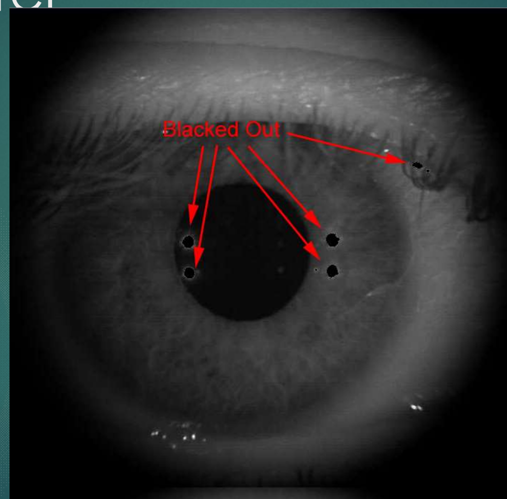
Evaluation for Refractive Surgery

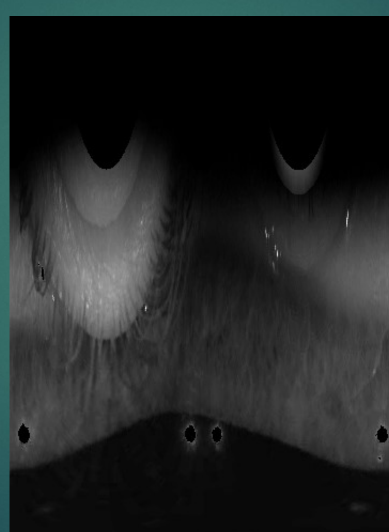
- Want to screen patients prior to performing a surgery that permanently alters the shape of their corneas to ensure they are suitable for the procedure.
- Pupil size is important, especially under dim conditions. Want to ensure the maximum pupil size is not much larger than the optical zone of the treatment.
- Corneal topography is important to ensure cornea irregularities do not exist that might suggest potential disease.

