Homework 3 Solutions

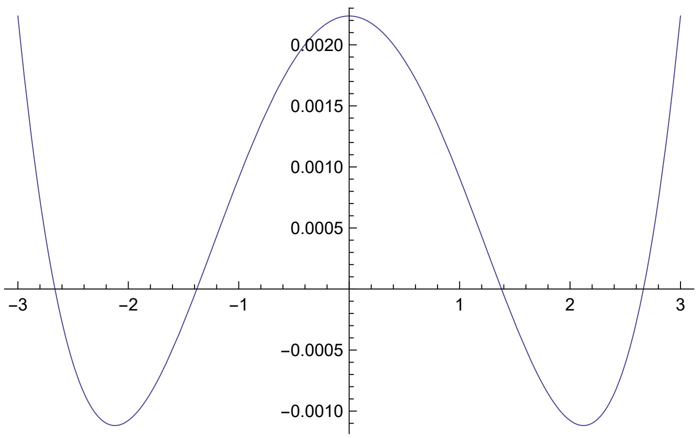
Problem 1

If we convert the wavefront to unnormalized coordinates (r,θ), then the wavefront is given by

$$W\left(\frac{r}{3}, \theta\right) = 0.001\, Z_4^0\left(\frac{r}{3}, \theta\right) = 0.001\ \sqrt{5}\ \left(6\left(\frac{r}{3}\right)^4 - 6\left(\frac{r}{3}\right)^2 + 1\right)$$

Plotting the wavefront gives

```
Plot[(Sqrt[5] / 1000) * (6 * (r / 3)^4 - 6 * (r / 3)^2 + 1), {r, -3, 3}]
```



We expect for a smaller diameter pupil that the wavefront remains unchanged, just the edges get clipped. Next, we want to normalize the coordinate system to the new pupil radius $r_{max} = 1.5$.

```
W = Apart[(Sqrt[5] / 1000) * (6 * (r / 3)^4 - 6 * (r / 3)^2 + 1) /. r → (3 / 2) * ρ]
```

$$\frac{1}{200\ \sqrt{5}} - \frac{3\,\rho^2}{400\ \sqrt{5}} + \frac{3\,\rho^4}{1600\ \sqrt{5}}$$

As in problem 1, this is a rotationally symmetric function with a maximum power of ρ of 4. We can therefore represent the wavefront as

$$W_{40} = c_{00}\, Z_0^0 + c_{20}\, Z_2^0 + c_{40}\, Z_4^0 = \frac{1}{200\ \sqrt{5}} - \frac{3\,\rho^2}{400\ \sqrt{5}} + \frac{3\,\rho^4}{1600\ \sqrt{5}}$$

$$W_{40} = c_{00} + c_{20}\ \sqrt{3}\ \left(2\,\rho^2 - 1\right) + c_{40}\ \sqrt{5}\ \left(6\,\rho^4 - 6\,\rho^2 + 1\right) = \frac{1}{200\ \sqrt{5}} - \frac{3\,\rho^2}{400\ \sqrt{5}} + \frac{3\,\rho^4}{1600\ \sqrt{5}}$$

Equating like powers of ρ gives

```
Solve[{c00 - c20 * Sqrt[3] + c40 * Sqrt[5] == 1 / (200 * Sqrt[5]),
  2 * Sqrt[3] * c20 - 6 * Sqrt[5] * c40 == -3 / (400 * Sqrt[5]),
  6 * Sqrt[5] * c40 == 3 / (1600 * Sqrt[5])}, {c00, c20, c40}]
```

$$\left\{\left\{c00 \to \frac{3}{1600\sqrt{5}},\ c20 \to -\frac{3\sqrt{\frac{3}{5}}}{3200},\ c40 \to \frac{1}{16000}\right\}\right\}$$

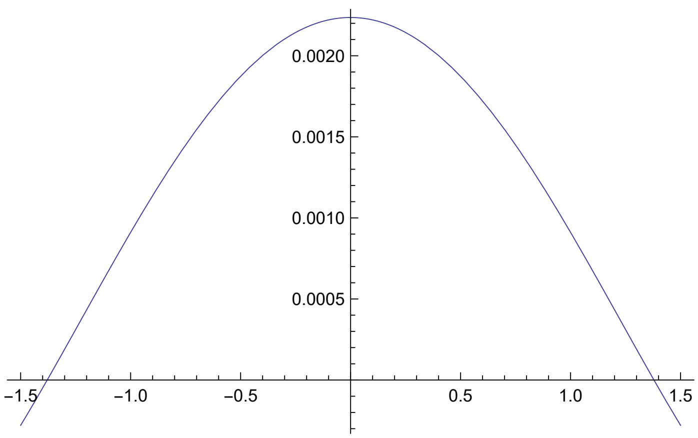
```
N[%]
```

{{c00 → 0.000838525, c20 → -0.000726184, c40 → 0.0000625}}

$$W = \text{Simplify}\left[c00 + c20\sqrt{3}\left(2\rho^2 - 1\right) + c40\sqrt{5}\left(6\rho^4 - 6\rho^2 + 1\right) /. \left\{c00 \to \frac{3}{1600\sqrt{5}},\ c20 \to -\frac{3\sqrt{\frac{3}{5}}}{3200},\ c40 \to \frac{1}{16000},\ \rho \to (2/3) * r\right\}\right]$$

$$\frac{27 - 18 r^2 + 2 r^4}{5400\sqrt{5}}$$

```
Plot[W, {r, -3 / 2, 3 / 2}]
```



