OPTI 435/535 Homework #4 Due: November 23, 2021

Undergrads do problems 1 through 3
Grads do all four problems

1. Show that a biconic surface can be written as:

$$(sag)\ f(r) = \frac{r^2/R_{eff}}{1 + \left[1 - \left(K_{eff} + 1\right) r^2/R_{eff}^2\right]^{1/2}}$$

where R_{eff} and K_{eff} are the effective Radius of Curvature and effective Conic Constant, respectively along a given meridian. What are the expressions for R_{eff} and K_{eff}?

The sag of a biconic surface is given by

$$(sag) f(x,y) = \frac{x^2/R_x + y^2/R_y}{1 + \left[1 - (K_x + 1)x^2/R_x^2 - (K_y + 1)y^2/R_y^2\right]^{1/2}}.$$

Converting to polar coordinates with $x = r\cos\theta$ *and* $y = r\sin\theta$ *gives*

$$(sag) f(r,\theta) = \frac{r^2\left(\cos^2\theta/R_x + \sin^2\theta/R_y\right)}{1 + \left[1 - (K_x + 1)r^2\cos^2\theta/R_x^2 - (K_y + 1)r^2\sin^2\theta/R_y^2\right]^{1/2}}.$$

Comparing the numerators in this expression and the target expression, it is clear that

$$R_{eff} = \frac{1}{\left(\cos^2\theta/R_x + \sin^2\theta/R_y\right)} = \frac{R_x R_y}{R_y\cos^2\theta + R_x\sin^2\theta}.$$

Comparing the denominators in the sag expression and the target expression gives

$$\left(K_{eff} + 1\right)/R_{eff}^2 = (K_x + 1)\cos^2\theta/R_x^2 + (K_y + 1)\sin^2\theta/R_y^2,$$

which simplifies to

$$K_{eff} = (K_x + 1)\frac{R_{eff}^2\cos^2\theta}{R_x^2} + (K_y + 1)\frac{R_{eff}^2\sin^2\theta}{R_y^2} - 1$$

2. An astigmatic cornea can be approximated as a biconic surface. Suppose the radius of curvature is 7.8 mm along the vertical meridian and 8.0 mm along the horizontal meridian, and the conic constant is –0.25 in all directions. Create color maps of the axial and instantaneous powers (in diopters) of this surface (NOTE: You can use the results of question 1 and just determine the

radial derivatives here). In addition, plot the difference in elevation (or sag) in mm between this surface and a sphere with a radius of curvature of 7.899 mm.

For the values defined above, $R_x = 8; R_y = 7.8; K_x = K_y = -0.25$. *The first and second radial derivatives of the sag expression are also needed:*

$$\frac{\partial f}{\partial r} = \frac{r}{\left[R_{eff} - \left(K_{eff}+1\right)r^2\right]^{1/2}} \quad and \quad \frac{\partial^2 f}{\partial r^2} = \frac{R_{eff}^2}{\left[R_{eff} - \left(K_{eff}+1\right)r^2\right]^{3/2}}.$$