Infrared Pupillometry

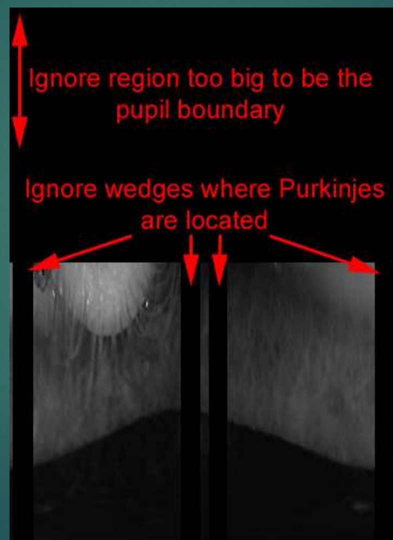
Locate Purkinje Images & Center



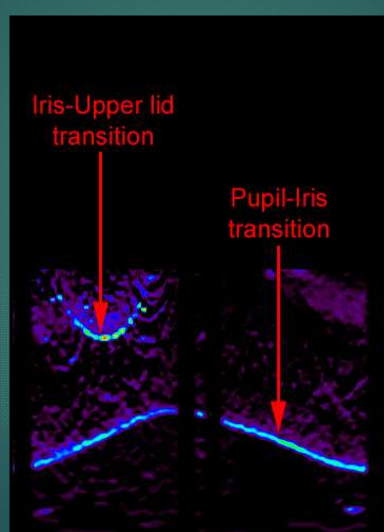
Polar Coordinates



Mask Purkinjes



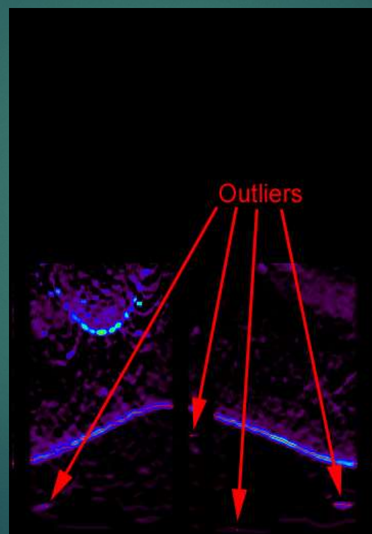
Edge Filter



-1	-1	-1	-1	-1
-1	-1	-1	-1	-1
0	0	0	0	0
1	1	1	1	1
1	1	1	1	1

Convolution

Find peaks near pupil

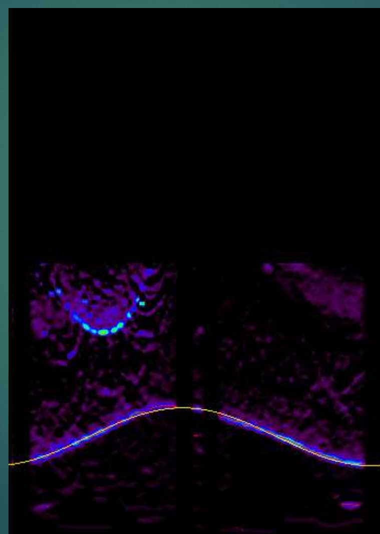


Fit points to sinusoid

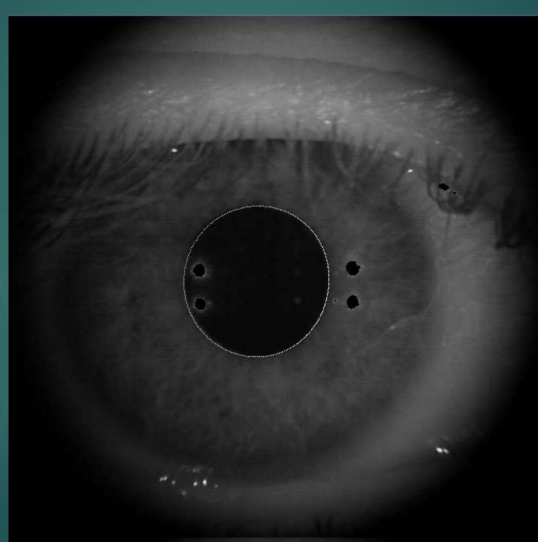
- Pupil margin is a shifted sinusoidal.
- Fit data to truncated Fourier series.
- Fit coefficients give center and dimensions of elliptical pupil.
- Iterate (3x) and eliminate outliers.

$$r_i = A0 + A1 \cdot \sin\left(\frac{2\pi\theta_i}{360}\right) + A2 \cdot \cos\left(\frac{2\pi\theta_i}{360}\right)$$

