OPTI 435/535 Midterm Due: 11:59pm November 7, 2021

Do all four problems and upload to D2L.

1. You are tasked with designing a near visual acuity test where letters are displayed on the screen of an iPad and the user responds by saying which letter is present. The letters should cover a LogMAR acuity ranging from $-0.1 \leq VA \leq 1.0$ in steps of 0.1 LogMAR. The test is designed so that the iPad is held 18" from the observer.

(a) What are the equivalent Snellen visual acuity steps need for this test?
See the chart in part (c). The LogMAR acuity LA is given by

$$LA = log_{10}\left(\frac{1}{S}\right)$$

where S is the Snellen fraction. Solving this for S gives

$$S = 10^{-LA}$$

To get the denominator of the Snellen fraction, calculate $20/S$. *For example,* $LA = -0.1$ *gives*

$$S = 10^{-(-0.1)} = 1.259$$

The denominator is $20/1.259 = 15.89 \cong 16$, *so the Snellen acuity is* $20/16$.

(b) What are the angular subtense in arcmin for each of these letter sizes?
See the chart in part (c). The angular subtense for the 20/20 letter is 5 arcmin, and scales linearly with the inverse of the Snellen fraction.

$$Angular\ Subtense = \frac{5arcmin}{S}$$

For example, the Snellen acuity $20/40$ *gives* $S = 0.5$, *and*

$$Angular\ Subtense = \frac{5arcmin}{0.5} = 10arcmin.$$

(c) What are the required sizes for each of these letters on the iPad screen?

The size of the letter on the iPad is given by $18tan\theta$ *in units of inches, and* θ *equal to angular subtense of the letter. The table below provides the results for parts (a)-(c).*

LogMAR	Snellen	Arcmin	iPad@18"(in)
-0.1	16	4	0.02094396
0	20	5	0.026179957
0.1	25	6.25	0.03272496
0.2	32	8	0.041887978
0.3	40	10	0.052360025
0.4	50	12.5	0.065450135
0.5	63	15.75	0.082467384
0.6	80	20	0.104720937
0.7	100	25	0.130902001
0.8	126	31.5	0.16493823
0.9	159	39.75	0.208139789
1	200	50	0.26181785

(d) If the minimum letter needs to be 25 pixels high to properly display the character, what does the screen resolution need to be in units of pixels per inch (ppi)?

To get the required resolution divide the number of pixels required by the size of the smallest letter.

$$\frac{25\ pixels}{0.021\ in} = 1190ppi$$

(e) For your answer to part (d), it this screen resolution feasible with modern iPad displays?

A quick Google search shows that the resolution of an iPad screen in 264ppi, so the current device cannot meet the requirement. In other words, the current resolution can only display the smallest character with $264ppi \times 0.021" \cong 5pixels$.

2. The wavefront error for an eye is given by $W(\rho,\theta) = W_{40}\rho^4 + W_{31}\rho^3 cos\theta$. Expand this wavefront into an equivalent series of Zernike polynomials.

The Zernike terms $Z_4^0(\rho,\theta)$ *and* $Z_3^1(\rho,\theta)$ *have terms dependent upon* ρ^4 *and* $\rho^3 cos\theta$*, so they need to be included. In addition,* $Z_2^0(\rho,\theta)$ *and* $Z_0^0(\rho,\theta)$*, will be needed to offset the* ρ^2 *and constant terms in* $Z_4^0(\rho,\theta)$*. Similarly,* $Z_1^1(\rho,\theta)$ *is needed to offset the* $\rho cos\theta$ *term in* $Z_3^1(\rho,\theta)$*. So,*

$$W(\rho,\theta) = W_{040}\rho^4 + W_{131}\rho^3 cos\theta$$
$$= a_{00}Z_0^0(\rho,\theta) + a_{20}Z_2^0(\rho,\theta) + a_{40}Z_4^0(\rho,\theta) + a_{11}Z_1^1(\rho,\theta) + a_{31}Z_3^1(\rho,\theta)$$

$$W_{040}\rho^4 + W_{131}\rho^3 cos\theta$$
$$= a_{00} + a_{20}\sqrt{3}(2\rho^2 - 1) + a_{40}\sqrt{5}(6\rho^4 - 6\rho^2 + 1) + a_{11}2\rho cos\theta$$
$$+ a_{31}\sqrt{8}(3\rho^3 - 2\rho)cos\theta$$

Grouping like terms

$$W_{040}\rho^4 + W_{131}\rho^3 cos\theta$$
$$= a_{00} - \sqrt{3}a_{20} + \sqrt{5}a_{40} + \left(2\sqrt{3}a_{20} - 6\sqrt{5}a_{40}\right)\rho^2 + 6\sqrt{5}a_{40}\rho^4$$
$$+ \left(2a_{11} - 2\sqrt{8}a_{31}\right)\rho cos\theta + 3\sqrt{8}a_{31}\rho^3 cos\theta$$

This gives us a system of equations

$$a_{00} - \sqrt{3}a_{20} + \sqrt{5}a_{40} = 0$$

$$2\sqrt{3}a_{20} - 6\sqrt{5}a_{40} = 0$$

$$6\sqrt{5}a_{40} = W_{040}$$

$$2a_{11} - 2\sqrt{8}a_{31} = 0$$

$$3\sqrt{8}a_{31} = W_{131}$$

Solving these gives

$$a_{31} = \frac{W_{131}}{3\sqrt{8}}$$

$$a_{40} = \frac{W_{040}}{6\sqrt{5}}$$

$$a_{11} = \sqrt{8}a_{31} = \frac{W_{131}}{3}$$

$$a_{20} = \frac{3\sqrt{5}}{\sqrt{3}}a_{40} = \frac{W_{040}}{2\sqrt{3}}$$

$$a_{00} = \sqrt{3}a_{20} - \sqrt{5}a_{40} = \frac{W_{040}}{2} - \frac{W_{040}}{6} = \frac{W_{040}}{3}$$

3. Measure the wavefront $W(x, y) = -0.002(x^2 - y^2)$ with a Shack Hartmann sensor for a 4 mm diameter pupil. Suppose the lenslets of the array have a focal length of 24 mm and a spacing of 1 mm.

