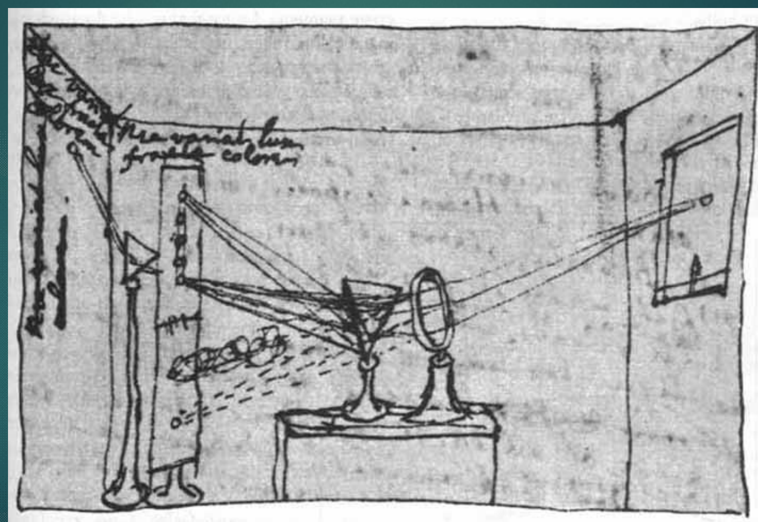
Newton's Color Experiments 1671



Newton's Conclusions

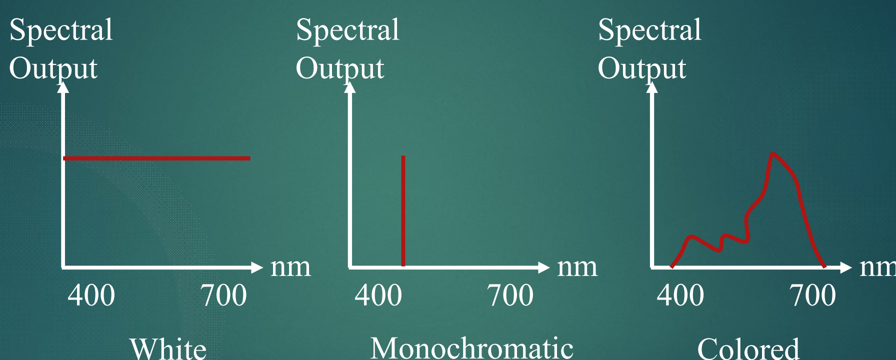
- White light has seven constituent components: red, orange, yellow, green, blue, indigo and violet.
- Dispersed light can be recombined to form white light.
- Magenta and purple can be obtained by combining only portions of the spectrum.

Color and Color Vision

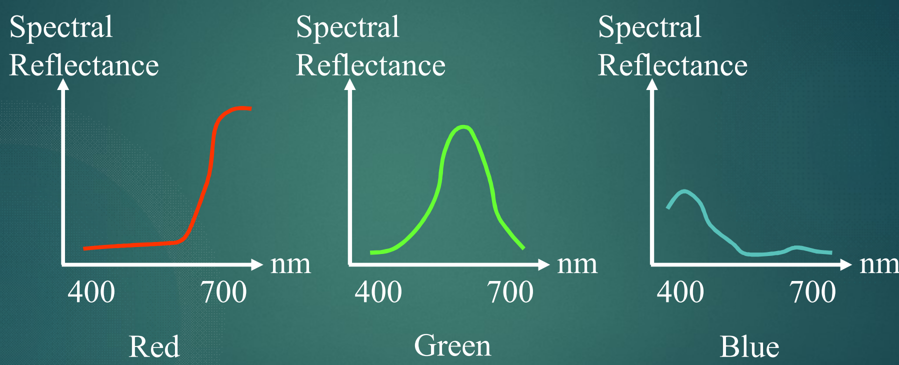
The perceived color of an object depends on four factors:

1. Spectrum of the illumination source
2. Spectral Reflectance of the object
3. Spectral response of the photoreceptors (including bleaching)
4. Interactions between photoreceptors

Light Sources

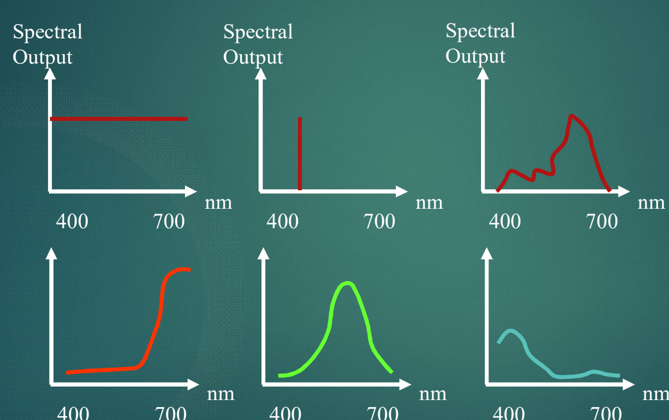


Pigments

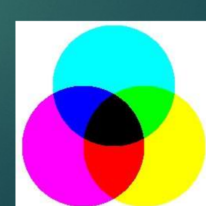


Subtractive Colors

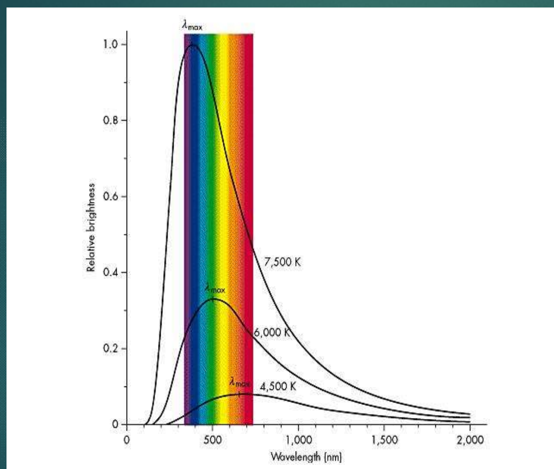
Pigments (e.g. paints and inks) absorb different portions of the spectrum



Multiply the spectrum of the light source by the spectral reflectivity of the object to find the distribution entering the eye.



Blackbody Radiation

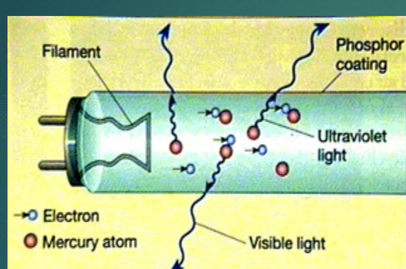


For lower temperatures, blackbodies appear red. As they heat up, the shift through the spectrum towards blue.

Our sun looks like a 6500K blackbody.

Incandescent lights are poor efficiency blackbodies radiators.

Gas-Discharge & Fluorescent Lamps



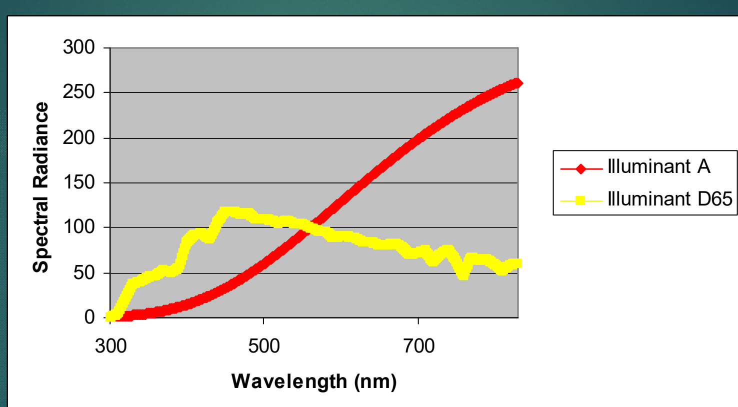
A low pressure gas or vapor is encased in a glass tube. Electrical connections are made at the ends of the tube. Electrical discharge excites the atoms and they emit in a series of spectral lines. We can use individual lines for illumination (e.g. sodium vapor) or ultraviolet lines to stimulate phosphors.

CIE Standard Illuminants

Illuminant A - Tungsten lamp looking like a blackbody of 2856 K
Illuminant B - (discontinued) Noon sunlight.
Illuminant C - (discontinued) Noon sunlight.

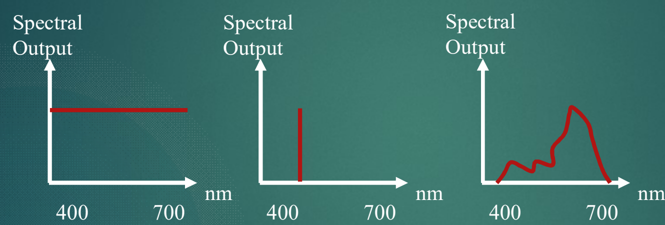
Illuminant D55 - (Occasionally used) 5500 K blackbody
Illuminant D65 - 6500 K blackbody, looks like average sunlight and replaces Illuminants B and C.
Illuminant D75 - (Occasionally used) 7500 K blackbody

Illuminants A and D_{65}

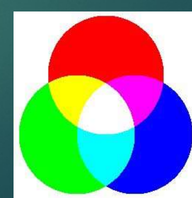


Additive Colors

Self-luminous Sources (e.g. lamps and CRT phosphors) emit different spectrums which combine to give a single apparent source.



Add the spectrums of the different light sources to get the spectrum of the apparent source entering the eye.



Color Models

- Attempt to put all visible colors in a ordered system.
- Mathematics based, art based and perceptually based systems.