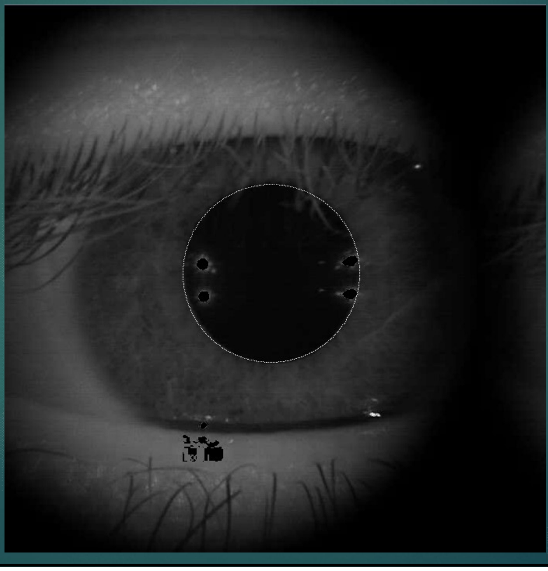
Pupil Margin



Pupil Margin

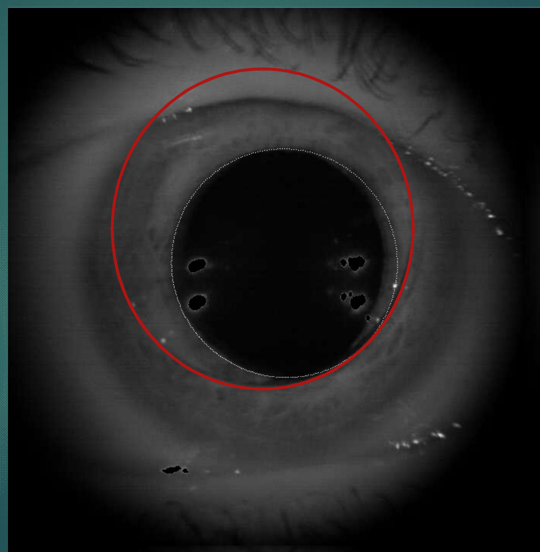


Eyelashes



Hair & Blur

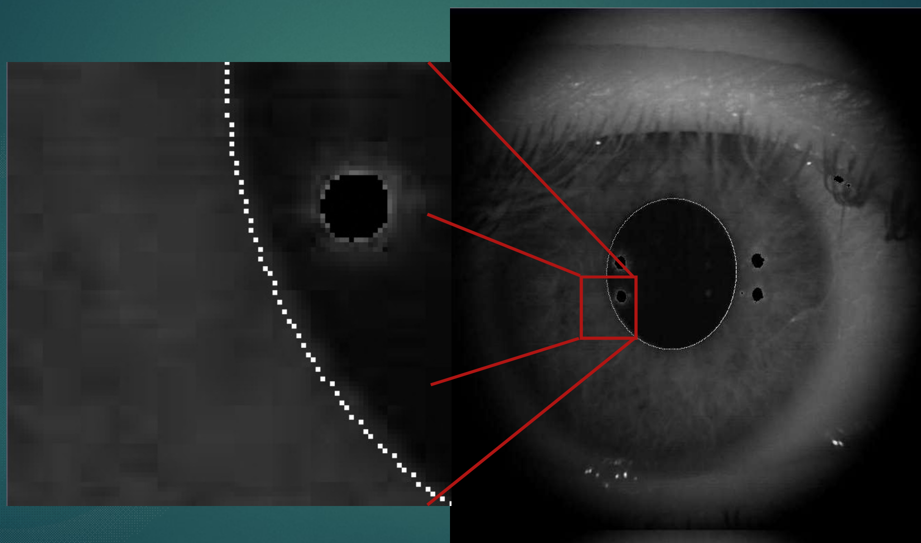
Contact Lens



Issues

- Saturation and/or small LCD screen make focusing difficult
- Refresh rate too slow. Currently 10 fps – Need 30 fps
- Need "Click" to provide feedback to user that picture was taken
- JPEG compression too high. Defeats Mpix advantage.
- Orient LEDs in vertical rectangle to leave sides open.
- Use entire imaging chip.

Image Compression

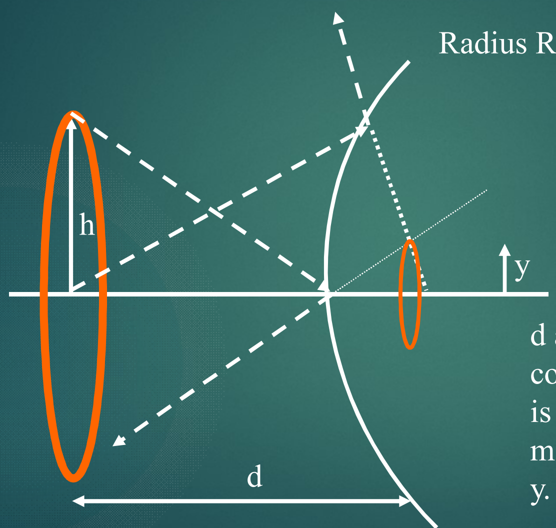


Keratometry

The keratometer is a device for measuring the radius of curvature of the anterior cornea along the flat and steep meridians.

Its primary function today is to determine corneal curvature for IOL implantation (SRK formula).

Keratometry

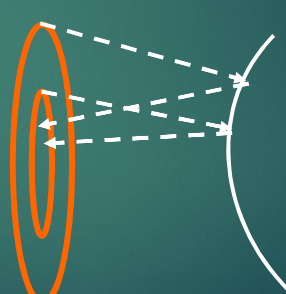


$$R = \frac{2dy}{h}$$

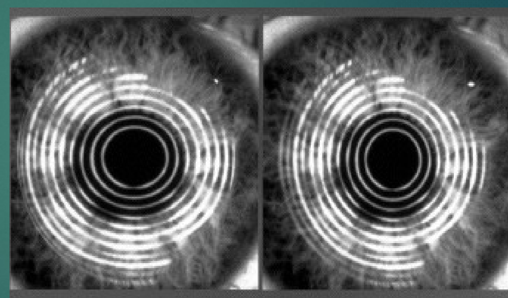
d and h are fixed in commercial systems and R is obtained from a calibrated measurement of the height y.

