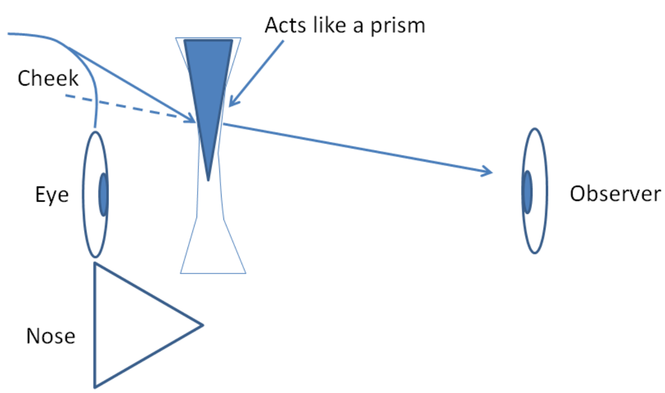
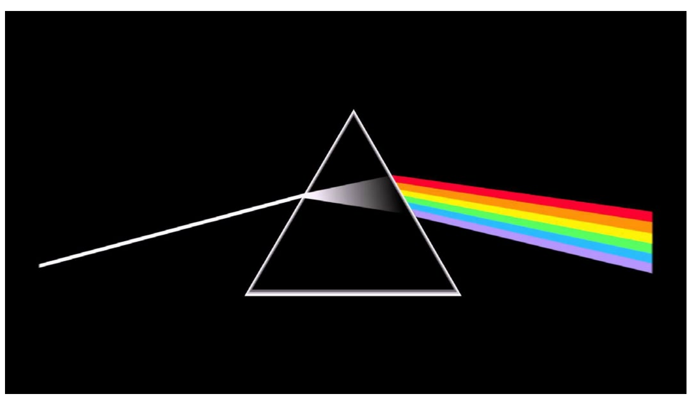
OPTI 415 Homework 2

1. Is this person below near-sighted or far-sighted? Justify your answer.

   The lens is a negative lens (meaning the person is near-sighted) since the side of his face appears to move in when looking through the glasses as compared to outside the glasses. The drawing below illustrates this effect.





2. Pink Floyd's The Dark Side of the Moon Album cover depicts an equilateral prism dispersing incident white light into a spectrum. The yellow band in the emerging spectrum is fairly realistic in terms of its angle of refraction leaving the prism. Unfortunately, the other colors do not obey the laws of optics, but we won't hold that against the designer Storm Thorgerson and artist George Hardie. With respect to the first surface normal, the angle of incidence of the white light is $\theta_1 = 45.3°$. With respect to the second surface normal, the

angle of refraction of the yellow light is $\theta_2' = -40.446°$. Using Snell's law and the fact that the internal angles satisfy $\theta_1' - \theta_2 = A$, where A is the apex angle of the prism, do the following:

(a) Find the refractive index of the prism.

Snell's law gives $sin\theta_1 = nsin\theta_1'$ *and* $nsin\theta_2 = sin\theta_2'$. *Using* $\theta_1' - \theta_2 = A$*, the second equation can be rewritten as*

$$sin\theta_2' = nsin(\theta_1' - A) = nsin\theta_1' cosA - ncos\theta_1' sinA.$$