RGB Color Model

A red, green and blue primary are mixed in different proportions to give a color

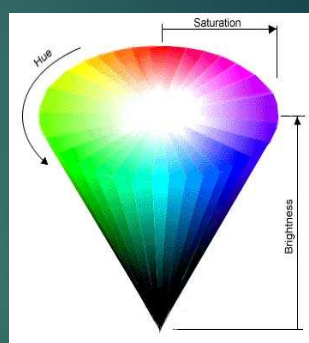
(1,0,0) is red
(0,1,0) is green
(0,0,1) is blue

24 bit color on computer monitors devote 8 bits (256 values) to each primary color (i.e. red can take on values (0…255) / 255)

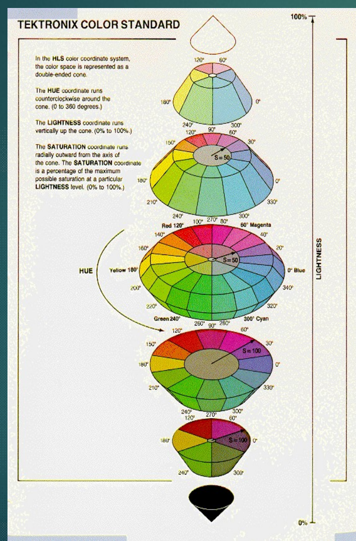
(1,1,1) is white
(0,0,0) is black

HSB (HSV) Color Model

Hue – color is represented by angle
Saturation – amount of white represented by radial position
Brightness (Value) – intensity is represented by the vertical dimension



HLS Color Model



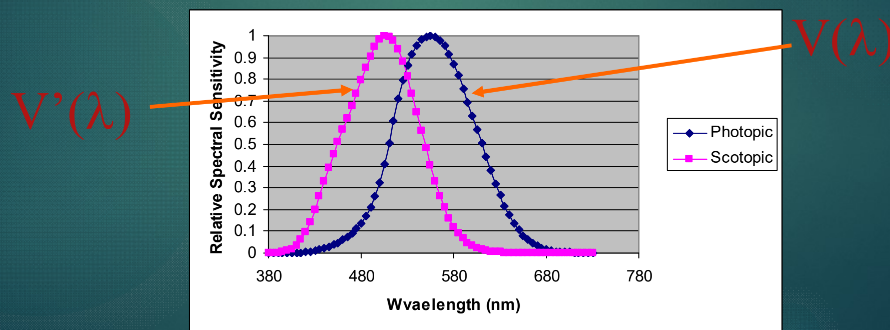
Hue – color is represented by angle
Lightness – intensity is represented by the vertical dimension
Saturation – amount of white represented by radial position

CIE

- 90 year old commission on color
- Recognized as the standards body for illumination & color.
- Has defined standard illuminants and human response curves.

Luminosity Functions

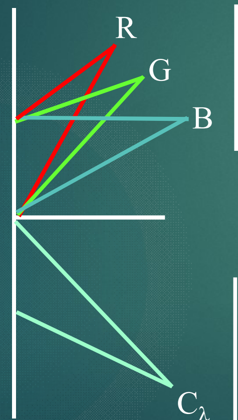
The spectral responses of the eye are called the luminosity functions. The The V(λ) curve (photpic response) is for cone vision and the V'(λ) curve (scoptopic response) is for rod vision. V(λ) was adopted as a standard by the CIE in 1924. There are some errors for λ<500 nm, that remain. The V'(λ) curve was adopted in 1951 and assumes an observer younger than 30 years old.



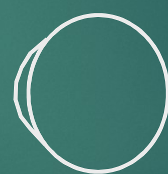
Purkinje Shift

Note how the brightness of reds and blues change with decreasing illumination. This is due to the sensitivity of the eye shifting from photopic to scoptic

Human Color Models



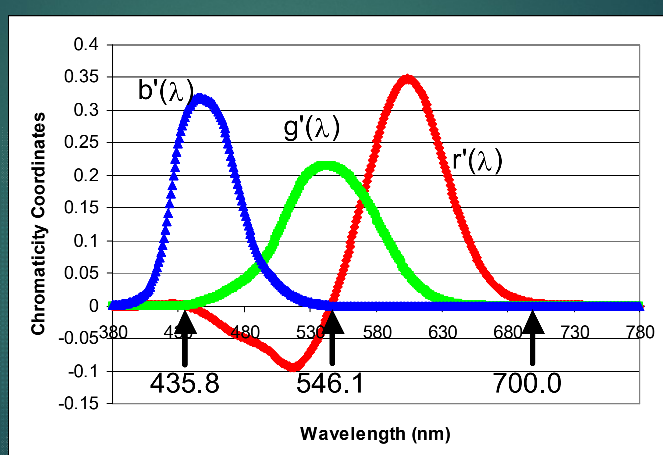
Observer adjusts the Luminances of R, G and B lights until they match C_λ



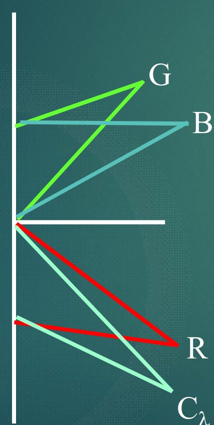
R = 700.0 nm
G = 546.1 nm
B = 435.8 nm

$$C_\lambda = r'(\lambda)R + g'(\lambda)G + b'(\lambda)B$$

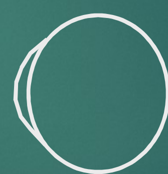
1931 CIE Color Matching Functions



Color Matching Functions



What does a negative value of the Color Matching Function mean? Bring one light to the other side of the field. Observer now adjusts, for example, the Luminances of G and B lights until they match C_λ + R

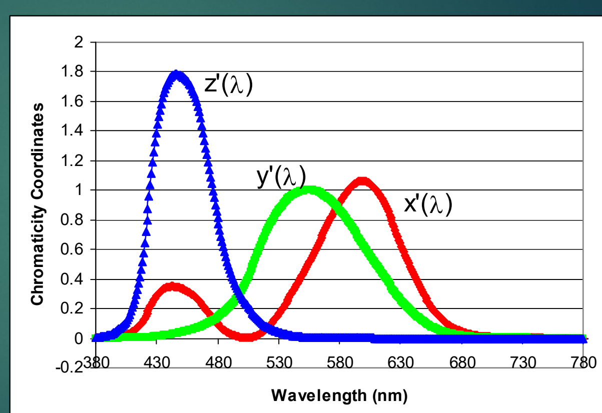


R = 700.0 nm
G = 546.1 nm
B = 435.8 nm

$$C_\lambda + r'(\lambda)R = g'(\lambda)G + b'(\lambda)B$$

1931 CIE Color Matching Functions

The CIE defined three theoretical primaries x', y' and z' such that the color matching functions are everywhere positive and the "green" matching function is the same as the photopic response of the eye.



Conversion x'y'z' to r'g'b'

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = \begin{bmatrix} 0.41846 & -0.15860 & -0.08283 \\ -0.09117 & 0.25243 & 0.01571 \\ 0.00092 & -0.00255 & 0.17860 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

The x', y', z' are more convenient from a book-keeping standpoint and the relative luminance is easy to determine since it is related to y'.

The consequence of this conversion is that the spectral distribution of the corresponding primaries now have negative values. This means they are a purely theoretical source and can not be made.

Tristimulus Values X, Y, Z

$$X = \int_0^\infty P(\lambda)x'(\lambda)d\lambda$$

$$Y = \int_0^\infty P(\lambda)y'(\lambda)d\lambda$$

$$Z = \int_0^\infty P(\lambda)z'(\lambda)d\lambda$$

The tristimulus values are coordinates in a three dimensional color space. They are obtained by projecting the spectral distribution of the object of interest $P(\lambda)$ onto the color matching functions.

Chromaticity Coordinates x, y, z

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z} = 1 - x - y$$

The chromaticity coordinates are used to normalize out the brightness of the object. This way, the color and the brightness can be spearated. The coordinate z is not independent of x and y, so this is a two dimensional space.

Chromaticity Coordinates

Light Source Spectral Distribution X Object Spectral Reflectance or Transmittance = Spectral Distribution of Light entering the eye.

↓ Project onto x', y', z'

Calculate Tristimulus Values X, Y, Z

← Normalize

Calculate Chromaticity Coordinates

←

Plot (x, y)

Example - Spectrally Pure Colors

Suppose $P(\lambda) = \delta(\lambda-\lambda_o)$

$$X = \int_0^\infty \delta(\lambda-\lambda_o)x'(\lambda)d\lambda = x'(\lambda_o)$$

$$Y = \int_0^\infty \delta(\lambda-\lambda_o)y'(\lambda)d\lambda = y'(\lambda_o)$$

$$Z = \int_0^\infty \delta(\lambda-\lambda_o)z'(\lambda)d\lambda = z'(\lambda_o)$$

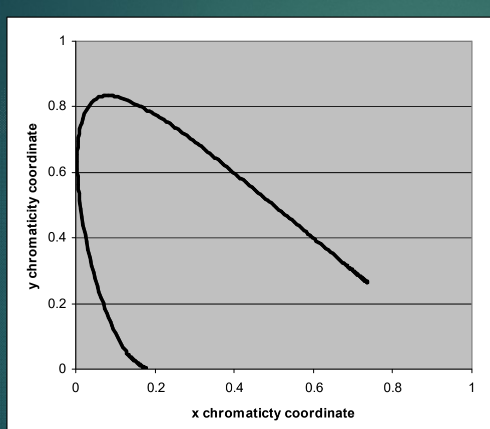
$$x = \frac{x'(\lambda_o)}{x'(\lambda_o)+y'(\lambda_o)+z'(\lambda_o)}$$