Placido Disk

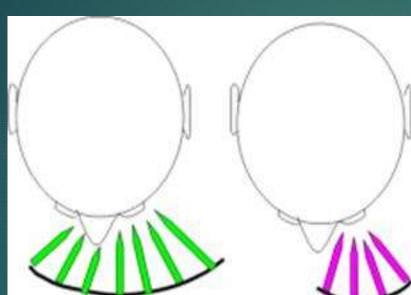
The Placido disk extends the keratometry concept to examine curvature at different points on the cornea.

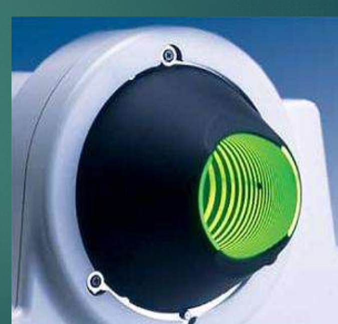


Corneal Topography

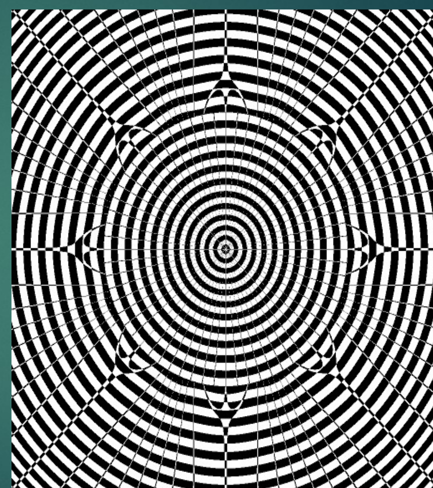


Variations on Placido Concept



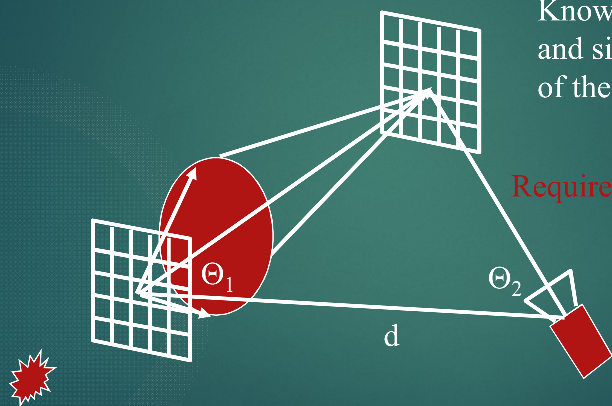


Variations on Placido Concept



Fringe Projection

Stereophotogrammetry

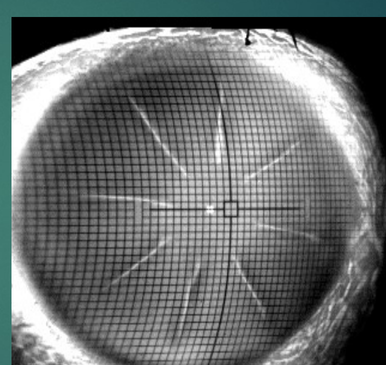


Knowledge of two angles and side tells where the tip of the triangle lies.

Requires a diffuse reflection!

Stereophotogrammetry

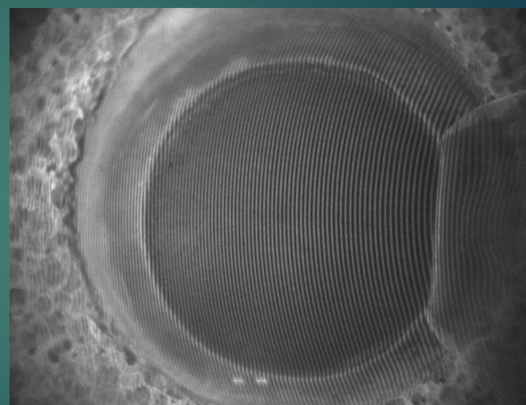
Fluorescein dye is instilled into the eye. When illuminated in blue light, the fluorescein fluoresces in the green.



