int_0^L dz\left[A_j^2 q_j(t)^2\sin^2(k_j z) + \frac{A_j^2}{k_j^2}\dot{q}_j(t)^2\cos^2(k_j z)\right] \\ &= \frac{1}{2}m_j\omega_j^2 q_j^2 + \frac{1}{2}m_j\dot{q}_j^2 \end{aligned}$$

Writing the field energy in this form is *highly suggestive*.

Harmonic Oscillator

$$H = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2m}p^2$$

Vs.

EM field in mode *j*

$$H = \frac{1}{2}m_j\omega_j^2 q_j^2 + \frac{1}{2}m_j\dot{q}_j^2$$

Quantum Theory

$$x \to \hat{x} = x$$
$$p \to \hat{p} = -i\hbar\frac{d}{dx}$$
$$\left[\hat{x},\hat{p}\right] = i\hbar$$

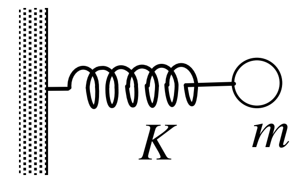
$$\hat{E}_x \propto q_j \to \hat{q}_j$$
$$\hat{B}_y \propto m_j\dot{q}_j \to \hat{p}_j$$
$$\left[\hat{q}_j,\hat{p}_j\right] = i\hbar,\ \left[\hat{E}_x,\hat{B}_y\right] \neq 0$$

Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator

Quick review of the Quantum Harmonic Oscillator

Standard paradigm: mass on a spring

$$\omega = \sqrt{K / m}$$



Observables: $\hat{x}, \hat{p}$

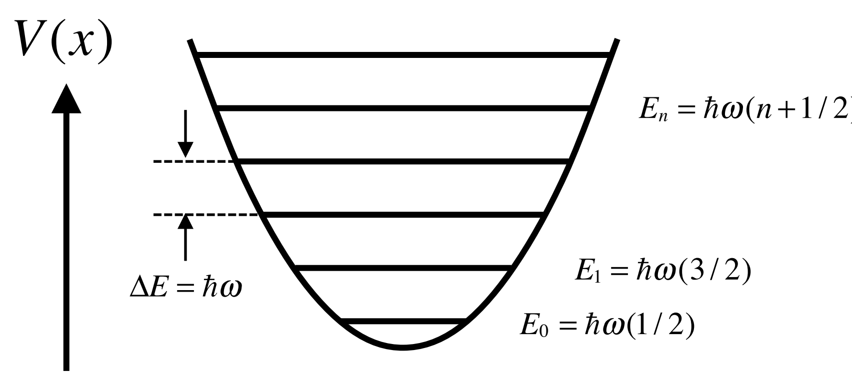
Hamiltonian: $\hat{H} = \frac{1}{2} m\omega^2 \hat{x}^2 + \frac{1}{2m}\hat{p}^2$

Energy eigenvalues and eigenfunctions: $\hat{H}\psi_n(z) = E_n\psi_n(z) \Rightarrow$

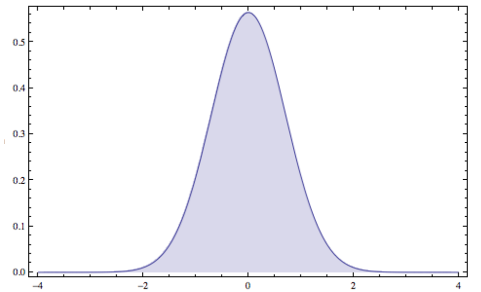
$$E_n = \hbar\omega(n + 1/2),\ n \geq 0$$