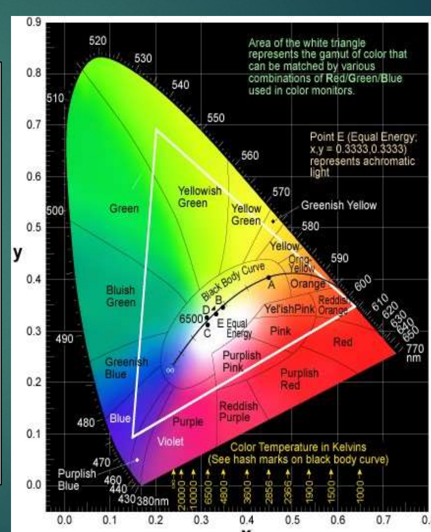
$$y = \frac{y'(\lambda_o)}{x'(\lambda_o)+y'(\lambda_o)+z'(\lambda_o)}$$

$$z = 1 - x - y$$

Plotting x vs. y for spectrally pure colors gives the boundary of color vision

CIE Chromaticity Chart





Example - White Light

Suppose P(λ) = 1

$$X = \int_0^\infty x'(\lambda)d\lambda = 1$$

$$Y = \int_0^\infty y'(\lambda)d\lambda = 1$$

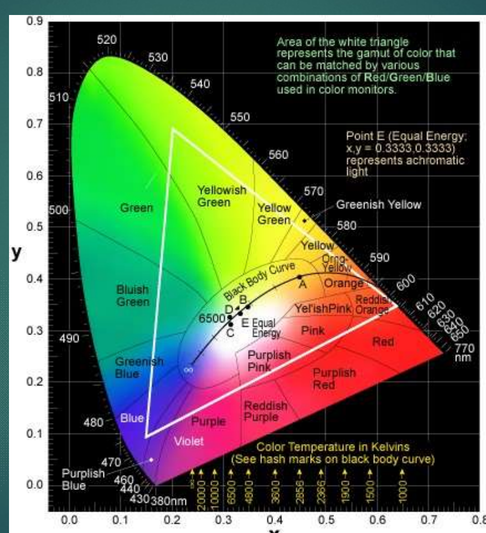
$$Z = \int_0^\infty z'(\lambda)d\lambda = 1$$

$$x = \frac{1}{1+1+1} = 0.33$$

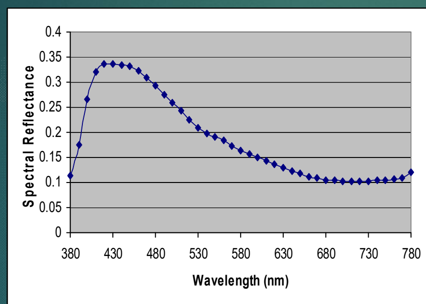
$$y = \frac{1}{1+1+1} = 0.33$$

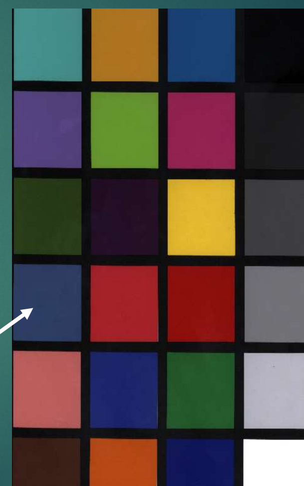
$$z = 1 - x - y = 0.33$$

CIE Chromaticity Chart



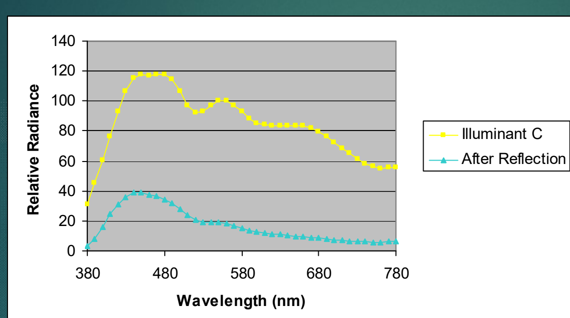
Example - MacBeth Color Checker





Blue Sky

Example - Blue Sky Patch



The Macbeth color checker assumes that Illuminant C is used for illumination.

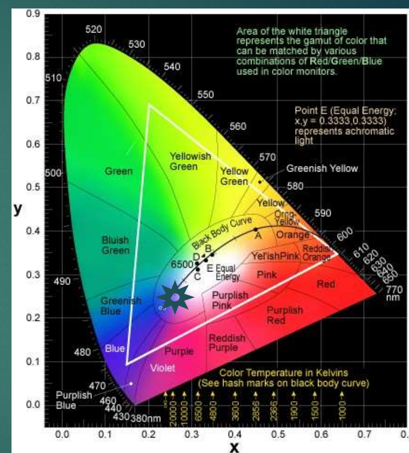
Multiply the spectral distribution of Illuminant C by the spectral reflectance of the color patch to get the light entering the eye.

Example Blue Sky Patch

$$X = \sum P(\lambda)x'(\lambda)\Delta\lambda = 188.1\Delta\lambda$$

$$Y = \sum P(\lambda)y'(\lambda)\Delta\lambda = 192.6\Delta\lambda$$

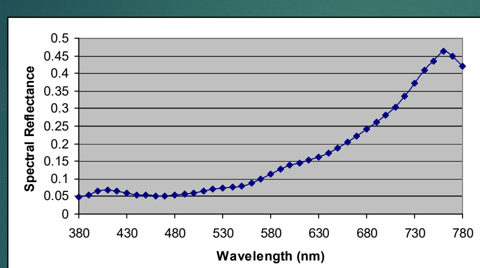
$$Z = \sum P(\lambda)z'(\lambda)\Delta\lambda = 379.9\Delta\lambda$$



$$x = 188.1/(188.1+192.6+379.9) = .247$$

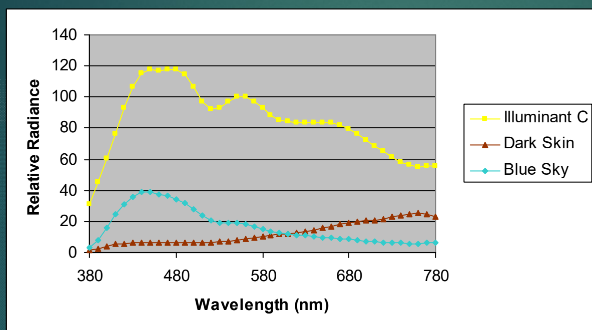
$$y = 192.6/(188.1+192.6+379.9) = .253$$

Example - MacBeth Color Checker



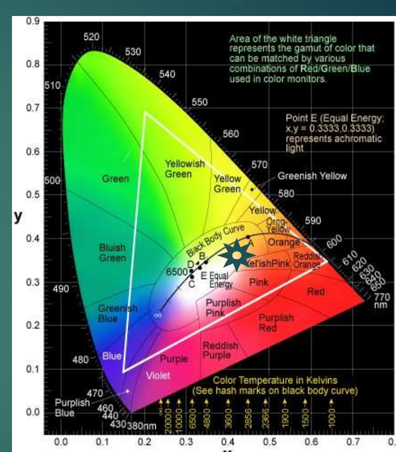
Dark Skin

Example - Dark Skin Patch



Dark skin is much darker than the blue sky. The chromaticity coordinates, however, remove the luminance factor and only look at color.

Example Dark Skin



$$X = \sum P(\lambda)x'(\lambda)\Delta\lambda = 113.6\Delta\lambda$$

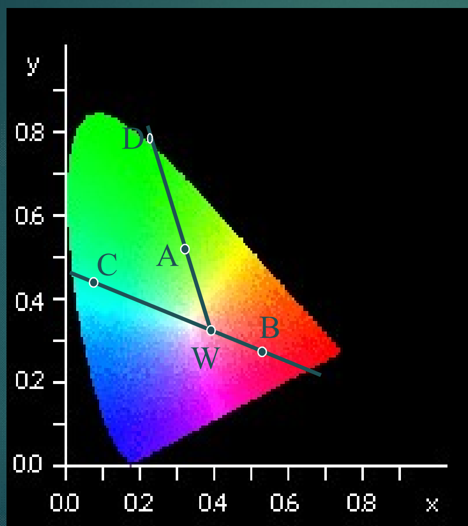
$$Y = \sum P(\lambda)y'(\lambda)\Delta\lambda = 98.8\Delta\lambda$$

$$Z = \sum P(\lambda)z'(\lambda)\Delta\lambda = 66.2\Delta\lambda$$

$$x = 188.1/(188.1+192.6+379.9) = 0.41$$

$$y = 192.6/(188.1+192.6+379.9) = 0.35$$

CIE Chromaticity Diagram



Think of this as a distorted version of the HSB color model.

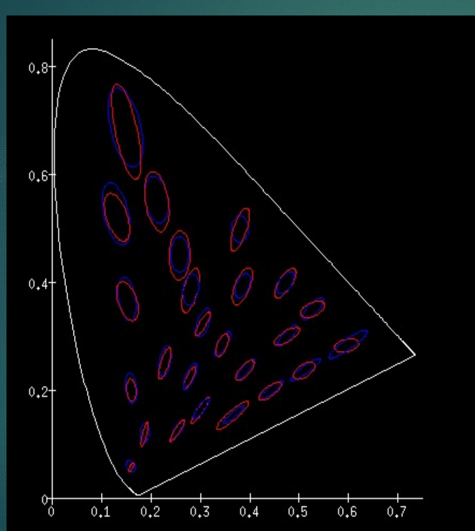
W = White Point (0.33,0.33)
D = Dominant Wavelength (hue)

$$p = \text{excitation purity} = \frac{WA}{WD}$$

C = Complimentary Color

$$p = \text{excitation purity} = \frac{WB}{WC}$$

MacAdam Ellipses



Just noticeable differences for two similar colors is nonlinear on the CIE diagram. Would like a color space that these ellipses become circles. (ellipses are 3x larger than actuality)

1976 CIELUV

Television and Video

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{1/3} - 16 \quad \text{for } \frac{Y}{Y_n} > 0.008856$$

$$L^* = 903.292\left(\frac{Y}{Y_n}\right) \quad \text{for } \frac{Y}{Y_n} \le 0.008856$$

$$u^* = 13L^*[u' - u_n]$$

$$v^* = 13L^*[v' - v_n]$$

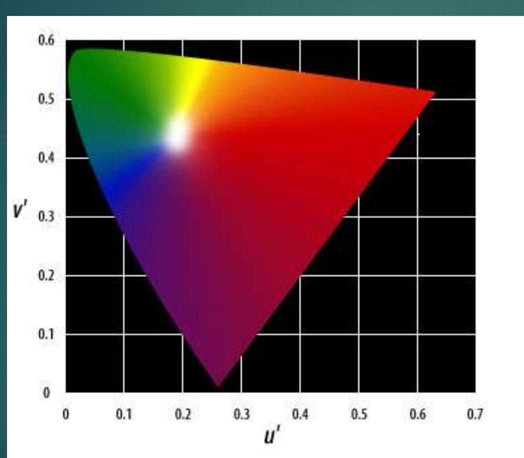
$$u' = \frac{4X}{X + 15Y + 3Z} = \frac{4x}{-2x + 12y + 3}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} = \frac{9y}{-2x + 12y + 3}$$

L* is related to the lightness and is nonlinear to account for the nonlinear response of the visual system to luminance. The u's and v's distort the CIE diagram to make the MacAdam ellipses more round.

