OPTI 415 Homework 4

1. Suppose we have a wavefront error $W(\rho,\theta) = 0.001Z_2^0(\rho,\theta) - 0.001Z_4^0(\rho,\theta)$, where $\rho = r/r_{max}$ and $r_{max} = 3\ mm$. If the pupil shrinks to $r_{max} = 2\ mm$, what is the new wavefront error in terms of Zernike polynomials? Plot the old and new wavefronts. They should look identical to the old wavefront over the central 2.0 mm. HINT: After applying the definitions of the polynomials, convert to real coordinates and then normalize to the new pupil size.

   The first step is to insert the definitions of the Zernike polynomials

$$0.001\sqrt{3}\left(2\rho^2 - 1\right) - 0.001\sqrt{5}\left(6\rho^4 - 6\rho^2 + 1\right)$$

   where $\rho = r/3$. *Revert to real coordinates so*

$$0.001\sqrt{3}\left(2\frac{r^2}{9} - 1\right) - 0.001\sqrt{5}\left(6\frac{r^4}{81} - 6\frac{r^2}{9} + 1\right).$$

   Next, define a new normalized coordinate $\rho' = r/2 \Rightarrow r = 2\rho'$ *and project the resultant expression onto the Zernikes to get the new expansion coefficients*

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fold[r_] := 0.001 * Sqrt[3] * (2 * (r / 3) ^ 2 - 1) - 0.001 * Sqrt[5] * (6 * (r / 3) ^ 4 - 6 * (r / 3) ^ 2 + 1);
foldnorm := fold[2 * ρ]

In[850]:= a00 = 2 * ∫_0^1 fold[2 * ρ] * ρ dρ
Out[850]= -0.00110028

In[852]:= a20 = 2 * ∫_0^1 fold[2 * ρ] * Sqrt[3] * (2 * ρ^2 - 1) * ρ dρ
Out[852]= 0.00140074

In[853]:= a40 = 2 * ∫_0^1 fold[2 * ρ] * Sqrt[5] * (6 * ρ^4 - 6 * ρ^2 + 1) * ρ dρ
Out[853]= -0.000197531
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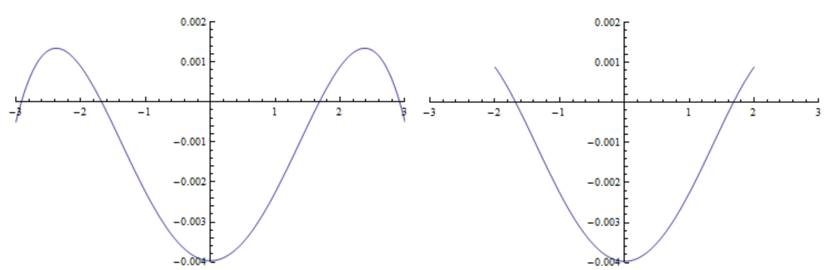
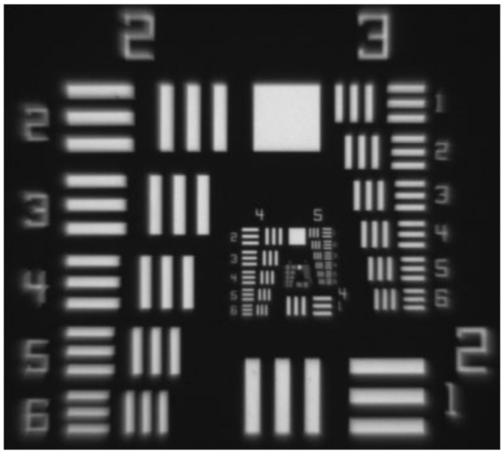
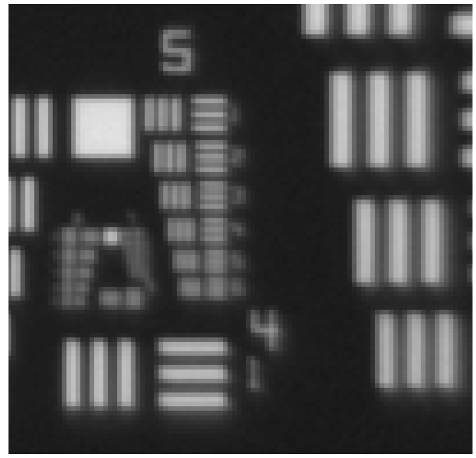
   Here are plots of the old and new wavefronts. They match over the central 4 mm.