$$\psi_n(z) \propto e^{-\beta x^2/2} H_n(\beta x)$$

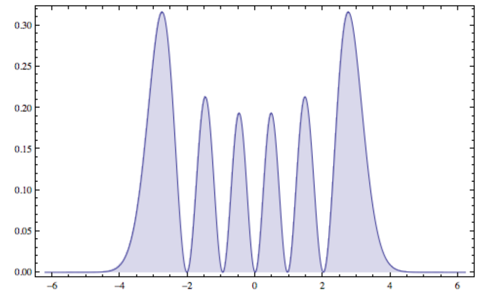
H_n: Hermite polynomial



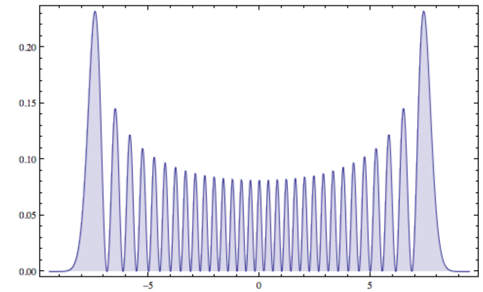
Energy eigenfunctions (stationary states): look at probability density $\left|\psi_n(x)\right|^2$



$n = 0$



$n = 5$



$n = 30$

Unique features: Mean position in state $\psi_n(x)$ $\qquad \langle \hat{x} \rangle = 0$

Quantum fluctuations $\qquad \Delta x_n = \sqrt{\frac{\hbar}{m\omega}}\sqrt{n+1/2}$

Heisenberg uncertainty relation $\qquad \Delta x \Delta p = \hbar(n+1/2)$

Creation and annihilation operators

Introduce dimensionless variables $\hat{X} = \hat{x}\sqrt{m\omega/2\hbar}, \quad \hat{P} = \hat{p}/\sqrt{2m\hbar\omega}$

Define	$\hat{a} = \hat{X} + i\hat{P}$	annihilation operator
	$\hat{a}^\dagger = \hat{X} - i\hat{P}$	creation operator

Can show that

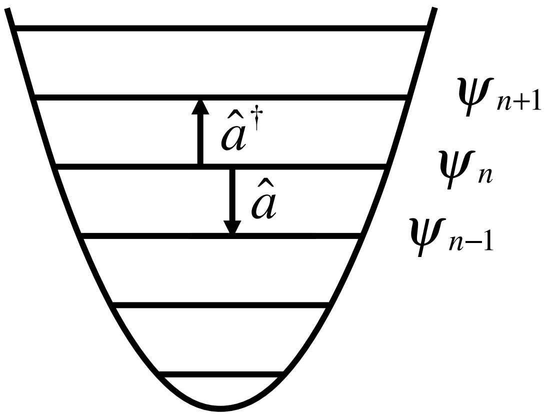
$$\psi_{n-1} \propto \hat{a}\,\psi_n$$

$$\psi_{n+1} \propto \hat{a}^\dagger\psi_n$$

annihilation/creation of an excitation

$$\hat{a}^\dagger\hat{a}\,\psi_n = n\,\psi_n$$

(# of excitations)



What is a "phonon"? A quantum of exitation in a Harmonic Oscillator

Back to…

Harmonic Oscillator

$$H = \frac{1}{2}m\omega^2 x^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2 x^2 + \frac{1}{2m}p^2$$

Vs.

EM field in mode *j*

$$H = \frac{1}{2}m_j\omega_j^2 q_j^2 + \frac{1}{2}m_j\dot{q}_j^2$$

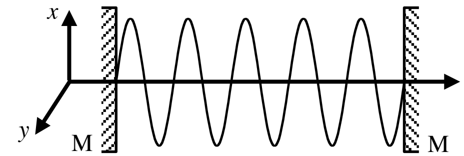
Quantum Theory

$$x \rightarrow \hat{x} = x$$
$$p \rightarrow \hat{p} = -i\hbar\frac{d}{dx}$$
$$\left[\hat{x}, \hat{p}\right] = i\hbar$$

$$\hat{E}_x \propto q_j \rightarrow \hat{q}_j$$
$$\hat{B}_y \propto m_j\dot{q}_j \rightarrow \hat{p}_j$$
$$\left[\hat{q}_j, \hat{p}_j\right] = i\hbar,\ \left[\hat{E}_x, \hat{B}_y\right] \neq 0$$

Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator

QED paradigm: normal mode j in cavity



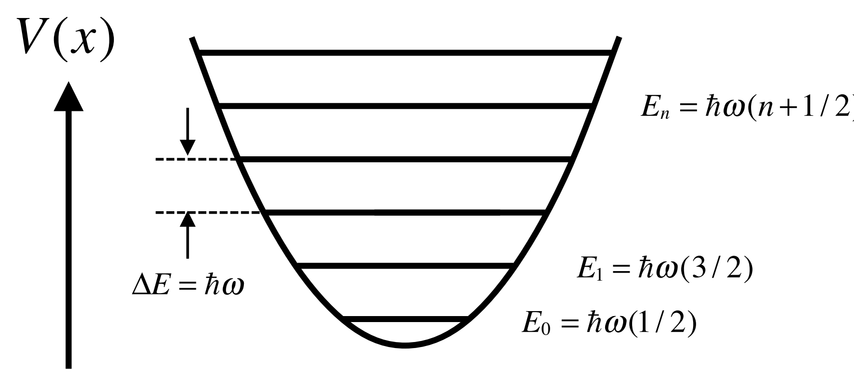
Observables: $\hat{q}\,(\propto \hat{E}_x),\ \hat{p}\,(\propto \hat{B}_y)$ **Hamiltonian:** $\hat{H} = \frac{1}{2} m\omega^2 \hat{q}^2 + \frac{1}{2m}\hat{p}^2$

Energy eigenvalues and eigenfunctions: $\hat{H}\psi_n = E_n\psi_n \ \Rightarrow$

$$E_n = \hbar\omega(n+1/2),\ n \geq 0$$

Number states ψ_n

No "wavefunction", use
Dirac notation $\psi_n \rightarrow |\psi_n\rangle$



Creation and annihilation operators

Introduce dimensionless variables $\hat{Q} = \hat{q}\sqrt{m\omega/2\hbar}, \quad \hat{P} = \hat{p}/\sqrt{2m\hbar\omega}$

Define	$\hat{a} = \hat{Q} + i\hat{P}$	annihilation operator
	$\hat{a}^\dagger = \hat{Q} - i\hat{P}$	creation operator

Can show that

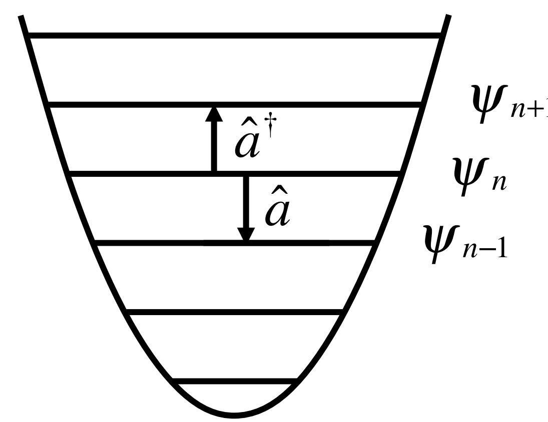
$$|\psi_{n-1}\rangle \propto \hat{a}|\psi_n\rangle$$