Replacing the cosine and bringing the index n *into the radical gives*

$$sin\theta_2' = nsin\theta_1' cosA - sinA\sqrt{n^2 - n^2 sin^2\theta_1'}.$$

Using Snell's law from the first surface leads to

$$sin\theta_2' = sin\theta_1 cosA - sinA\sqrt{n^2 - sin^2\theta_1}.$$

Finally, solving for n *gives*

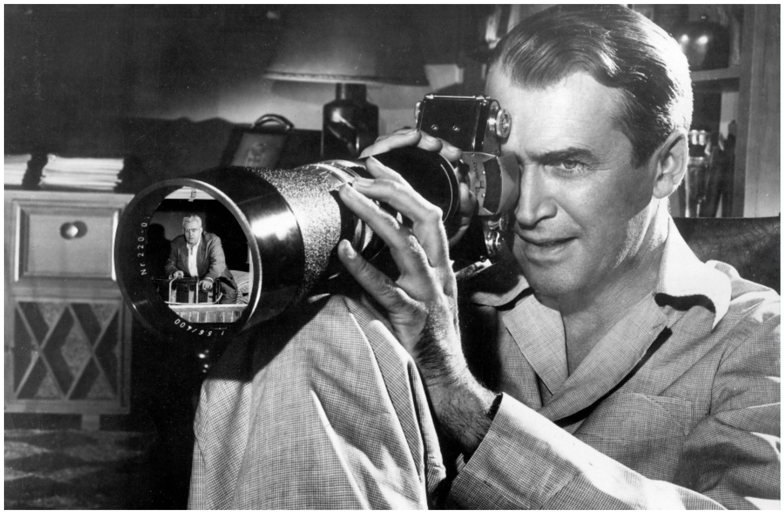
$$n = \left[\frac{(sin\theta_1 cosA - sin^2\theta_2')^2}{sin^2 A} + sin^2\theta_1\right]^{1/2}.$$

Since $A = 60°$ *for an equilateral prism, the refractive index is* $n = 1.36$.

(b) Find a suitable optical material that has this refractive index.

There are several fluids with this refractive index such as vodka (essentially ethyl a1cohol) and acetone.

3. In Alfred Hitchcock's movie *Rear Window*, Jimmy Stewart witnesses a murder in a neighboring apartment. In the scene shown below, he is using a 35 mm film camera, and a lens with a focal length $f = 400mm$ and $F/\# = 5.6$. Based on this information, answer the following questions:



(a) Estimate the resolution limit of the camera.

Assuming a wavelength of $\lambda = 0.5876\mu m$*, the Rayleigh criterion says the resolution limit if the system is diffraction limited is* $1.22\lambda\, F/\# = 4.0\mu m$.

(b) Estimate the horizontal and vertical full field of view?

The dimensions of the frame in 35mm file is 36mm x 24mm. The horizontal full field is approximately

$$\theta_h = \frac{36}{400}\, rad = 5.2°$$

The vertical full field of view is approximately

$$\theta_h = \frac{24}{400}\, rad = 3.4°$$

(c) What is the entrance pupil diameter of the camera?

$$EPD = \frac{f}{F/\#} = \frac{400mm}{5.6} = 71.43mm$$

(d) What is the power of the lens?

$$\Phi = \frac{1}{f} = \frac{1}{400mm} = 0.0025mm^{-1}$$

(e) Let's assume the lens is a telephoto design consisting of two *thin* lenses separated by a distance $t = 100mm$. The paraxial raytrace of the marginal and chief rays is shown below. Where is the aperture stop located?

The stop is located at the first lens since the chief ray height is zero at that point.

		Lens 1		Lens 2		Image
-phi		-0.005		0.005		
t			100		200	
y(marg)		35.71429		17.85714		0
u(marg)	0		-0.17857		-0.08929	
y(chief)		0		5.4125		21.65
u(chief)	0.054125		0.054125		0.081188	

(f) What is the distance from the front of the telephoto lens to the image plane?

The total distance is $100mm + 200mm = 300mm$.

(g) Where is the rear principal plane located relative to the first lens?

The rear principal plane is 400mm to the left of the image plane, so this means that it is located 100mm to the left of the first lens.

(h) What is the Lagrange Invariant of the system?

$$|u\bar{y} - \bar{u}y| = 1.93$$

4. An optical element consists of a spherical front surface with radii of curvature $R_1 = 42\ mm$ and a flat back surface. The back surface is located at the rear focal point of the first surface. The index of refraction of the element is 1.50.

(a) What is the thickness of the optical element?