Y_n, u_n and v_n are for white

1976 CIELUV



CIELUV Color Difference

$$\Delta E = \sqrt{(\Delta L^*)^2 + (\Delta u^*)^2 + (\Delta v^*)^2}$$

Polar Coordinates

$$C_{uv}^* = \sqrt{u^{*2} + v^{*2}}$$

$$h_{uv} = \frac{180}{\pi} tan^{-1}\left(\frac{v^*}{u^*}\right)$$

1976 CIELAB

Plastic, Textile & Paint

$$L^* = 116 f\left(\frac{Y}{Y_n}\right) - 16$$

$$a^* = 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right]$$

$$b^* = 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right]$$

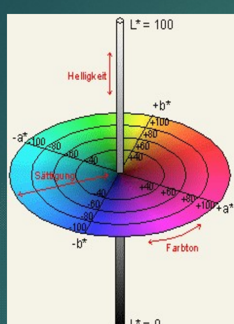
where $f(s) = s^{1/3}$ for $s > 0.008856$

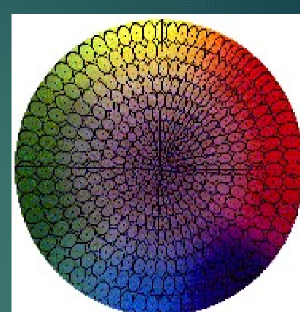
$f(s) = 7.787s + 16/116$ for $s \le 0.008856$

L* is related to the lightness and is nonlinear to account for the nonlinear response of the visual system to luminance. The a's and b's distort the CIE diagram to make the MacAdam ellipses more round.

X_n, Y_n and Z_n are for white

1976 CIELAB





Polar Coordinates

$$C^*_{ab} = \sqrt{a^{*2} + b^{*2}}$$

$$h_{ab} = \frac{180}{\pi} tan^{-1}\left(\frac{b^*}{a^*}\right)$$

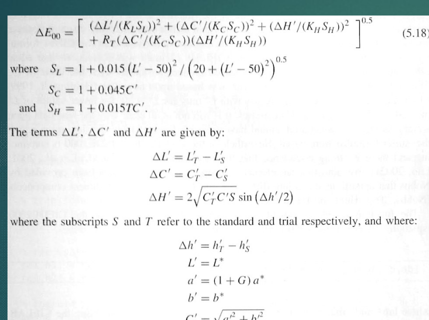
Color Difference

$$\Delta E^*_{ab} = \sqrt{\left(\Delta L^*\right)^2 + \left(\Delta a^*\right)^2 + \left(\Delta b^*\right)^2}$$

Color Difference Formulas

- DE = 1 is approximate threshold for Just Noticeable Difference.
- CIELAB used primarily today instead of CIELUV
- CIELAB DE > 5, but some variation for smaller differences.
- CIEDE2000 performs better for small color differences.

CIEDE2000



$$\Delta E_{00} = \left[\begin{array}{l} (\Delta L'/(K_L S_L))^2 + (\Delta C'/(K_C S_C))^2 + (\Delta H'/(K_H S_H))^2 \\ + R_T(\Delta C'/(K_C S_C))(\Delta H'/(K_H S_H)) \end{array} \right]^{0.5} \quad (5.18)$$

where $S_L = 1 + 0.015\,(L' - 50)^2 / \left(20 + (L' - 50)^2\right)^{0.5}$

$S_C = 1 + 0.045C'$

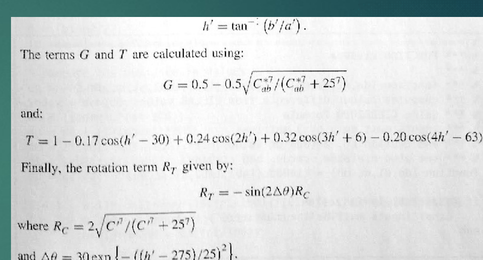
and $S_H = 1 + 0.015TC'$.

The terms $\Delta L'$, $\Delta C'$ and $\Delta H'$ are given by:

$$\Delta L' = L'_T - L'_S$$
$$\Delta C' = C'_T - C'_S$$
$$\Delta H' = 2\sqrt{C'_T C'S}\,\sin\left(\Delta h'/2\right)$$

where the subscripts S and T refer to the standard and trial respectively, and where:

$$\Delta h' = h'_T - h'_S$$
$$L' = L^*$$
$$a' = (1 + G)\,a^*$$
$$b' = b^*$$
$$C' = \sqrt{a'^2 + b'^2}$$



$$h' = \tan^{-1}\left(b'/a'\right).$$

The terms G and T are calculated using:

$$G = 0.5 - 0.5\sqrt{C^{*7}_{ab}/(C^{*7}_{ab} + 25^7)}$$

and:

$$T = 1 - 0.17\cos(h' - 30) + 0.24\cos(2h') + 0.32\cos(3h' + 6) - 0.20\cos(4h' - 63)$$

Finally, the rotation term R_T given by:

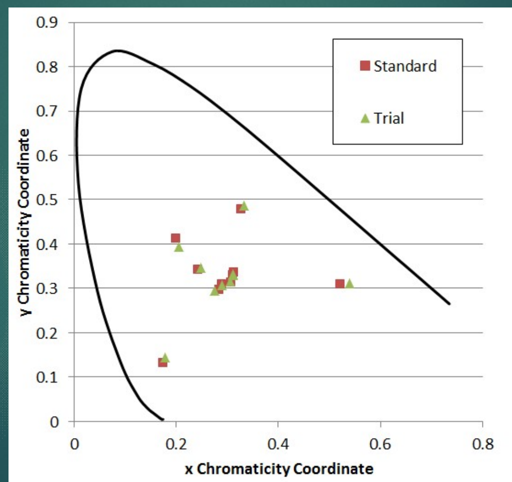
$$R_T = -\sin(2\Delta\theta)R_C$$

where $R_C = 2\sqrt{C'^7/(C'^7 + 25^7)}$

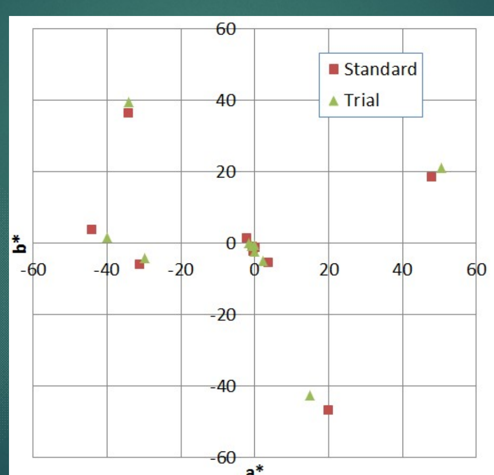
and $\Delta\theta = 30\exp\left\{-\left((h' - 275)/25\right)^2\right\}$.