2. Fit the points below to a 2nd order Zernike expansion (i.e. n ≤ 2) for a normalization radius of 3 mm. The raw data is available on the website for download.

See HW4 Q2 Zernike Fit.xlsx for solution

3. Assume that an imaging system with a magnification m = -0.5 is used to capture an image of a 1951 USAF target (original on-line). What is the resolution limit of the system in cyc/mm?

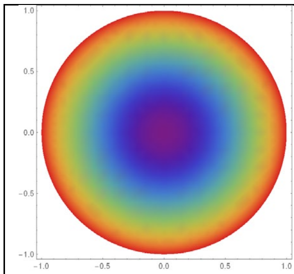
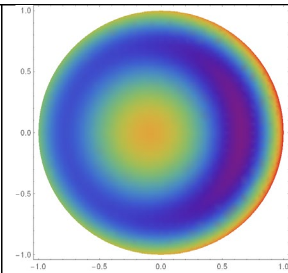
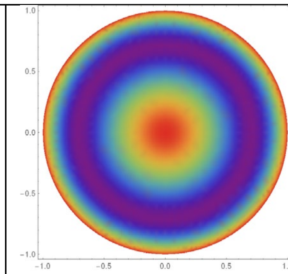
If we zoom in on the image, the contrast of the bars appears to go to zero for Element 5 in Group 5. This target corresponds to 50.8 cyc/mm in object space. We need to scale by 1/m to convert to image space spatial frequency, so the cutoff frequency of this system is 101.6 cyc/mm.

4. You are testing an optical system and measure a wavefront given by the following Zernike expansion

$W(\rho,\theta) = 1.5Z_1^{-1}(\rho,\theta) + 2.0Z_2^0(\rho,\theta) + 0.23Z_3^1(\rho,\theta) + 0.5Z_4^0(\rho,\theta)$

where the expansion coefficients are in units of μm.

   a) What are the peak-to-valley errors for the wavefront spherical approximation, the wavefront irregularity and the rotationally invariant wavefront?

		
Wavefront spherical approximation	*Wavefront irregularity*	*Rotationally invariant wavefront*

From the plots above of the minimum of the wavefront spherical approximation occurs at the center of the map and the maximum occurs at the edge. The PV error in this case then is $A = 2.0Z_2^0(1,\theta) - 2.0Z_2^0(0,\theta) = 4\sqrt{3} = 6.928\mu m$.

For the wavefront irregularity, the maximum occurs at the right edge of the map $(\rho,\theta) = (1,0)$ *and the minimum occurs at roughly* $(\rho,\theta) = (0.656,0)$. *The difference between the values at these points gives the PV Error B = 1.76857 + 0.829027 = 2.598 μm.*

For the rotationally invariant wavefront, the maximum occurs at the edge of the map $(\rho,\theta) = (1,\theta)$ *and the minimum occurs at* $(\rho,\theta) = \left(1/\sqrt{2},\theta\right)$. *The difference between the values at these points gives the PV Error C = 1.11803 + 0.559017 = 1.677 μm.*

b) Calculate the values for RMSt, RMSi and RMSa.

All of the RMS values ignore the tilt term. We can use the result that the wavefront variance is the sum of the squares of the Zernike coefficients.

$$RMSt = \sqrt{2.0^2 + 0.23^2 + 0.5^2} = 2.074\ \mu m$$

$$RMSi = \sqrt{0.23^2 + 0.5^2} = 0.550\ \mu m$$

$$RMSa = \sqrt{0.23^2} = 0.23\ \mu m$$

c) The ISO 10110 specification for the wavefront deformation of the system is 13/ 2(0.5) RMSa < 0.05; λ = 632.8 nm. Does the system meet the specification?

Both PV Errors and the RMSa exceed the specification for the wavefront deformation.