## OPTI 415 Homework 5

1. Suppose we have an infinite conjugate optical system that operates at $\mathrm{F} / \#=4$ and has a wavelength of $\lambda=0.5 \mu \mathrm{~m}$. Answer the following:
(a) What is the cutoff frequency?
(b) The irradiance of the object for the system is given by
$o(x, y)=\frac{1}{2}+\frac{1}{4} \cos \left(2 \pi \xi_{o} x\right)+\frac{1}{10} \cos \left(4 \pi \xi_{o} x\right)+\frac{1}{10} \cos \left(8 \pi \xi_{o} x\right)$,
where $\xi_{o}=250 \mathrm{cyc} / \mathrm{mm}$. If the $\mathrm{OTF}=0.5$ at the frequency $\xi_{o}=250 \mathrm{cyc} / \mathrm{mm}$, what is the irradiance of the image?
(c) What is the contrast of the final image?
2. The file EdgeBlur256.bmp shows the image of a blurred edge formed by an optical system. The size of each pixel in the image is 2 microns, and the image has dimensions of $512 \times 512$ pixels.
(a) Plot the Edge Spread Function (ESF) for the system. This can be accomplished by taking a slice through the image. Also, normalize the maximum value of this plot to 1.0 .
(b) Plot the Line Spread Function (LSF) for the system. This can be accomplished by using finite differences to calculate the derivative of the ESF. We can write

$$
\operatorname{LSF}[i]=0.5(E S F[i+1]-E S F[i-1])
$$

Of course the doesn't work at the very edges of the LSF, so set these values to 0.0 .
(c) Plot the Modulation Transfer Function for the system. Fourier transform the LSF to get the Optical Transfer Function(OTF) and then MTF $=|\mathrm{OTF}|$. Be aware that the Fourier transform routines usually shifts the pattern so that the MTF data will be at the edge of the array.
(d) Finally, everything has been done in pixel coordinates so far. We'd like to know the spatial frequency size of each pixel in Fourier space in units of cyc/mm. If $d$ is the size of a single pixel in image space in units of mm , then the width of the entire array in Fourier space is $1 / d$ in units of cyc $/ \mathrm{mm}$. How big is a single pixel in Fourier space?
3. The file asphere515.txt (available online) contains a set of data points from a stylus profilometer measurement of an aspheric surface. The data $\left\{r_{i}, z_{i}\right\}$ with $i=1 \cdots N$ is in two columns, where $r_{i}$ is the lateral distance from the origin and $z_{i}$ is the measured surface sag at that point. Compute the apical radius $R$ and conic constant $K$ for a fit to this data using the following technique.

For an conoid, the sag equation is

$$
z=\frac{R-\sqrt{R^{2}-(K+1) r^{2}}}{(K+1)} .
$$

Rearranging gives

$$
(K+1) z^{2}-2 R z=-r^{2} .
$$

For the data, we can write a system of equations as $\left(\begin{array}{cc}z_{1}^{2} & -2 z_{1} \\ \vdots & \vdots \\ z_{N}^{2} & -2 z_{N}\end{array}\right)\binom{K+1}{R}=\left(\begin{array}{c}-r_{1}^{2} \\ \vdots \\ -r_{N}^{2}\end{array}\right)$
This matrix equation can be solved using a least squares technique as $\binom{K+1}{R}=\left[A^{T} A\right]^{-1} A^{T} b$
where $A=\left(\begin{array}{cc}z_{1}^{2} & -2 z_{1} \\ \vdots & \vdots \\ z_{N}^{2} & -2 z_{N}\end{array}\right)$ and $b=\left(\begin{array}{c}-r_{1}^{2} \\ \vdots \\ -r_{N}^{2}\end{array}\right)$. Plot the fit error between the original data and the conoid.
4. The following page has a drawing of an element with aspheric surfaces. Fill out the sag values $z$ in the sag table. What type of surfaces are used for this lens?