$$|\psi_{n+1}\rangle \propto \hat{a}^\dagger|\psi_n\rangle$$

annihilation/creation
of an excitation

$$\hat{a}^\dagger\hat{a}|\psi_n\rangle = n|\psi_n\rangle$$

↑ number states

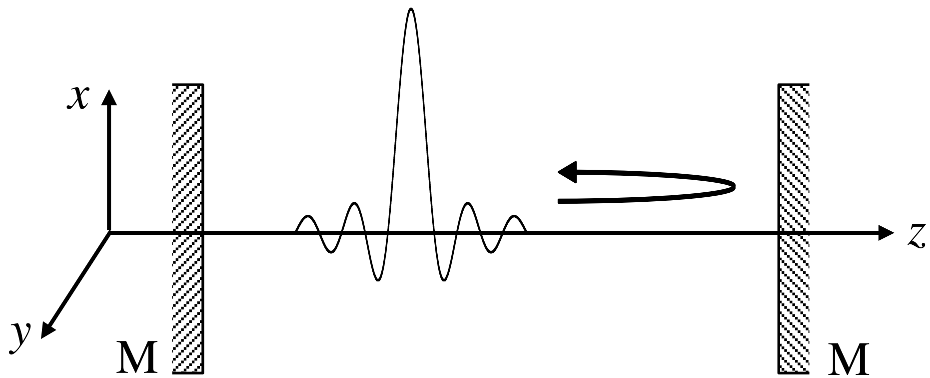


What is a "photon"?	A quantum of exitation in a Normal Mode of the EM field

Photons as particles

Standing wave normal modes $\Rightarrow$ photons are *delocalized* in space

We can make superpositions of standing waves that correspond to *wavepackets* & use these as our normal modes $\Rightarrow$ photons become localized in space



It is in this sense than we can talk about, e. g, a photon traveling along a specific path in an interferometer, as in the first part of the lecture.

More about number states (Foch states):

Mean field in state $\left|\psi_n\right\rangle$ $\qquad$ $\left\langle \hat{E}_x \right\rangle \propto \langle \hat{q} \rangle = 0$

Quantum fluctuations

$$\Delta\hat{q}_n = q_0\sqrt{n+1/2} \;\Rightarrow$$
$$(\Delta\hat{E}_x)_n = E_0\sqrt{n+1/2}$$

Vacuum fluctuations

$$\Delta\hat{q}_{n=0} = q_0\,/\,2 \;\Rightarrow$$
$$(\Delta\hat{E}_x)_{n=0} = E_0\,/\,2$$

Does a laser emit light in a number state with a well defined number of photons?

Back to…

Harmonic Oscillator

$$H = \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m v^2$$
$$= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2m} p^2$$

Vs.

EM field in mode *j*

$$H = \frac{1}{2} m_j \omega_j^2 q_j^2 + \frac{1}{2} m_j \dot{q}_j^2$$

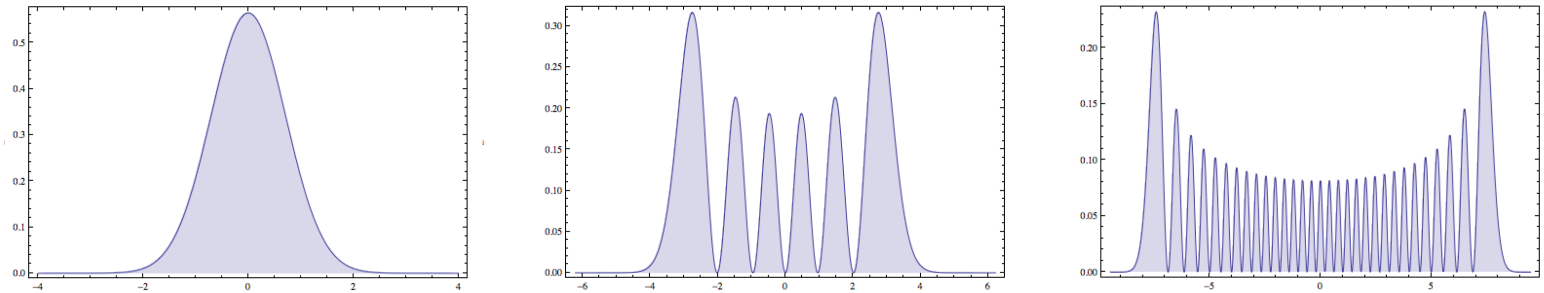
Quantum Theory

$$x \rightarrow \hat{x} = x$$
$$p \rightarrow \hat{p} = -i\hbar \frac{d}{dx}$$
$$\left[\hat{x}, \hat{p}\right] = i\hbar$$

$$\hat{E}_x \propto q_j \rightarrow \hat{q}_j$$
$$\hat{B}_y \propto m_j \dot{q}_j \rightarrow \hat{p}_j$$
$$\left[\hat{q}_j, \hat{p}_j\right] = i\hbar, \ \left[\hat{E}_x, \hat{B}_y\right] \neq 0$$