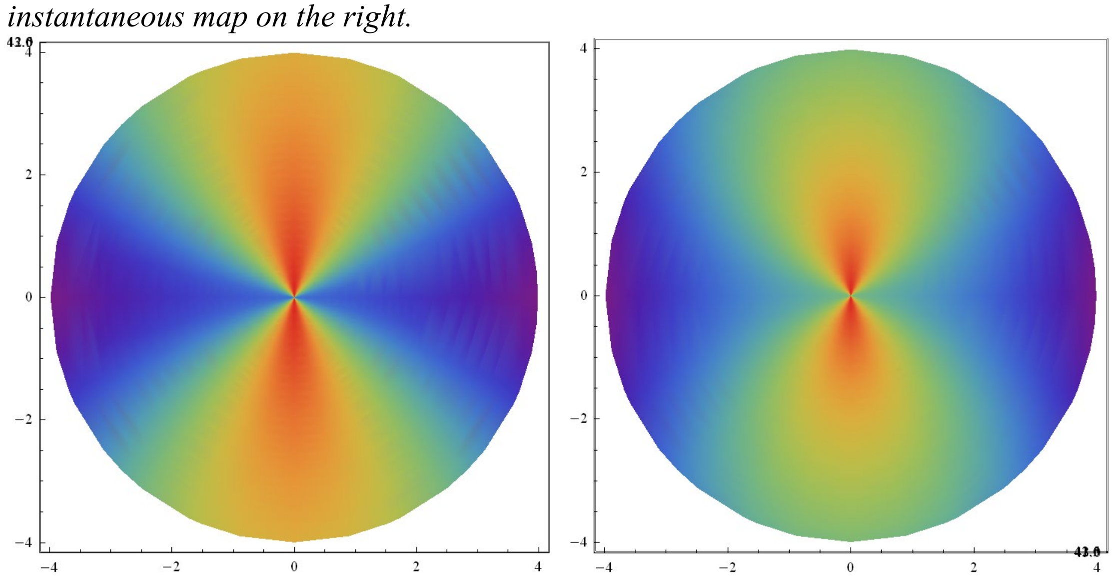
From the definition of axial power

$$\phi_a = \frac{\left(n_k - 1\right)\frac{\partial f}{\partial r}}{r\sqrt{1+\left(\frac{\partial f}{\partial r}\right)^2}} = \frac{\left(n_k - 1\right)}{\left[R_{eff}^2 - K_{eff} r^2\right]^{1/2}}.$$

From the definition of instantaneous power

$$\phi_i = \frac{\left(n_k - 1\right)\frac{\partial^2 f}{\partial r^2}}{\left[1+\left(\frac{\partial f}{\partial r}\right)^2\right]^{3/2}} = \frac{\left(n_k - 1\right)R_{eff}^2}{\left[R_{eff}^2 - K_{eff} r^2\right]^{3/2}}$$

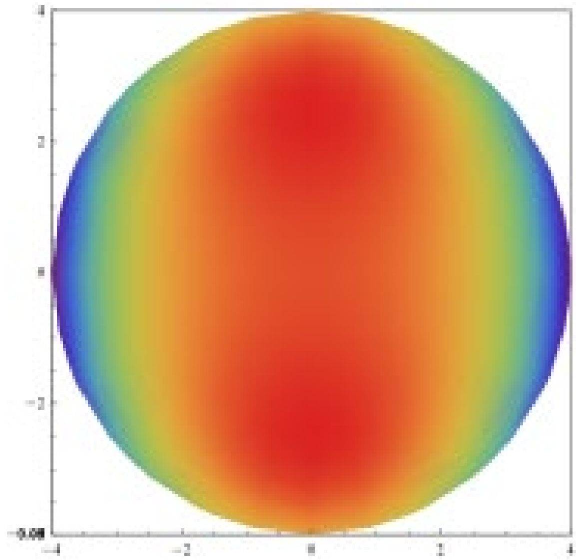
Below are maps of the axial and instantaneous power. The colors range from 41.86D (violet) to 43.26D (Red) for the axial map on the left and from 41.21D (violet) to 43.26D (Red) for the instantaneous map on the right.



Finally, we look at the sag relative to a sphere of radius 7.899 mm. Plotting

$$\text{sag difference} = \frac{r^2/R_{eff}}{1+\left[1-\left(K_{eff}+1\right)r^2/R_{eff}^2\right]^{1/2}} - \frac{r^2/7.899}{1+\left[1-r^2/(7.899)^2\right]^{1/2}}$$

gives



3. Suppose a myopic person with a prescription of -3.00D wants to get LASIK. When they are evaluated, their anterior corneal radius is measured to be 8.00 mm. What does the radius of the cornea need to be after surgery to correct their refractive error? Based on the Munnerlyn formula, how deep is the ablation for a treated area 6 mm in diameter?

Before surgery, the cornea has a power of

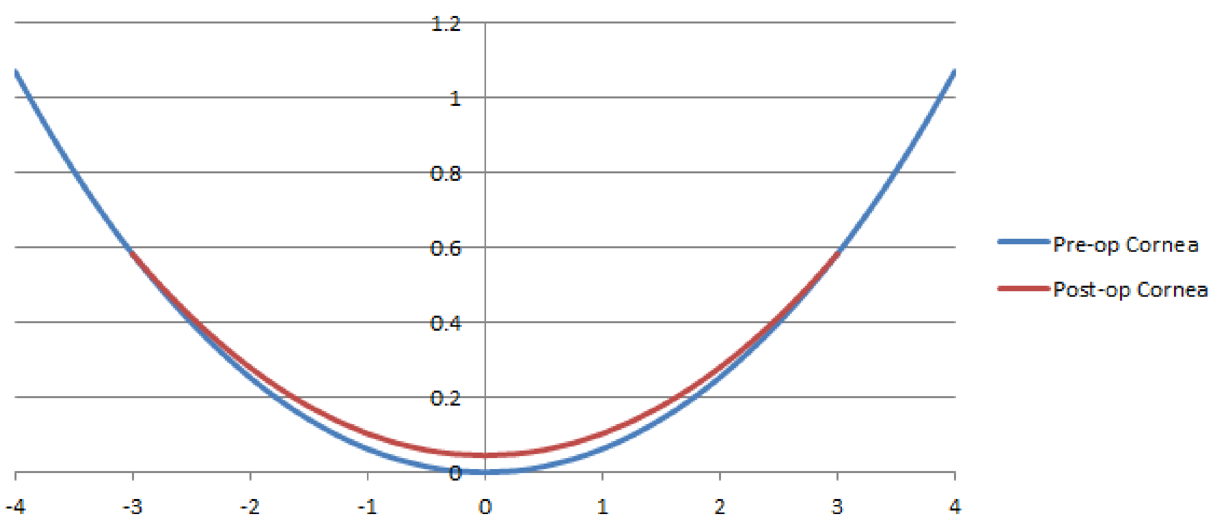
$$\phi_{pre} = 1000\frac{0.377}{8.00mm} = 47.125D$$

