OPTI 415 Homework 3

1. The image below is of the Big Dipper (The original is on the website). Estimate the PSF of the imaging system used. If all of the stars are point sources, why are some stars bigger than others?


The stars are at a sufficient distance where they are all point sources. We can think of the object for this image as an array of delta functions of varying heights. The height of each delta function is related to the magnitude of the star. The PSF of the system will be approximately the image of one of the bright stars. The image essentially replaces each delta function with the PSF. As far as the width goes, the camera will have some black level below which everything appears black. For dimmer stars, the tails of the PSF get cut off and the diameter of the PSF appears narrower. See the drawing below.

Object   Image
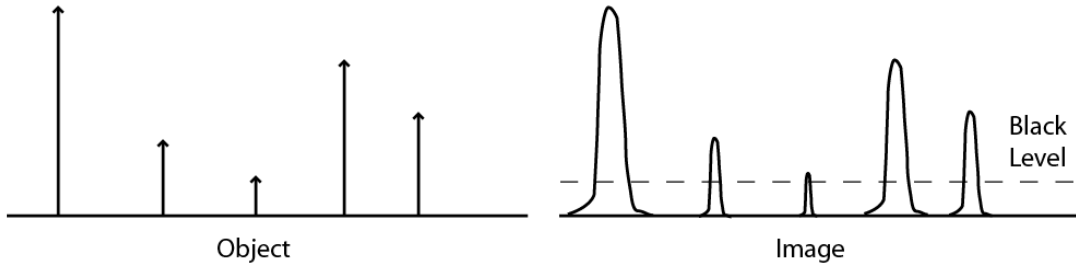
2. Suppose we have a wavefront error $W(\rho_x, \rho_y) = W_{222}h^2\rho^2 \cos^2\psi + W_{131}h\rho^3 \cos\psi$. What are

the components of the transverse ray error ε_x and ε_y? Create 500 random values of

$-1 \le \rho_x \le 1$ and $-1 \le \rho_y \le 1$. Exclude any points where $\rho_x^2 + \rho_y^2 > 1$. Plot ρ_x vs. ρ_y to see

how random your random number generator truly is. Create a spot diagram by plotting ε_x

vs. ε_y for these random pupil coordinates. Assume h = 1, W_{222} = −0.001 mm,

W_{131} = 0.001 mm, reference sphere radius R = 100 mm and the exit pupil diameter is 20

mm.

The wavefront error is given by $W(\rho_x, \rho_y) = -0.001h^2\rho_y^2 + 0.001h(\rho_x^2\rho_y + \rho_y^3)$. *The transverse*

ray error components are given by

$\varepsilon_x(\rho_x, \rho_y) = -0.02\rho_x\rho_y$ *and* $\varepsilon_y(\rho_x, \rho_y) = 0.02\rho_y - 0.01(\rho_x^2 + 3\rho_y^2)$.
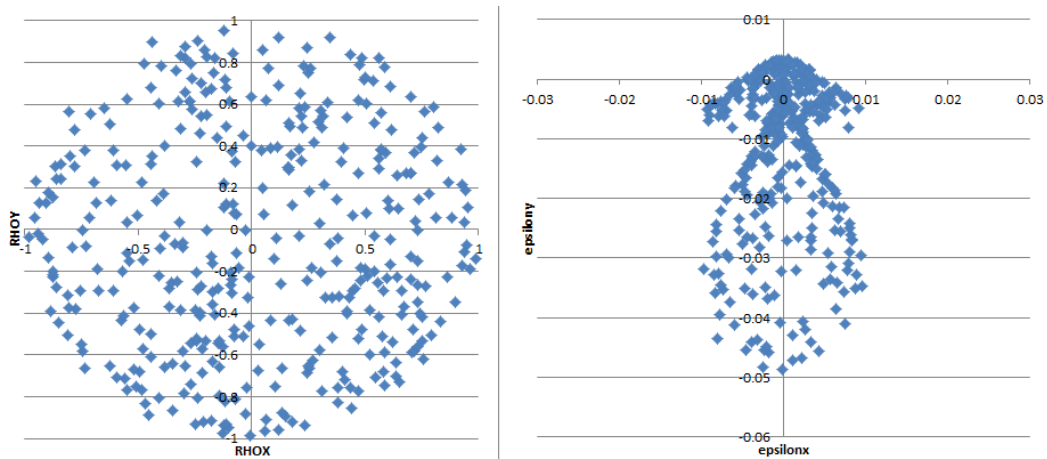
3. Calculate the Percent Distortion in the image below (original available on line). HINT: In class, we defined %Distortion=$\dfrac{y' - y'_p}{y'_p}$, where $y' - y'_p = \varepsilon_y$ (i.e. the transverse ray error for distortion) and $y'_p =$ paraxial image height. The original image has a 4:3 aspect ratio and $h = 1$ along the diagonal of the image.