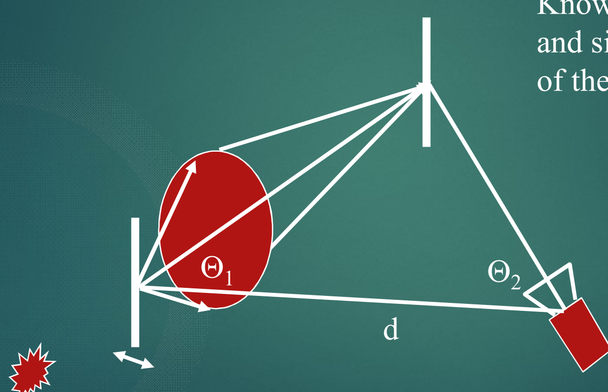
Stereophotogrammetry

This system is being linked to excimer lasers in refractive surgery. The ultraviolet laser light causes the cornea cells to fluoresce in the blue.

Eliminates the need for drops and can be done during surgery.



Scanning Slit



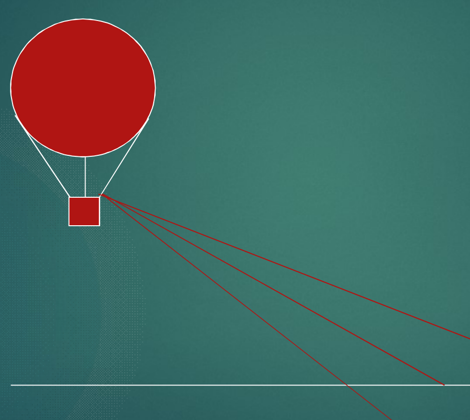
Knowledge of two angles and side tells where the tip of the triangle lies.

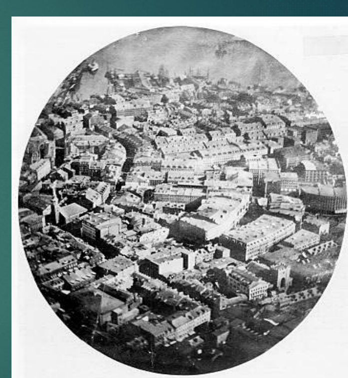
Scanning Slit

Technique uses white light scatter from cornea and crystalline lens.

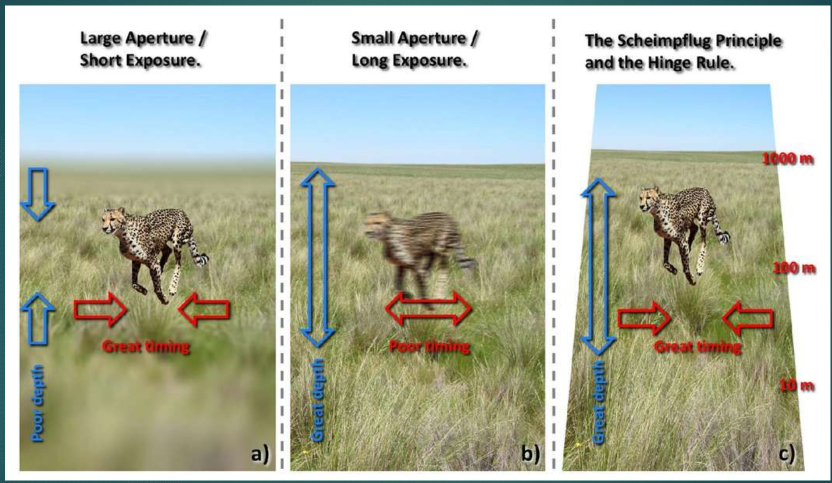
By refracting through each surface, this technique can measure anterior and posterior corneal shape, as well as anterior crystalline lens shape.