The surgery needs to reduce this by 3 diopters to correct for the myopia. The post-operative corneal power should therefore be 44.125D. This gives a post-operative radius of

$$R = 1000\frac{0.3375}{44.125D} = 8.544mm$$

These two spherical surfaces must be continuous at the edge of the ablation zone, which requires the central depth of the ablation to be about 39.7 μm. Also, the approximation

$$\frac{OZ^2}{3}\Delta\phi = 36\ \mu\text{m}$$

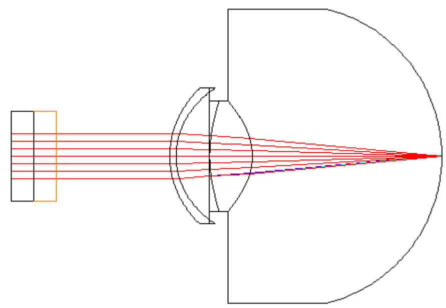


***************************Grads Only**

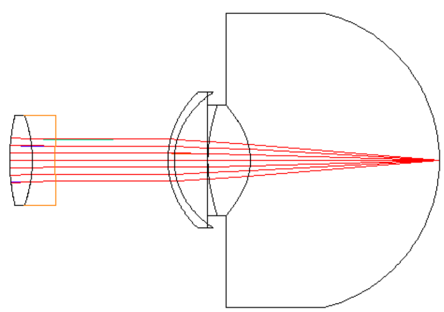
4. Design a doublet corrects the longitudinal chromatic aberration of the eye. The doublet should have zero power so that an object at infinity still focuses on the retina. However, the dispersions of the glasses in the doublet should be chosen to help cancel the chromatic aberration of the eye. Plot the residual chromatic aberration of the eye/doublet system.

Normally, we design achromatic doublets to have a positive power and have a blue and red wavelength focus at the same point. Here, the doublet design varies slightly in that we want to introduce a specific amount of chromatic aberration to compensate for what is in the eye and the power of the lens is zero so that as a person looks through it, distant rays still focus on the retina. There are an infinite number of solutions, but I'll go through how I approached the problem.

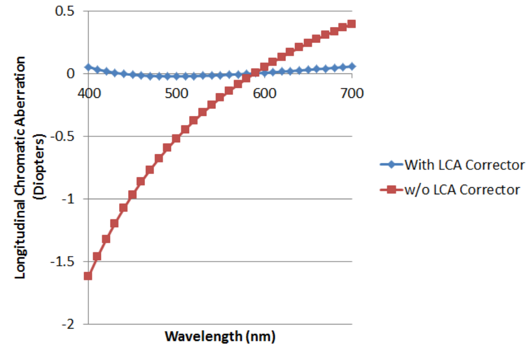
First, let's pick high and low dispersion glasses for the doublet. I chose BK7 and F5 since they are common glasses. Starting with the Arizona Eye in Zemax, I added two parallel plates of these materials, each 2 mm thick in front of the eye as shown below.



Next, I set the Zemax model wavelengths to 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7 μm to cover the visible region. I created a merit function to target the y height of a paraxial ray on the retina to be zero for each of these wavelengths. I set each surface of the plates to be variable. The plates are assumed in contact, so the middle surface will have the same shape ultimately for each lens. Optimizing this arrangement gives



where the radii of the doublet are 17.290, -11.627 and 24.752 mm respectively. The longitudinal chromatic aberration of eye with and without the doublet is shown below.



0.5
0
-0.5
-1
-1.5
-2
400
500
600
700
Longitudinal Chromatic Aberration (Diopters)
Wavelength (nm)
With LCA Corrector
w/o LCA Corrector