Consider the upper right quadrant of the image as illustrated in the figure below. The dashed lines are the boundaries of the undistorted image. The distance d is the difference in height between the center top of the picture and the corner of the picture. I measured it in Photoshop and d=15 pixels. The distance y'_p is unknown, but we know that

$y' = \varepsilon_y + y'_p = 400$ pixels *since the original image is 640x480 making the full diagonal 800 pixels. Furthermore, the image has a 4:3 aspect ratio due to these pixel values. The full field*
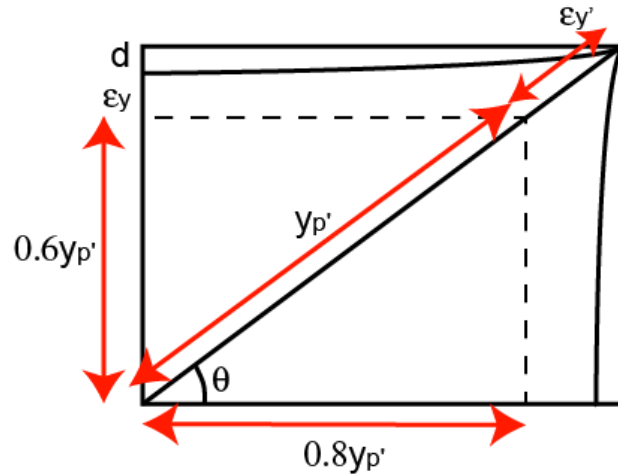
(h = 1) occurs along the diagonal and $\varepsilon_{y'}$ is the transverse ray error for h = 1. The transverse ray error ε_y occurs along the vertical direction for h = 0.6. For the illustration, we see that

$$\sin\theta = \frac{0.6y'_p}{y'_p} = \frac{d + \varepsilon_y}{\varepsilon_{y'}}.$$

The expressions for transverse ray error for distortion give

$$\varepsilon_y = -\frac{R}{r_{max}}W_{311}(0.6)^3 \text{ and}$$

$$\varepsilon_{y'} = -\frac{R}{r_{max}}W_{311}(1)^3 \Rightarrow \varepsilon_y = 0.216\varepsilon_{y'}$$

Plugging in to the expressions above leads to

$$\varepsilon_{y'} = \frac{d}{0.6 - 0.216} = 39.06 \text{ pixels } and \ y'_p = 360.96 \text{ pixels}.$$

Finally,

$$\%\text{Distortion} = \frac{y' - y'_p}{y'_p} = \frac{39.06}{360.96} = 0.0108 = 10.8\%$$

4. Write a program to remove the distortion from the image in question 1. Show your resulting image. Don't just download a distortion correction software from the web, but create and implement your own algorithm.

   From problem 3, we saw that

   $$\varepsilon_{y'} = 39.06 = -\frac{R}{r_{max}}W_{311}$$

Which means that the transverse ray error anywhere in the field is $\varepsilon_y = 39.06\text{h}^3$. The

basic algorithm for removing the distortion is as follows:

1. Create a new array to hold the undistorted image.

2. For each pixel in the new array calculate the Cartesian coordinates (x, y) and the

polar coordinates (r, θ) relative to the center of the array.

3. Calculate the normalized field coordinate $h = r / y'_p$, where $y'_p = 360.96$ pixels from

the first question.

4. Calculate $\varepsilon_y = 39.06\text{h}^3$.

5. If the location $(x + \varepsilon_y \cos\theta, y + \varepsilon_y \sin\theta)$ lies within the distorted image, then get the pixel

value at that location and set the pixel in the new array to this value.

Here's my image