Westland – Computational Colour Science using Matlab

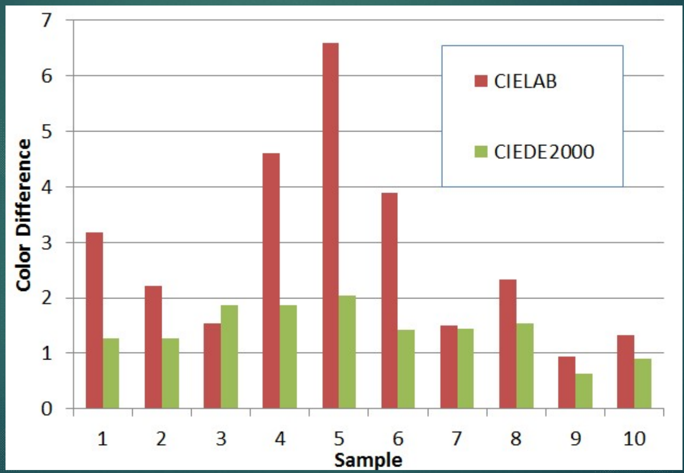
Comparison of 10 Samples



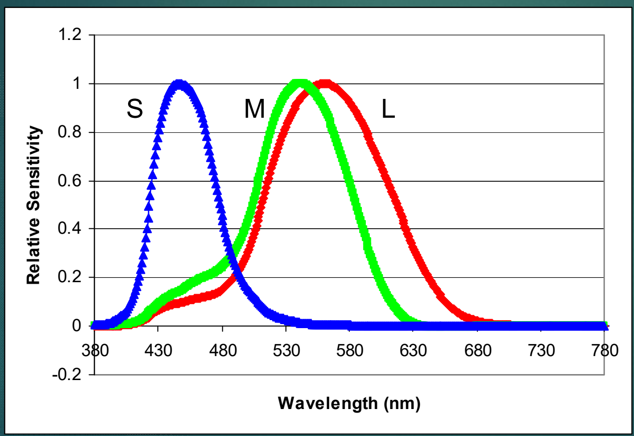
CIELAB Space



Color Difference Comparison

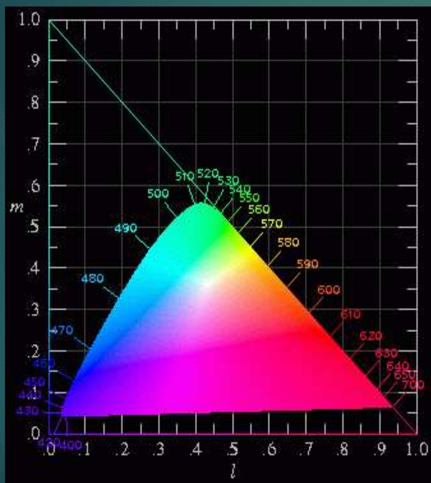


Human Cone Sensitivities



Only during the 1990s were researcher able to distinguish the cone sensitivities

LMS Color Space

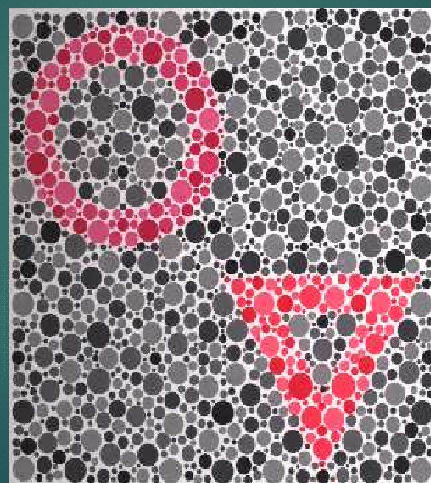


Project P onto LMS color matching functions. The (l,m) is analogous to (x,y) chromaticity coordinates.

Color Blindness

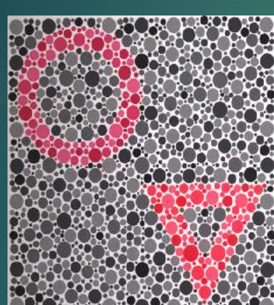
- Protanopia - No L Cones
  - 1% men, rare in women
- Deutranopia - No M Cones
  - 1% men, 0.01% women
- Tritanopia - No S Cones
  - rare

Color Blindness

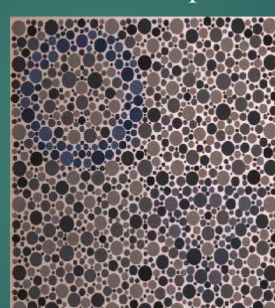


Color Blindness

Normal



Protonopia

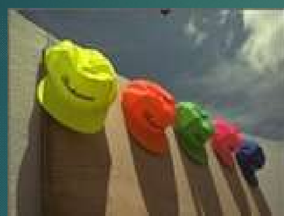


No L Cones

Deuteranopia

No M Cones

Color Blindness Examples



Normal

Deuteranopia

Protanopia

Color Space Conversions

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

This is a conversion from an RGB color model to XYZ trichromatic coordinates. The 709 refers to a standard set of phosphors used in most displays. RGB colors are assumed to range from 0..1. However, in the computer they usually range from 0..255. Simply divide the computer value by 255 to normalize.

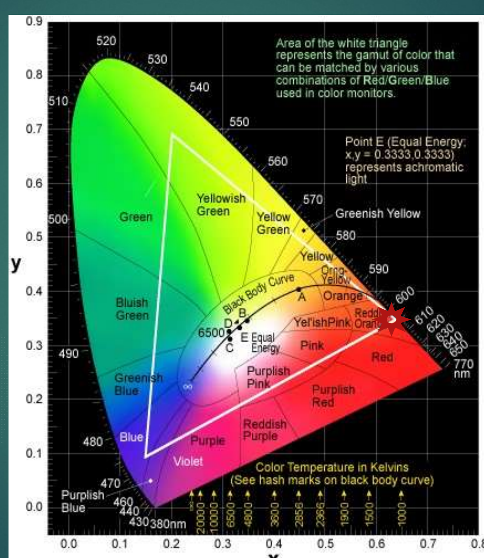
Color Space Conversions - Example

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

X = 0.412453
Y = 0.212671
Z = 0.019334

x = 0.412453/(0.412453+0.212671+0.019334)
x = 0.64
y = 0.212671/(0.412453+0.212671+0.019334)
y = 0.33

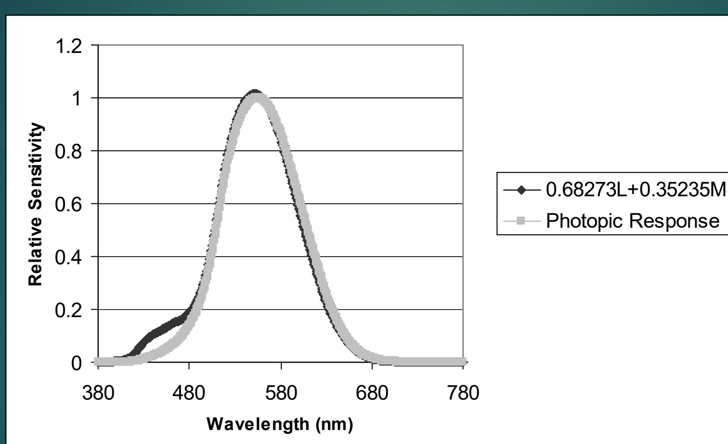
Example (1, 0, 0)



Color Space Conversions

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

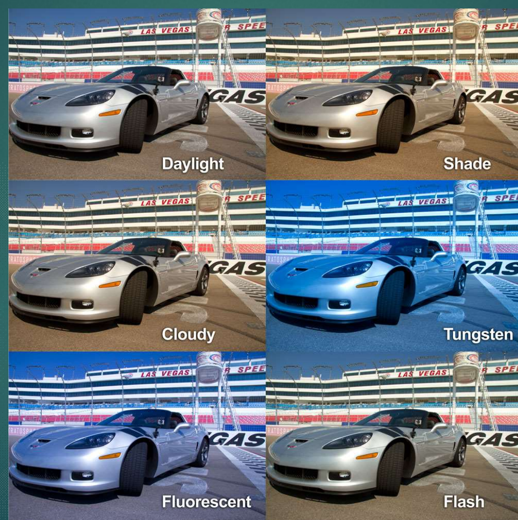
L+M Channels



Opponent Processes

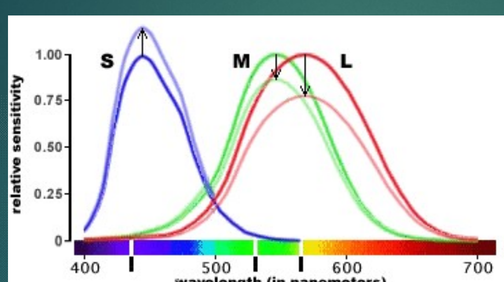
Opponent Colors

Camera White Balance



Photofocus.com

Von Kries Transform



Sometimes called "wrong" von Kries transform when done on XYZ instead of LMS.

$$\begin{pmatrix} L_2 \\ M_2 \\ S_2 \end{pmatrix} = \begin{pmatrix} L_{W2}/L_{W1} & 0 & 0 \\ 0 & M_{W2}/M_{W1} & 0 \\ 0 & 0 & S_{W2}/S_{W1} \end{pmatrix} \begin{pmatrix} L_1 \\ M_1 \\ S_1 \end{pmatrix}$$

Chromatic Adaptation Transform

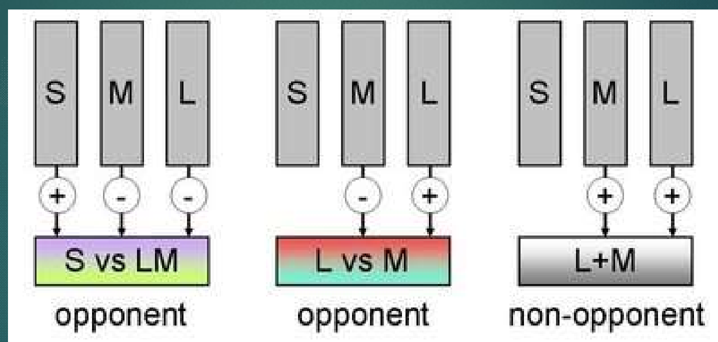
- Von Kries is a little too simple due to interaction between the channels.
- CATs try to match real data

$$\begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = M_{CAT}^{-1} \begin{pmatrix} X_{W2}/X_{W1} & 0 & 0 \\ 0 & Y_{W2}/Y_{W1} & 0 \\ 0 & 0 & Z_{W2}/Z_{W1} \end{pmatrix} M_{CAT} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}$$

Questions on Trichromatic Theory

- What does bluish-yellow look like?
- What does greenish-red look like?
- Why do colorblind people either lose red-green or yellow-blue colors in pairs?
- Red, green and blue appear to be pure colors (i.e. not a mixture of other colors). Why does yellow also appear as a pure color?

Opponent Process



Opponent Process Test

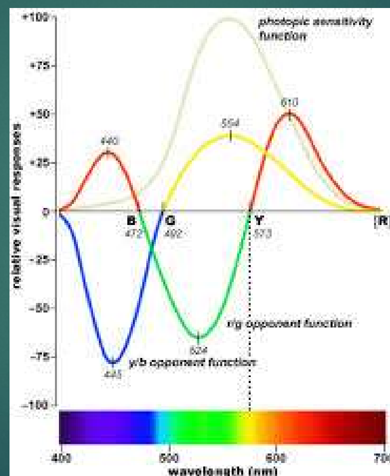
How much yellow is needed to cancel blue tint?

How much blue is needed to cancel yellow tint?