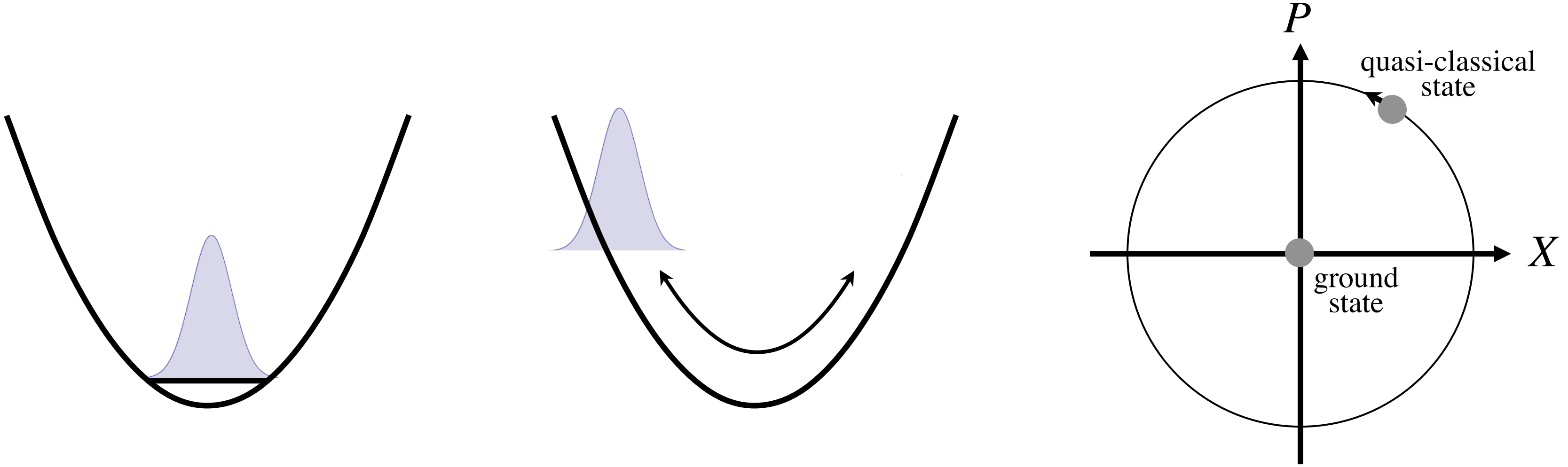
Postulate (leap of faith): EM field in a normal mode is a harmonic oscillator

Number states are highly non-classical – look at the probability density $|\psi_n(x)|^2$



$n = 0$ $n = 5$ $n = 30$

A *quasi-classical* state is a minimum-uncertainty oscillating wavepacket



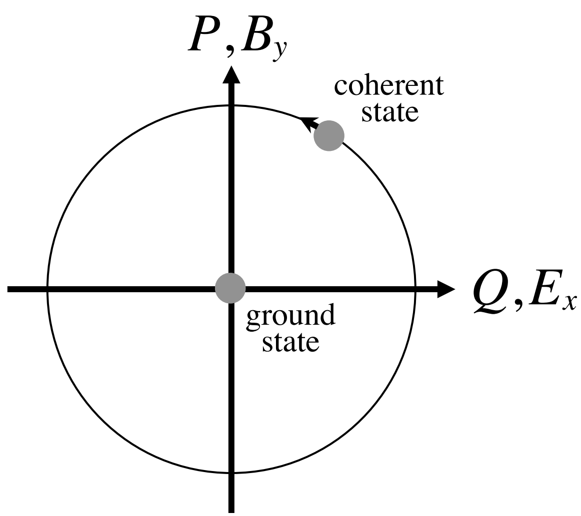
We can make a quasi-classical state $\psi_\alpha(t)$ as a superposition of number states

$$\psi_\alpha(t) = e^{-|\alpha|^2/2} \sum_n \frac{(\alpha e^{i\omega t})^n}{\sqrt{n!}} \psi_n \quad \Rightarrow \quad \langle \hat{X} \rangle \propto \cos(\omega t), \ \ \langle \hat{P} \rangle \propto \sin(\omega t)$$