Problem 2

To use astigmatic decomposition, first the prescription written as S / C x θ needs to be converted to spherical equivalent, M, and two crossed cylinders, J0 and J45, where $M = S + \frac{1}{2}C$, $J0 = \frac{-C}{2}\cos(2\theta)$, and $J45 = \frac{-C}{2}\sin(2\theta)$

In[26]:=
```
header = {"S", "C", "Axis", "M", "J0", "J45"};
rx1 = {-3.00, 1.00, 30};
rx2 = {-2.25, 2.00, 50};
ad1 = {rx1[[1]] + 0.5 * rx1[[2]],
   -0.5 * rx1[[2]] * Cos[2 * rx1[[3]] * π / 180.], -0.5 * rx1[[2]] * Sin[2 * rx1[[3]] * π / 180.]};
ad2 = {rx2[[1]] + 0.5 * rx2[[2]], -0.5 * rx2[[2]] * Cos[2 * rx2[[3]] * π / 180.],
   -0.5 * rx2[[2]] * Sin[2 * rx2[[3]] * π / 180.]};
adTotal = ad1 + ad2;
tableData = {header, Join[rx1, ad1], Join[rx2, ad2],
   {"", "", "", "", "", ""}, Join[{"", "", "Total"}, adTotal]};
Grid[
 tableData]
```

Out[33]=

S	C	Axis	M	J0	J45
-3.	1.	30	-2.5	-0.25	-0.433013
-2.25	2.	50	-1.25	0.173648	-0.984808
		Total	-3.75	-0.0763518	-1.41782

Next, the spherical equivalents and the crossed cylinders and summed to give a net spherical equivalent M = -3.75 and net crossed cylinders J0 = -0.0763518 and J45 = -1.41782. Finally, these values are converted back to S / C x θ form.

In[34]:=
```
m = adTotal[[1]];
j0 = adTotal[[2]];
j45 = adTotal[[3]];
cyl = 2 * Sqrt[j0^2 + j45^2];
sph = m - 0.5 * cyl;
axis = -ArcTan[j45, 0.5 * cyl + j0];
axis = (180 / π) * axis;
If[axis ≤ 0, axis = axis + 180,];
Print[sph, " / ", cyl, " x ", axis, "°"]
```

```
-5.16987 / 2.83975 x 43.4588°
```

Problem 3

Part (a)

The Tscherning ellipse describes solutions for the front surface power $\phi 1$ given the total power ϕ of the spectacle lens, the refractive index of the spectacle lens material (n = 1.4914 for PMMA), and the distance between the spectacle and the center of rotation of the eye (27 mm in this case). Using these values and solving for $\phi 1$ gives

```
n = 1.4914;
ϕ = 4;
q = 0.027;
```

```
Solve[(ϕ1^2) * (n + 2) - ϕ1 * ((2 / q) * (n^2 - 1) + ϕ * (n + 2)) + n * (ϕ + (n - 1) / q)^2 == 0, ϕ1]
{{ϕ1 → 11.2332}, {ϕ1 → 18.7412}}
```

So front surface powers of $\phi 1$ = 11.23D and $\phi 1$ = 18.74D are solutions for zero astigmatism. For question 4, it will also be useful to calculate the radii of these surfaces.

```
(n - 1) / 11.2332
(n - 1) / 18.7412
0.0437453

0.0262203
```

The corresponding front surface radii which give zero astigmatism are 43.75 mm and 26.22 mm.

Part (b)

The Tscherning ellipse derivation assume a thin lens. Consequently, the total lens power is just the sum of the front and back surface powers. From this, the back surface powers are

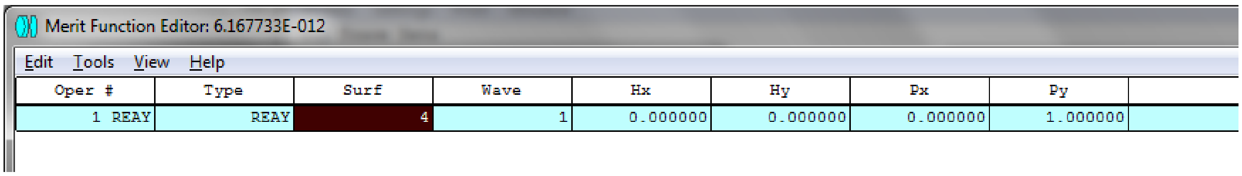
```
ϕ - 11.2332
ϕ - 18.7412
-7.2332

-14.7412
```

So back surface powers of $\phi 2$ = -7.23D and $\phi 2$ = -14.74D are needed for zero astigmatism.

Question 4

This problem seeks to create a +4.00 D spectacle lens that minimizes astigmatism. The basic setup is described in the problem. I added two features to the layout to facilitate the calculation. First, I made radius of the posterior spectacle lens surface a variable and second I added the merit function operand shown below, which drives the height of the on-axis marginal ray to zero at the image plane.



Oper #	Type	Surf	Wave	Hx	Hy	Px	Py
1 REAY	REAY	4	1	0.000000	0.000000	0.000000	1.000000

I then wrote a little macro to export the astigmatism values:

```
for R1=10,100,1
   radi(1)=R1
   optimize
   update all
   code=ocod("ASTI")
   astig=opev(code,0,1,0,1,0,0)
   print R1,",",astig
next
```

The for loop sets the radius of the anterior spectacle lens surface to R1 and then optimizes the system based on the merit function above. The optimization effectively sets the posterior radius to a value that keeps the lens power constant. The ocod and opev lines in the macro extract the Seidel astigmatism from Zemax and finally, the results are printed. A plot of the results is shown below. The approximate solutions for R1 are 29 mm and 48.5 mm which are close to the values in Q3.