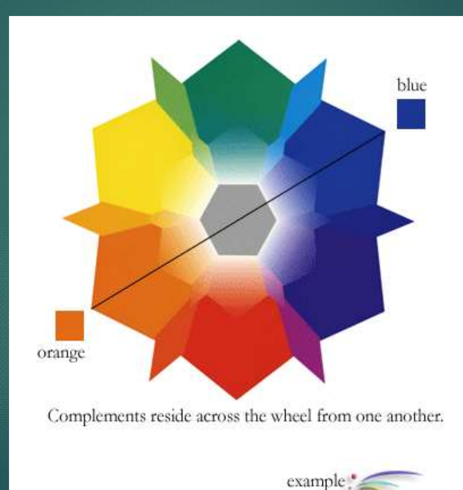
How much red is needed to cancel green tint?

How much green is needed to cancel red tint?

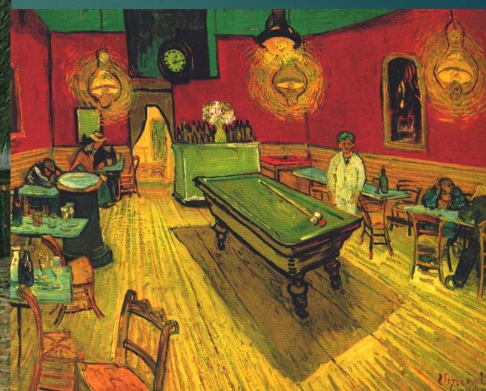
Opponent Process



Opponent Process

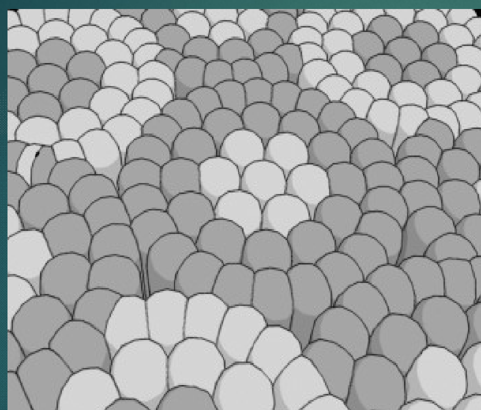


Opponent Process



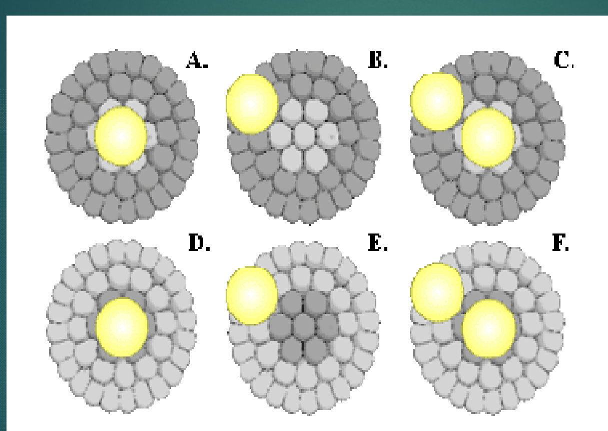
Opponent Colors

Receptive Fields



The lighter photoreceptors are the ones that activate in response to light while the others are the photoreceptors that activate in the absence of light. As you can see, some of the dark photoreceptors encircle the lighter ones and vice versa. In reality, however, these two types of photoreceptors look the same.

