
OPTI 435/535 Homework 2 Solutions

■ Problem 1

The wavefront error is given by

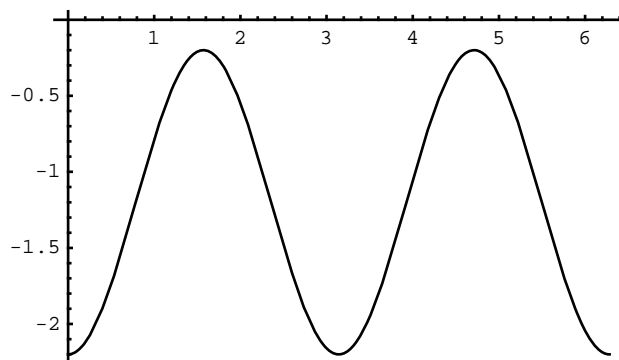
$$W = -0.001 * r^2 - 0.0005 * r^2 * \text{Cos}[2 * \theta] + 0.00005 * r^4$$

$$-0.001 r^2 + 0.00005 r^4 - 0.0005 r^2 \text{Cos}[2 \theta]$$

$$d\phi[r_, \theta_] = \text{Simplify}[(1000 / r) * D[W, r]]$$

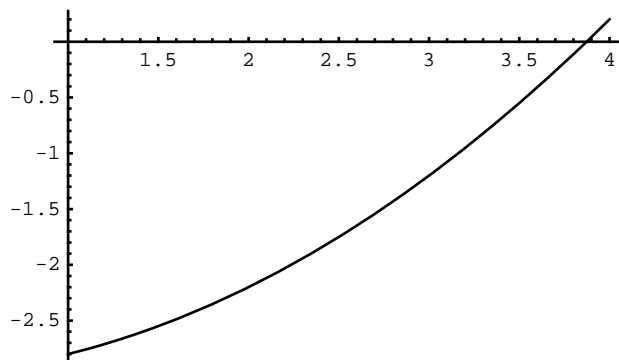
$$-2. + 0.2 r^2 - 1. \text{Cos}[2 \theta]$$

$$\text{Plot}[d\phi[2, \theta], \{\theta, 0, 2 * \pi\}]$$



- Graphics -

$$\text{Plot}[d\phi[r, 0], \{r, 1, 4\}]$$



- Graphics -

■ Problem 2

The sag of a paraboloid is given by

$$z = (x^2 + y^2) / (2 * R)$$

$$\frac{x^2 + y^2}{2R}$$

First, calculate the first and second derivatives

$$\begin{aligned} dzdx &= D[z, x] \\ dzdy &= D[z, y] \\ d2zdx2 &= D[dzdx, x] \\ d2zdy2 &= D[dzdy, y] \\ d2zdxdy &= D[dzdx, y] \end{aligned}$$

$$\begin{aligned} &x \\ &- \\ &R \\ &Y \\ &- \\ &R \\ &1 \\ &- \\ &R \\ &1 \\ &- \\ &R \\ &0 \end{aligned}$$

Next, calculate the First and Second Fundamental Forms

$$\begin{aligned} E0 &= 1 + dzdx^2 \\ F0 &= dzdx * dzdy \\ G0 &= 1 + dzdy^2 \\ L0 &= d2zdx2 / Sqrt[E0 * G0 - F0^2] \\ M0 &= d2zdxdy / Sqrt[E0 * G0 - F0^2] \\ N0 &= d2zdy2 / Sqrt[E0 * G0 - F0^2] \end{aligned}$$

$$1 + \frac{x^2}{R^2}$$

$$\frac{xy}{R^2}$$

$$1 + \frac{y^2}{R^2}$$

$$\frac{1}{R \sqrt{-\frac{x^2 y^2}{R^4} + \left(1 + \frac{x^2}{R^2}\right) \left(1 + \frac{y^2}{R^2}\right)}}$$

0

$$\frac{1}{R \sqrt{-\frac{x^2 y^2}{R^4} + \left(1 + \frac{x^2}{R^2}\right) \left(1 + \frac{y^2}{R^2}\right)}}$$

Calculate the Mean Curvature

$H0 = \text{Simplify}[(E0 * N0 + G0 * L0 + 2 * F0 * M0) / (2 * (E0 * G0 - F0^2))]$

$$\frac{2R^2 + x^2 + y^2}{2R^3 \left(\frac{R^2 + x^2 + y^2}{R^2} \right)^{3/2}}$$

$$H0 = \frac{2R^2 + x^2 + y^2}{2(R^2 + x^2 + y^2)^{3/2}}$$

$$\frac{2R^2 + x^2 + y^2}{2(R^2 + x^2 + y^2)^{3/2}}$$

Finally, evaluate the Mean Curvature at the origin

$\text{Simplify}[H0 /. \{x \rightarrow 0, y \rightarrow 0\}, \{R \in \text{Reals}, R > 0\}]$

$$\frac{1}{R}$$

which is the same as the curvature of a sphere with radius R

■ Problem 3

In the original image, the height of the storage container in pixels is 442 pixels. Furthermore, we can estimate the angular subtense θ of the storage container from $\tan\theta = \frac{8 \text{ feet}}{30 \text{ feet}}$

In[205]:= $\theta = N[(180 / \pi) * \text{ArcTan}[8 / 30]]$

Out[205]= 14.9314

This means each pixel subtends $\frac{14.9314}{442} = 0.0338$ degrees/pixel.