(a) What does the *unaberrated* Shack Hartmann pattern look like?

See part (c). The unaberrated spots are formed directly behind each of the lenslets. With a 4mm diameter pupil and a spacing of 1mm between lenslets, only 13 spots are formed.

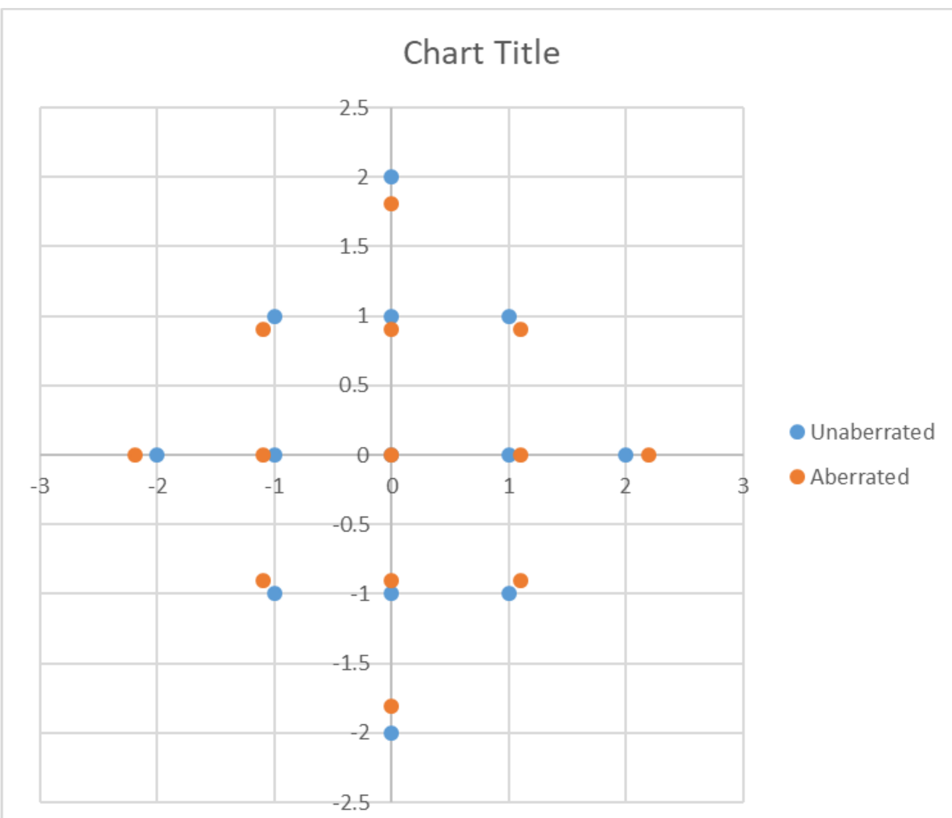
(b) What are the focal spot shifts Δx and Δy for each spot?

The table below shows the coordinates of each of the lenslets, as well as the spot displacements calculated from $\Delta x = -f\ dW/dx$ *and* $\Delta y = -f\ dW/dy$.

x	y	Δx	Δy	x+Δx	y+Δy
0	2	0	0.192	0	2.192
-1	1	0.096	0.096	-0.904	1.096
0	1	0	0.096	0	1.096
1	1	-0.096	0.096	0.904	1.096
-2	0	0.192	0	-1.808	0
-1	0	0.096	0	-0.904	0
0	0	0	0	0	0
1	0	-0.096	0	0.904	0
2	0	-0.192	0	1.808	0
-1	-1	0.096	-0.096	-0.904	-1.096
0	-1	0	-0.096	0	-1.096
1	-1	-0.096	-0.096	0.904	-1.096
0	-2	0	-0.192	0	-2.192

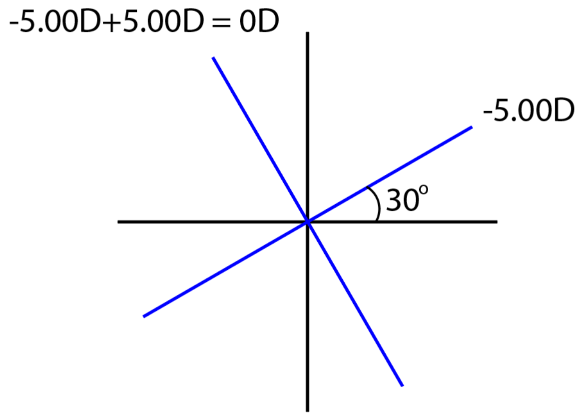
(c) What does the Shack Hartmann pattern look like for the wavefront?

The spots are expanded in the x-direction and compressed in the y-direction.



4. For a lens with the prescription $-5.00 / +5.00 \times 30°$, do the following:

   (a) Draw the principal meridia and label the powers along each of the principal meridia



(b) Convert the prescription to minus cylinder form

The new spherical power is the sum of the old sphere and cylinder powers, so $-5.00D + 5.00D = 0D$.

The new cylinder power is the negative of the old cylinder power, so $-(+5.00D) = -5.00D$.

Finally, the axis is rotated by $90°$*, so the new axis is* $120°$. *The prescription in minus cylinder form is* $0D/{-5.00D} \times 120°$.

(c) What type of lens is this?

Negative cylinder lens since no spherical power in the design.