Receptive Fields

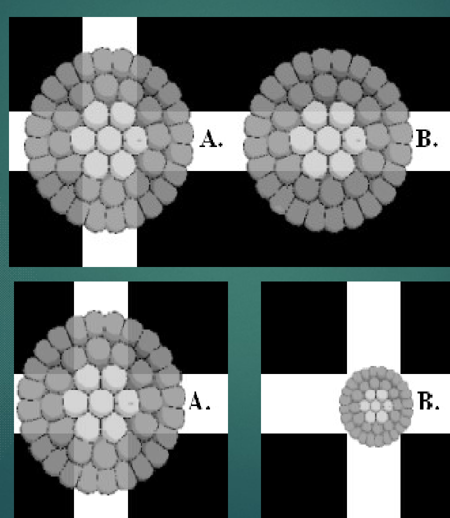


Center-ON

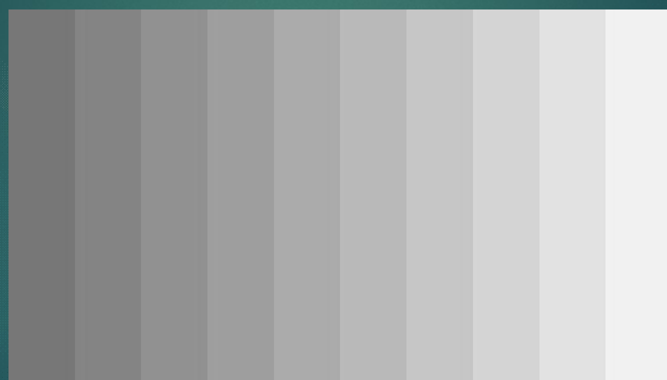
Center-OFF

Hermann's Grid

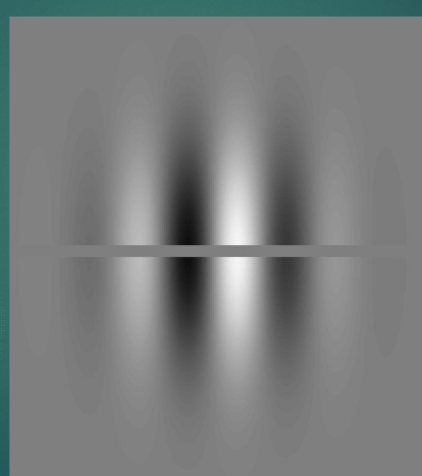
Hermann's Grid



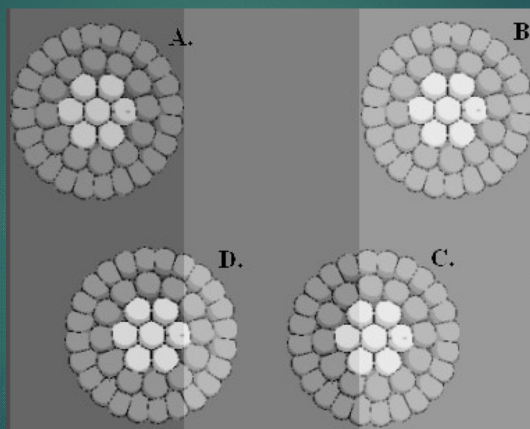
Mach Bands



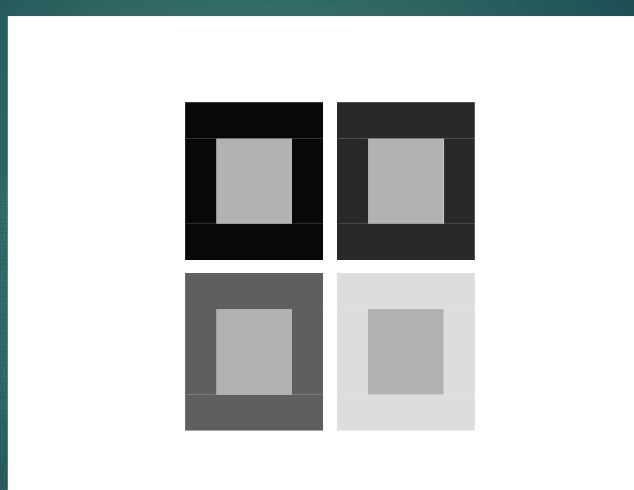
Mach Bands



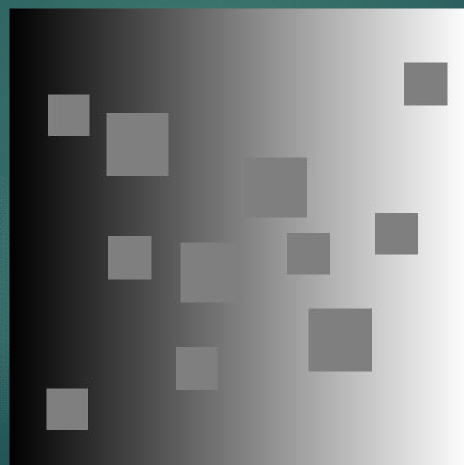
Mach Bands



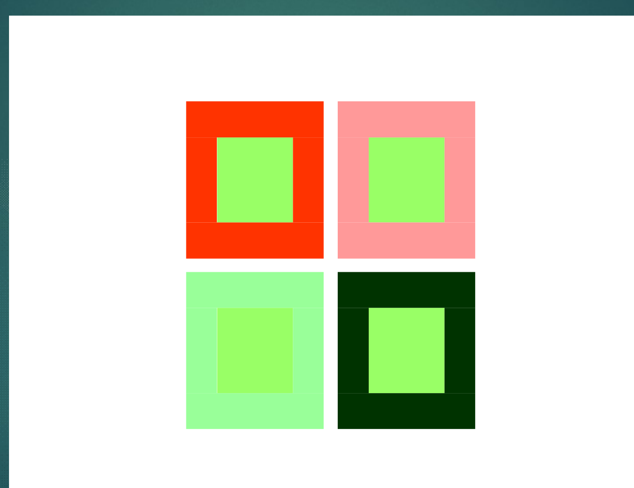
Lightness Contrast



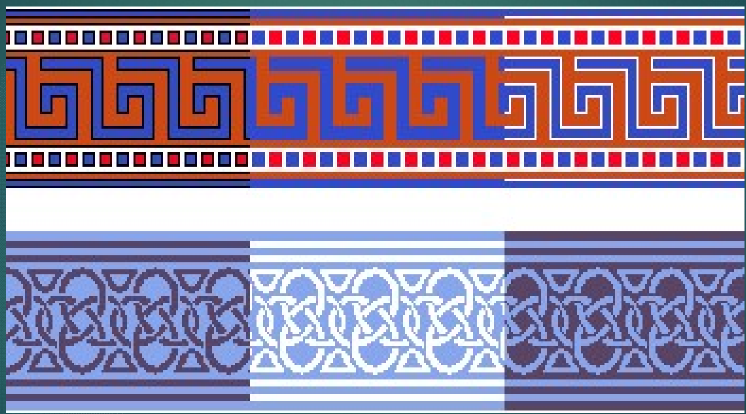
Lightness Contrast



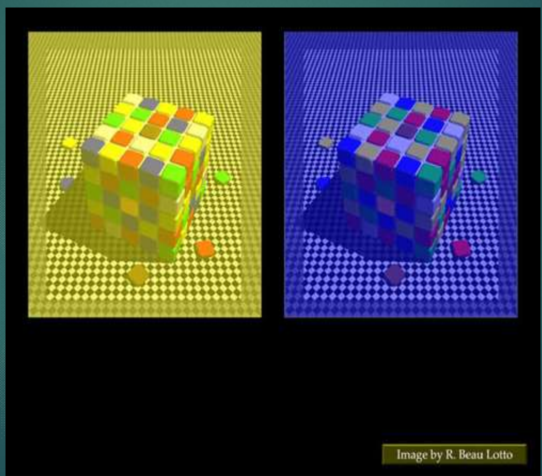
Color Contrast



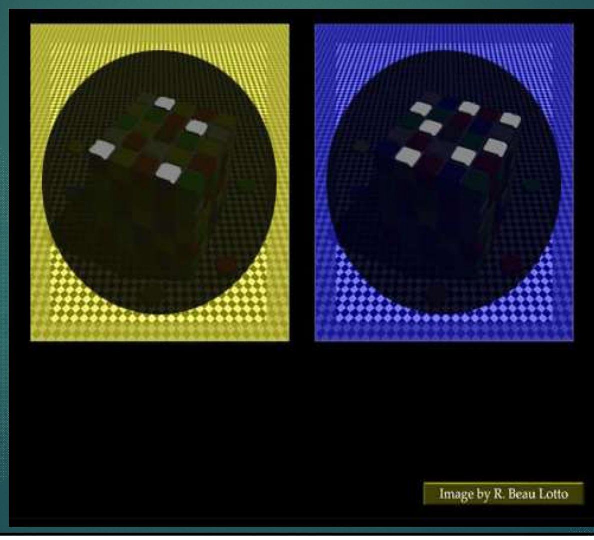
Complex Color Shifts



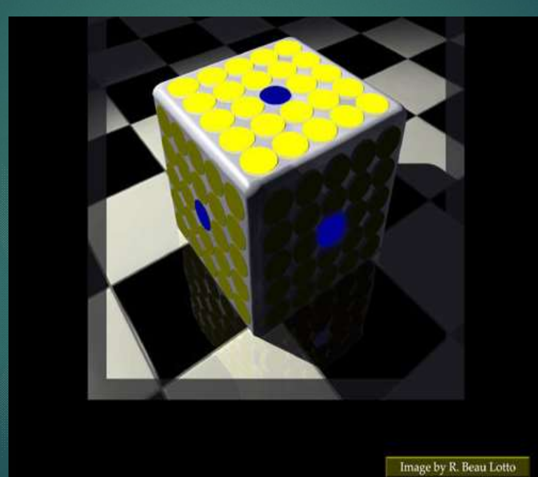
Rubik's Cube??



Rubic's Cube



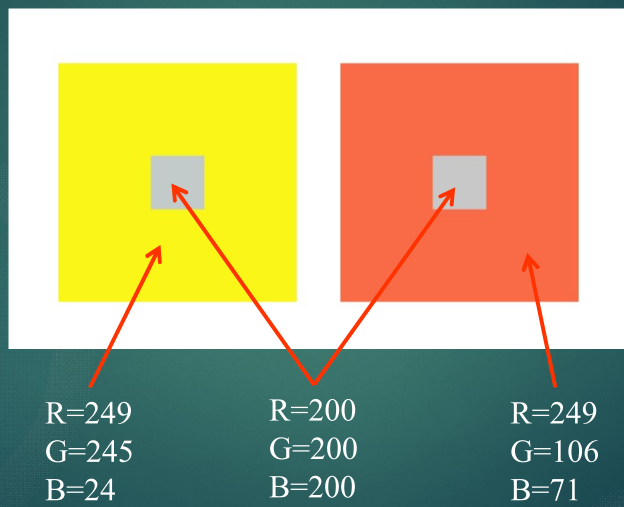
Blue Dot



Blue Dot

Color Mach Bands

Color Contrast Example



Color Contrast

sRGB				
	Grey	Yellow	Orange	White
R	200	249	249	255
G	200	245	106	255
B	200	24	71	255
Rnorm	0.78431373	0.97647059	0.97647059	1
Gnorm	0.78431373	0.96078431	0.41568627	1
Bnorm	0.78431373	0.09411765	0.27843137	1
RL	0.57758044	0.94730654	0.94730654	1
GL	0.57758044	0.91309865	0.14412847	1
BL	0.57758044	0.00913406	0.06301002	1
X	0.5489648	0.71887323	0.45362534	0.950456
Y	0.57758044	0.85513546	0.30908692	1
Z	0.62884301	0.13582962	0.09536815	1.088754
x	0.31273127	0.42043346	0.52865132	0.31273127
y	0.32903287	0.5001265	0.36020741	0.32903287
X100	54.8964795	71.8873233	45.3625338	95.0456
Y100	57.758044	85.5135456	30.9086916	100
Z100	62.8843015	13.5829621	9.53681495	108.8754

Rnorm = R/255

$$C_L = \begin{cases} C/12.92 & \text{for } C \le 0.04045 \\ \left(\dfrac{C+0.055}{1.055}\right)^{2.4} & \text{for } C > 0.04045 \end{cases} \approx C^{2.2}$$

where C corresponds to R, G, or B. Finally, the Tristimulus values are obtained with the following matrix operation.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R_L \\ G_L \\ B_L \end{bmatrix}$$

Scale XYZ to make $Y_w = 100$

CIECAM02 Output

- Brightness is the subjective appearance of how bright an object appears given its surroundings and how it is illuminated.
- Lightness is the subjective appearance of how light a color appears to be.
- Colorfulness is the degree of difference between a color and grey.
- Chroma is the colorfulness relative to the brightness of another color that appears white under similar viewing conditions. This allows for the fact that a surface of a given chroma displays increasing colorfulness as the level of illumination increases.
- Saturation is the colorfulness of a color relative to its own brightness.
- Hue is the degree to which a stimulus can be described as similar to or different from stimuli that are described as red, green, blue, and yellow.

Source: Wikipedia

Lightness Contrast

Sample Color

54.8964795
57.758044
62.8843015

White

95.0456
100
108.8754

Background Luminance (Y_b)

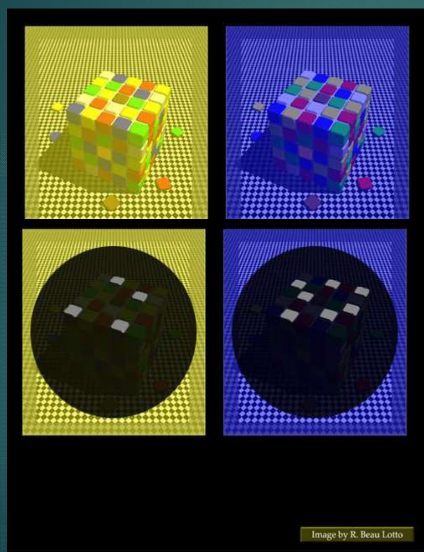
85.5135456	30.9086916

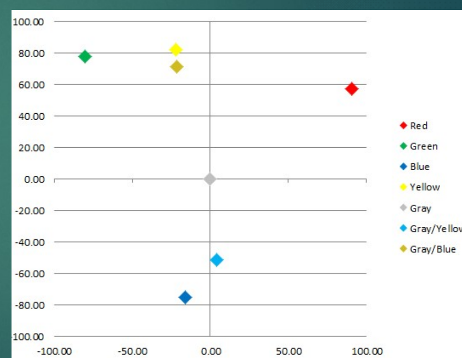
CIECAM02

J	69.10	73.13	Lightness
Q	193.06	238.79	Brightness
t	0.10	0.12	
s	2.56	2.29	saturation
C	0.12	0.12	chroma
M	0.13	0.13	colorfulness
	211.9	211.9	hue

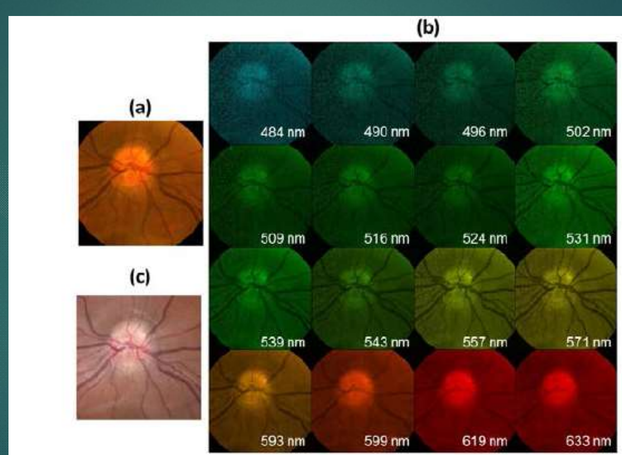
Chroma & Hue unchanged, Brightness increased

CIECAM02 Example





Hyperspectral Imaging



Snapshot hyperspectral retinal camera with the Image Mapping Spectrometer (IMS)

Liang Gao,[1] R. Theodore Smith,[2,*] and Tomasz S. Tkaczyk[1,3,*]

Singular Value Decomposition (SVD)

A matrix A_{mxn} with m rows and n columns can be decomposed into

$$A = USV^T$$

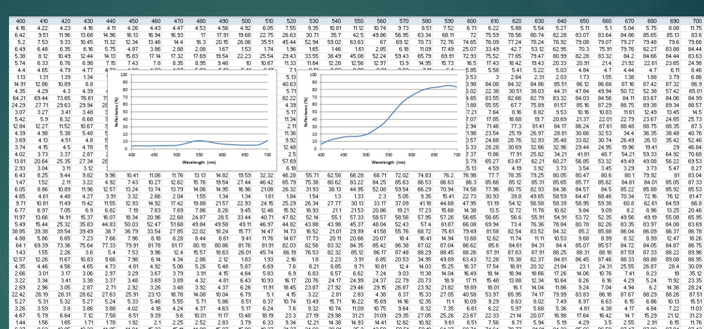
where $U^TU = I$, $V^TV = I$ (i.e. orthogonal) and S is a diagonal matrix.