A *coherent state* is the equivalent superposition of photon number states

$$|\psi_\alpha(t)\rangle = e^{-|\alpha|^2/2} \sum_n \frac{(\alpha e^{i\omega t})^n}{\sqrt{n!}} |\psi_n\rangle \ \Rightarrow$$

$$\langle \hat{E}_x \rangle \propto \langle \hat{Q} \rangle \propto \cos(\omega t), \ \ \langle \hat{B}_y \rangle \propto \langle \hat{P} \rangle \propto \sin(\omega t)$$

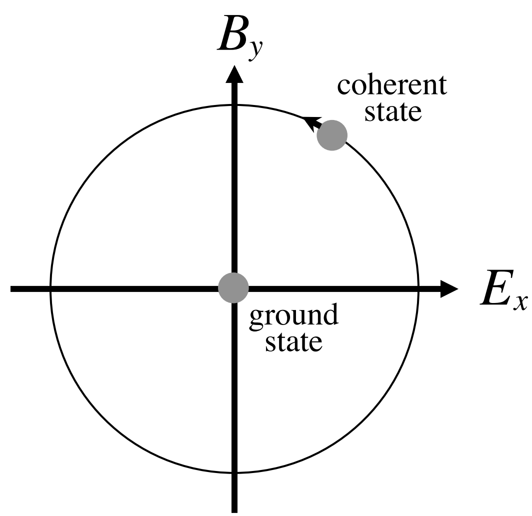


Probability of detecting *n* photons

$$P(n) = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} \qquad \text{(shot noise)}$$

An ideal laser comes very close to emitting a *coherent state*.

This is the closest we can come to a classical, monochromatic light field.



Other Interesting Topics in Quantum Optics

– The quantum beam splitter (photons are bosons)

– Quantum theory of interferometers

– How to make number states, squeezed states, coherent states, etc.