The power of the front surface is

$$\phi_1 = \frac{n_1' - 1}{R_1} = \frac{0.5}{42\ mm} = \frac{n_1'}{f_1'},$$

so the focal length of the front surface, and therefore the thickness of the optical element, is $f_1' = 42\ mm\ \times 3 = 126\ mm.$

(b) Using the real raytracing equations developed in class, calculate where a ray with origin $\vec{r}_o = (0, 12.5, -10)$ and direction $\vec{r}_d = (0, 0, 1)$ intersects the back surface. For convenience, put the origin at the vertex of the first surface.

The sphere is then describes by its radius $R = 42\ mm$ *and the vertex is* $z_s = 0$. *The intersection coefficients are*

$$A = x_d^2 + y_d^2 + z_d^2 = 1$$

$$B = 2[x_d x_o + y_d y_o + z_d(z_o - R)] = 2(-10 - 42) = -104$$

$$C = x_o^2 + y_o^2 + (z_o - R)^2 - R^2 = 12.5^2 + (-52)^2 - 42^2 = 1096.25$$

and

$$t = \frac{-104 \pm \sqrt{10816 - 4(1096.25)}}{2}$$

so the two solutions are $t_1 = 11.9032$ *and* $t_2 = 92.0968$. *Since* t_1 *is smaller, the ray must hit the spherical surface here first. So, the point of intersection is*

$$\vec{r}_1 = \vec{r}_o + t_1\vec{r}_d = (0, 12.5, 1.9032).$$

The normal at this point is

$$\vec{n} = \left(\frac{x_1}{R}, \frac{y_1}{R}, \frac{z_1 - (z_s + R)}{R}\right) = (0, 0.297619, -0.954685)$$

The magnitude of the normal vector is $\lceil\vec{n}\rceil = 1$, *so this is a unit vector*

$$\hat{n} = (0, 0.297619, -0.954685).$$

Next, it is useful to calculate the cosine of the angle of incidence of the ray. The is given by (Note: $\vec{r}_d = \hat{r}_d$, the direction vector is already a unit vector.

$$cos\theta = -\hat{r}_d \cdot \hat{n} = 0.954685.$$

The ratio of the refractive indices on either side of the interface is useful as well.

$$\frac{n_1}{n_2} = \frac{1}{1.5} = 0.666667$$

Snell's law in vector form is

$$\vec{r}_d' = \left(\frac{n_1}{n_2}\right)\hat{r}_d + \left[\left(\frac{n_1}{n_2}\right)cos\theta - \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (1 - cos^2\theta)}\right]\hat{n} = (0, -0.10228, 0.994756).$$

Also, double check that this is a unit vector. The new ray after refraction is now given by

$$\vec{r}_1 + t\vec{r}_d$$

and we must find the value t that causes this ray to intersect the planar back surface of the element. From part (a), the vertex of the back surface is located at $\vec{r}_p = (0, 0, 126\ )$. The plane is perpendicular to the optical axis, so is unit normal is $\hat{n} = (0, 0, 1)$. Planes are given by the formula

$$Ax + By + Cz + D = 0$$

where A, B, and C are the components of the normal vector, and

$$D = -\hat{n} \cdot \vec{r}_p = -126,$$

so the plane in this case is given by $z = 126$. The intersection of the ray with this plane is given by

$$t = -\frac{Ax_o + By_o + Cz_o + D}{Ax_d + By_d + Cz_d} = -\frac{1.9032 - 126}{0.994756} = 124.751.$$

The intersection point on the back surface is given by

$$\vec{r}_1 + t\vec{r}_d = (0, 12.5, 1.9032) + 124.751(0, -0.10228, 0.994756)$$

$$= (0, -0.25958, 126)$$

So, the ray strikes the second surface about 260 microns below the optical axis.

(c) The incident ray is parallel to the optical axis, but doesn't intersect the optical axis at the back surface. Why?

The front surface introduces spherical aberration, so the ray doesn't focus to the paraxial focus located at the vertex of the back surface.