The ribs in the side of the storage container and the horizontal and vertical mortar lines subtend 52 pixels, 12 pixels and 30 pixels, respectively in the original image. In terms of spatial frequency, this gives

$$\text{Ribs} \rightarrow \frac{1}{52 \text{ pixels} * 0.0338 \text{ degrees / pix}} = 0.57 \text{ cyc / deg}$$

$$\text{Horizontal Mortar Lines} \rightarrow \frac{1}{12 \text{ pixels} * 0.0338 \text{ degrees / pix}} = 2.47 \text{ cyc / deg}$$

$$\text{Vertical Mortar Lines} \rightarrow \frac{1}{30 \text{ pixels} * 0.0338 \text{ degrees / pix}} = 0.99 \text{ cyc / deg}$$



■ Problem 4

Below is a zpl macro that calculates the spherical aberration of an eye model in Zemax. Similar macros can be constructed in other raytracing program, or the procedure can be done manually.

```

nsurf=nsurf()
n=indx(nsurf-1)
print "Pupil Diameter  LSA"
for i=1,8,1
    atyp=0
    aval=i
    getsystemdata 1
    pf=n*vec1(7)
    pv=pf-thic(nsurf-1)
    raytrace 0,0,0,1
    tanu=tang(acos(rayn(nsurf-1)))
    y=rayy(nsurf-1)
    z=rayz(nsurf-1)
    vm=y/tanu+z
    pm=pv+vm
    lsa=1000*n*(1/pm-1/pf)
    print,i,"      ",lsa
next

```

We can dissect the lines of this macro as follows:

Get the refractive index of the vitreous, `nsurf` calculates the total number of surfaces in the eye model and `indx` retrieves the refractive index

```

nsurf=nsurf()
n=indx(nsurf-1)

```

Loop through the various pupil diameters from 1 mm to 8 mm in steps of 1mm. This is equivalent to pupil radii from 0.5 mm to 4 mm.

```

for i=1,8,1
next

```

Set the aperture type to Entrance Pupil Diameter

```

atyp=0

```

Set the diameter of the Entrance Pupil to the loop variable *i*

aval=i

Get a copy of the System Data. This places a bunch of useful data regarding the system into an array called *vec1*

getsystemdata 1

The 7th item in the array is the focal length. Remember to scale by the index of the vitreous to get P'F'

pf=n*vec1(7)

Calculate the position of the rear principal plane relative to the posterior crystalline lens

pv=pf-thic(nsurf-1)

Raytrace a marginal ray. Its height is indirectly set by the current Entrance Pupil diameter

raytrace 0,0,0,1

Calculate the marginal ray angle and intersection coordinates at the posterior crystalline lens

tanu=tang(acos(rayn(nsurf-1)))

y=rayy(nsurf-1)

z=rayz(nsurf-1)

Calculate the position of the marginal focus relative to the posterior crystalline lens

vm=y/tanu+z

Calculate the position of the marginal focus relative to the rear principal plane. This is P'M'

pm=pv+vm

Finally, calculate and print the longitudinal spherical aberration

lsa=1000*n*(1/pm-1/pf)

print,i," ",lsa

For the Gullstrand-LeGrand Model, this macro gives the following result

```
LSA1 = {{1.0000, 0.0904},
        {2.0000, 0.3656},
        {3.0000, 0.8386},
        {4.0000, 1.5336},
        {5.0000, 2.4911},
        {6.0000, 3.7780},
        {7.0000, 5.5100},
        {8.0000, 7.9068}}

{{1., 0.0904}, {2., 0.3656}, {3., 0.8386},
 {4., 1.5336}, {5., 2.4911}, {6., 3.778}, {7., 5.51}, {8., 7.9068}}
```

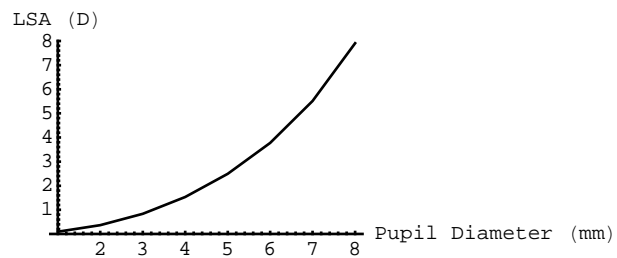
For the Arizona Eye Model, the macro gives

Pupil Diameter LSA

```
1.0000 0.0275
2.0000 0.1111
3.0000 0.2620
4.0000 0.4965
5.0000 0.8363
6.0000 1.3083
7.0000 1.9463
8.0000 2.7940
```

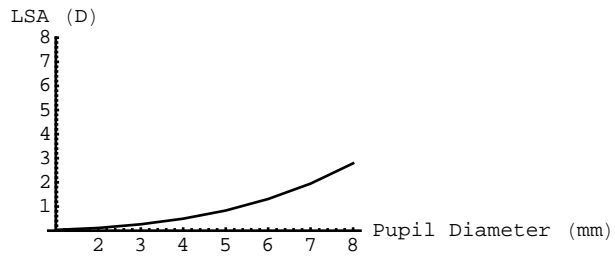
```
LSA2 = {{1.0000, 0.0275},
        {2.0000, 0.1111},
        {3.0000, 0.2620},
        {4.0000, 0.4965},
        {5.0000, 0.8363},
        {6.0000, 1.3083},
        {7.0000, 1.9463},
        {8.0000, 2.7940}}
{{1., 0.0275}, {2., 0.1111}, {3., 0.262}, {4., 0.4965},
 {5., 0.8363}, {6., 1.3083}, {7., 1.9463}, {8., 2.794}}
```

```
ListPlot[LSA1, PlotJoined → True,
  AxesLabel → {"Pupil Diameter (mm)", "LSA (D)"}, PlotRange → {0, 8}]
```



- Graphics -

```
ListPlot[LSA2, PlotJoined → True,  
  AxesLabel → {"Pupil Diameter (mm)", "LSA (D)"}, PlotRange → {0, 8}]
```



- Graphics -