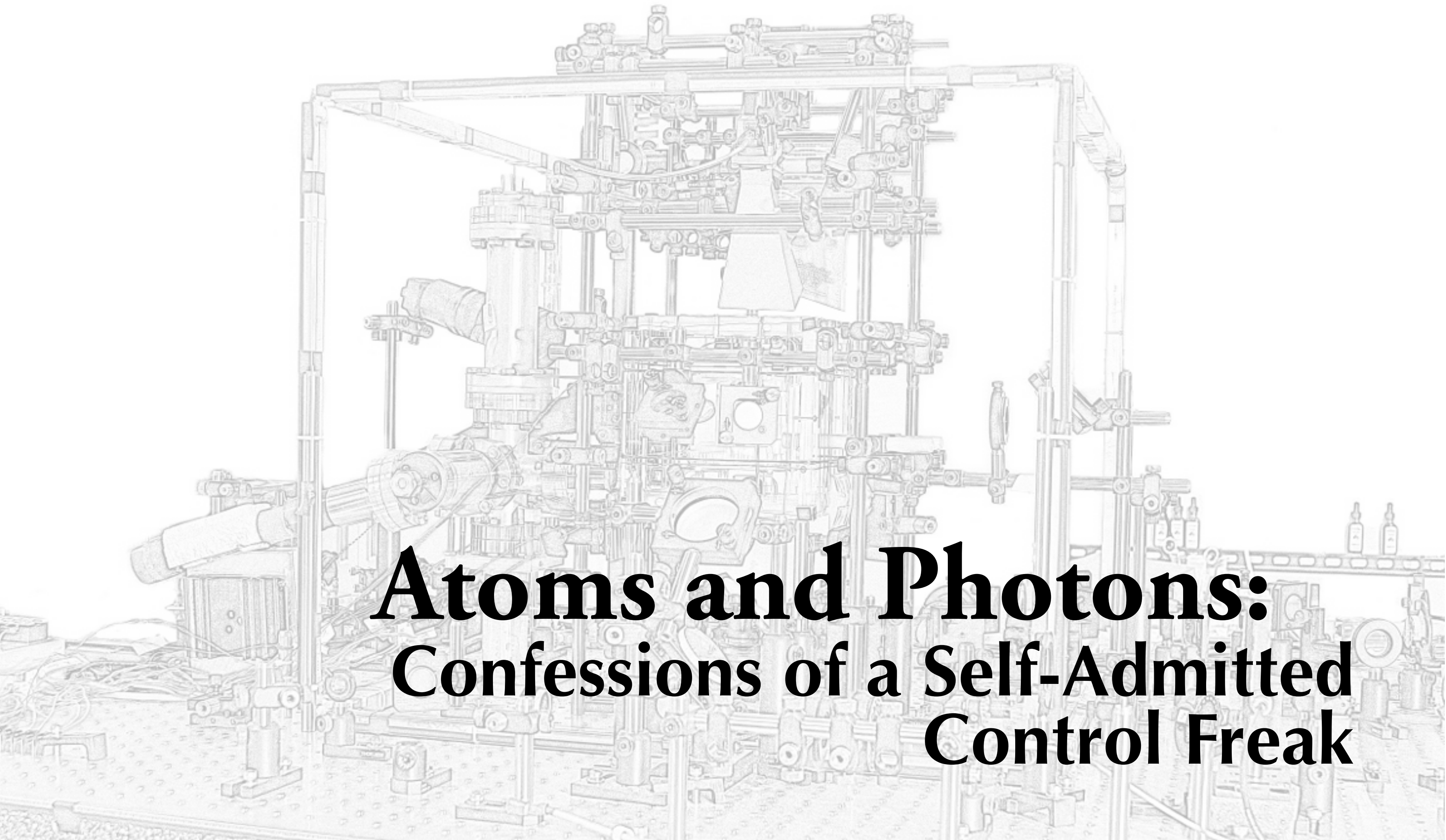
– Two-level atoms in single-mode cavities (Jaynes-Cummings model)