If Rank(A) = p, then U_{mxp}, V_{nxp} and S_{pxp}

OK, but what does this mean is English?

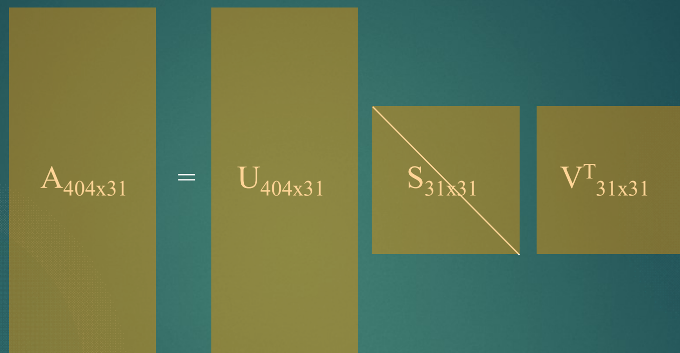
SVD by Example

A_{404x31}=



Keele Data on Reflectance of Natural Objects
m = 404 rows of different objects
n = 31 columns, wavelengths 400-700 nm in 10 nm steps
Rank(A) = 31 means at least 31 independent rows

SVD by Example



$U^TU = I$ means dot product of two different columns of U equals zero.
$V^TV = I$ means dot product of two different columns of V (rows of V^T) equals zero.

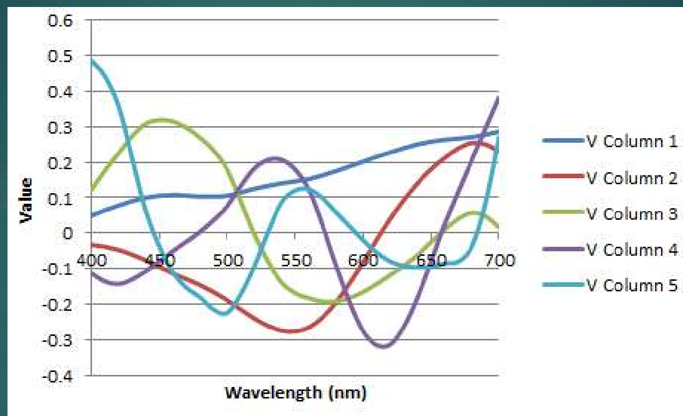
Basis Functions

V_{31x31}=

0.049	-0.033	0.1212	-0.112	0.4869	-0.302	0.0796	-0.281	0.3339	-0.264	0.3548	-0.296	0.1109
0.0644	-0.038	0.1759	-0.136	0.4432	-0.214	0.0394	-0.109	0.0406	0.0351	-0.079	0.1171	-0.063
0.0781	-0.047	0.2253	-0.143	0.3584	-0.111	-0.002	0.0334	-0.154	0.1904	-0.279	0.2819	-0.124
0.0904	-0.06	0.2727	-0.131	0.2146	0.0246	-0.053	0.1591	-0.249	0.1686	-0.179	0.123	-0.028
0.1	-0.076	0.3078	-0.107	0.0651	0.1408	-0.09	0.233	-0.249	0.0689	0.0249	-0.125	0.0969
0.1056	-0.094	0.3192	-0.08	-0.036	0.1828	-0.09	0.2073	-0.137	-0.015	0.1641	-0.263	0.1332
0.1078	-0.114	0.3146	-0.05	-0.11	0.182	-0.07	0.1331	0.0265	-0.099	0.2309	-0.267	0.0917
0.1059	-0.13	0.2979	-0.024	-0.153	0.153	-0.037	0.051	0.2217	-0.204	0.1227	0.088	-0.15
0.1032	-0.146	0.2692	0.0035	-0.18	0.0899	0.0068	-0.055	0.3459	-0.227	-0.042	0.3905	-0.307
0.1029	-0.166	0.233	0.0365	-0.215	-0.027	0.0572	-0.226	0.2463	0.019	-0.219	0.1406	0.0446
0.1056	-0.189	0.1796	0.0762	-0.225	-0.142	0.1081	-0.342	0.0559	0.2561	-0.287	-0.171	0.3609
0.1147	-0.215	0.0908	0.1323	-0.173	-0.206	0.1602	-0.256	-0.186	0.2067	0.0194	-0.136	0.0321
0.1254	-0.24	-0.004	0.1826	-0.093	-0.225	0.1878	-0.087	-0.35	0.0574	0.3144	-0.007	-0.319
0.1334	-0.261	-0.08	0.2067	0.0028	-0.177	0.1594	0.119	-0.219	-0.132	0.1993	0.1238	-0.115
0.1405	-0.275	-0.139	0.2069	0.0864	-0.1	0.0941	0.2795	-0.012	-0.259	-0.02	0.1851	0.1664
0.1461	-0.277	-0.169	0.1797	0.1204	-0.028	-0.008	0.2676	0.1199	-0.142	-0.182	0.0025	0.1903
0.1527	-0.266	-0.183	0.1231	0.124	0.0422	-0.121	0.1693	0.1954	0.0431	-0.252	-0.191	0.1014
0.1628	-0.238	-0.192	0.0234	0.0992	0.1079	-0.242	0.0044	0.167	0.2009	-0.07	-0.186	-0.153
0.1749	-0.196	-0.193	-0.089	0.0597	0.1556	-0.306	-0.155	0.082	0.2718	0.142	-0.085	-0.318
0.1881	-0.142	-0.183	-0.194	0.0194	0.1714	-0.215	-0.221	-0.052	0.1003	0.1883	0.1554	0.0075
0.2019	-0.081	-0.166	-0.277	-0.02	0.1545	-0.065	-0.221	-0.164	-0.119	0.1371	0.3137	0.331
0.2151	-0.019	-0.144	-0.315	-0.054	0.0885	0.0931	-0.139	-0.173	-0.254	-0.073	0.052	0.1575
0.2278	0.0406	-0.118	-0.314	-0.08	0.0072	0.2233	-0.018	-0.112	-0.276	-0.248	-0.236	-0.106
0.24	0.0939	-0.091	-0.263	-0.092	-0.058	0.2734	0.1221	0.0498	-0.031	-0.14	-0.192	-0.204
0.2506	0.141	-0.061	-0.181	-0.096	-0.116	0.2416	0.2258	0.1946	0.2286	0.0384	-0.047	-0.186
0.258	0.1803	-0.025	-0.081	-0.095	-0.176	0.0748	0.2157	0.188	0.261	0.1681	0.1046	0.0789
0.2635	0.2123	0.0099	0.0255	-0.085	-0.215	-0.124	0.1433	0.1112	0.177	0.2144	0.1925	0.2957
0.267	0.2395	0.0409	0.1177	-0.08	-0.237	-0.315	0.0112	-0.033	-0.061	0.016	0.0469	0.0849
0.2708	0.2547	0.0577	0.2061	-0.038	-0.16	-0.362	-0.113	-0.143	-0.243	-0.173	-0.109	-0.157
0.2777	0.2508	0.048	0.2948	0.084	0.1084	-0.093	-0.153	-0.105	-0.151	-0.127	-0.085	-0.127
0.2867	0.2302	0.016	0.3824	0.2707	0.5383	0.4337	-0.135	0.0442	0.1449	0.0785	0.0549	0.0755

Columns of V are basis functions that can be used to represent the original Reflectance curves.

Basis Functions



First column handles most of the variance, then the second column etc.

Singular Values

S_{31x31}=

4041.4	0	0	0	0	0	0	0	0	0	0	0	0
0	1143.8	0	0	0	0	0	0	0	0	0	0	0
0	0	752.56	0	0	0	0	0	0	0	0	0	0
0	0	0	348	0	0	0	0	0	0	0	0	0
0	0	0	0	242.15	0	0	0	0	0	0	0	0
0	0	0	0	0	142.88	0	0	0	0	0	0	0
0	0	0	0	0	0	116.85	0	0	0	0	0	0
0	0	0	0	0	0	0	85.144	0	0	0	0	0
0	0	0	0	0	0	0	0	59.404	0	0	0	0
0	0	0	0	0	0	0	0	0	43.579	0	0	0
0	0	0	0	0	0	0	0	0	0	28.987	0	0
0	0	0	0	0	0	0	0	0	0	0	23.311	0
0	0	0	0	0	0	0	0	0	0	0	0	17.551

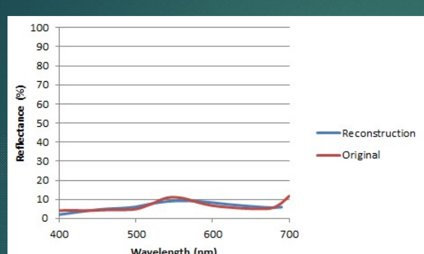
The square of diagonal elements of S describe the variance accounted for by each of the basis functions.

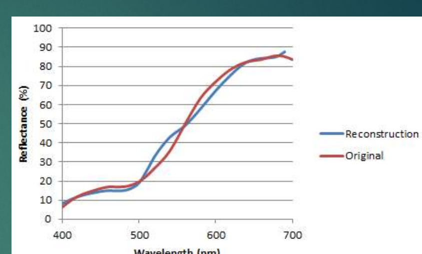
SVD Approximation

The original matrix can be approximated by taking the first d columns of U, reducing S to a d x d matrix and using the first d rows of V^T.

SVD Reconstruction

Three Basis Functions





Five Basis Functions