– Generalizations of the Jaynes-Cummings model

– Excited atoms interacting with the vacuum, decoherence & decay

– Quantum theory of photodetection

– Quasi-probability distributions and non-classical light



Atoms and Photons:
Confessions of a Self-Admitted Control Freak

Poul Jessen

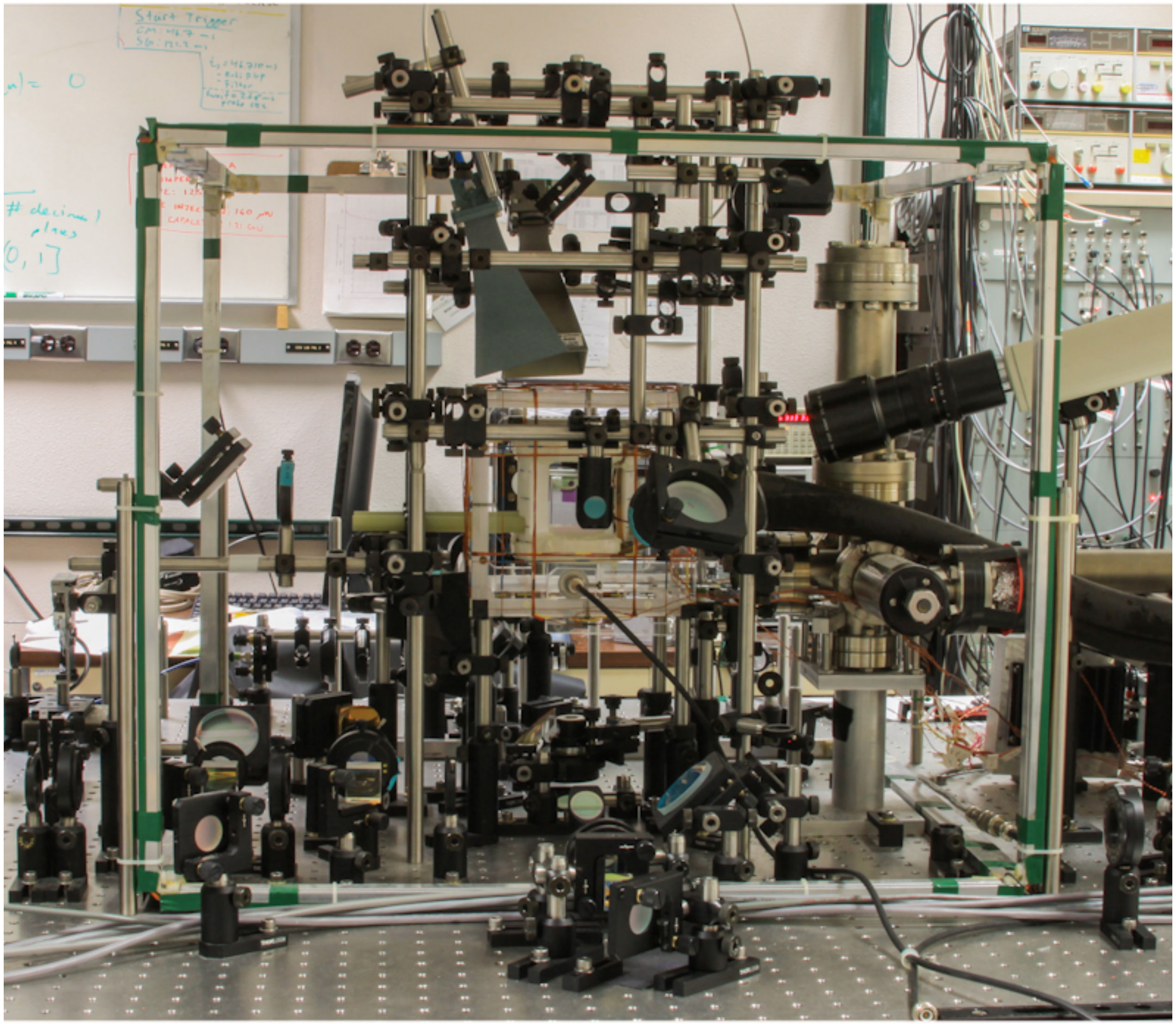
Daniel Hemmer
Nathan Lysne
David Melchior
Enrique Montano
Hector Sosa-Martinez
Kyle Taylor

College of Optical Sciences
The University of Arizona

Light, Atoms and Quantum Control

- **Quantum Mechanics is our reigning theory of everything**
  - behavior of light and matter on scales from micro- to macroscopic ✓
- **Quantum Computing, Quantum Information Science**
  - quantum mechanics is a resource that allows us to do different things ✓
- **Central Challenge: Quantum Control**
  - how to make quantum systems do what we want, not what comes naturally ?

Experimental Setup – "NMR" w/Cold Atoms



Quantum Control for Fun and Science

naturenews

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News

The butterfly effect gets entangled

Cold-atom experiments show chaotic fingerprints in the quantum world.

Zeeya Merali

A hidden partnership between two of the hottest topics in physics — quantum entanglement and chaos theory — may have been uncovered by a series of ingenious experiments with caesium atoms. The relationship could provide clues about where the quantum realm ends and the classical world begins.

The 'butterfly effect' has now been seen at the quantum level.

Millard H. Sharp / Science Photo Library

Chaos theory describes how the slightest change in the starting conditions of a system can have dramatic effects on how it develops. It's usually explained using the 'butterfly effect', in which the atmospheric changes caused by the beating of a butterfly's

kicking this 'quantum top' caused the spin to c
chaotically.

“They've brought together two sexy concepts in physics that are usually thought to operate in completely different regimes.”

Nir Davidson
Weizmann Institute of Science

According to the tean
quantum top behavec
kicking it should proc
outcomes depending
its spin. If the top's ir
one of three subsets c
dubbed islands of sta
kick would knock the
sending it around a s
island. If, however, tl
outside these islands,
spin should jump aro
unpredictably.

When they did the experiment, they found alm