Scheimpflug Imaging



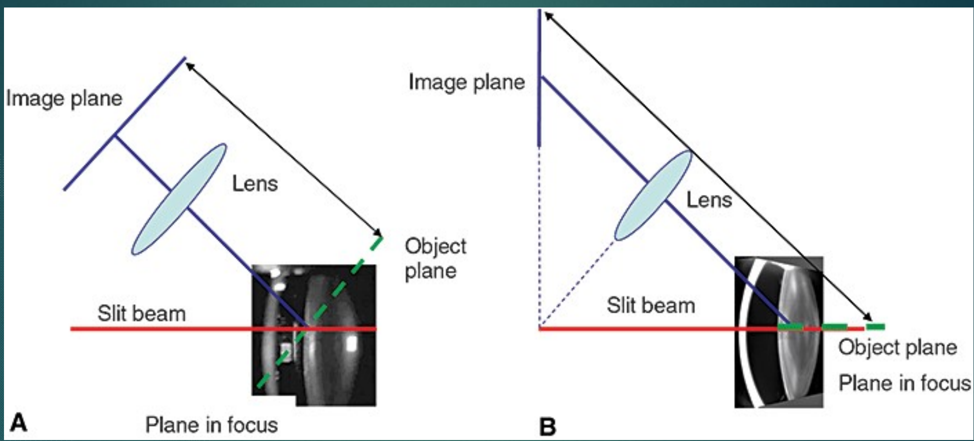


Scheimpflug Imaging



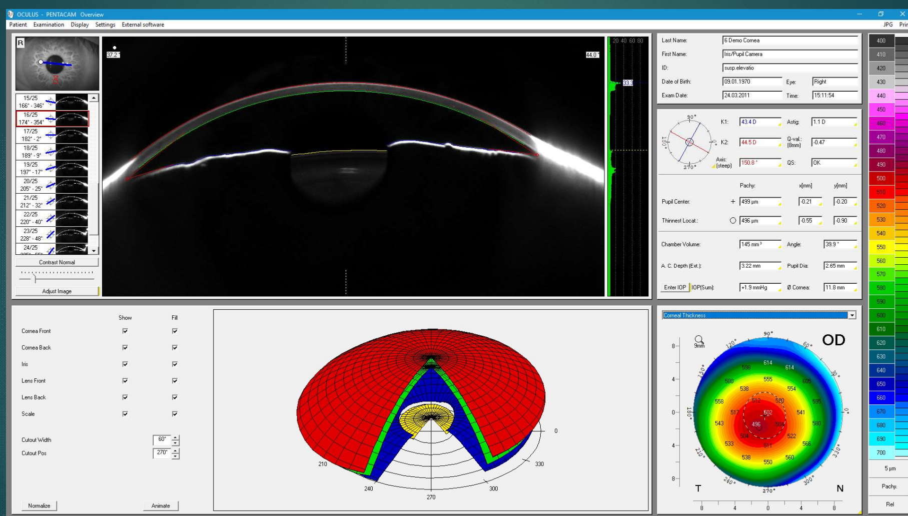
The Scheimpflug lidar method
Mikkel Brydegaard, Elin Malmqvist, Samuel Jansson, Jim Larsson, Sandra Török, Guangyu Zhao

Scheimpflug

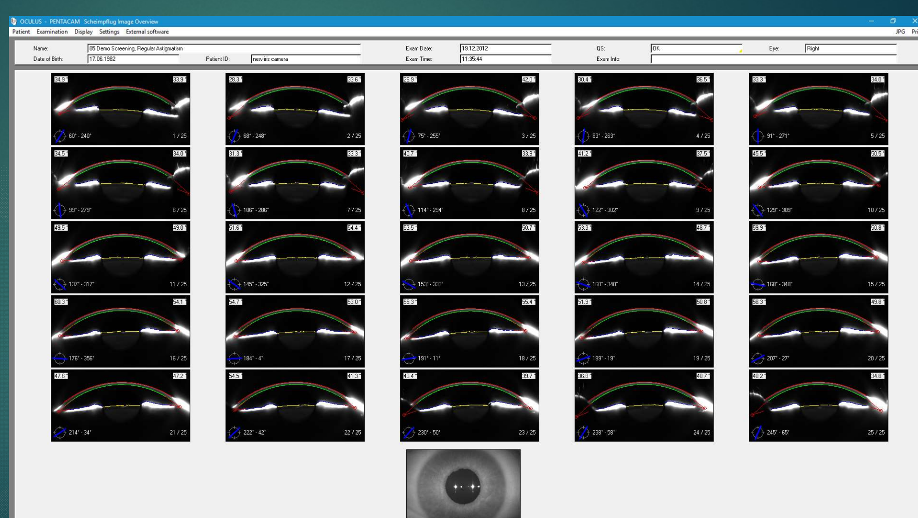


Intraocular lens alignment from Purkinje and Scheimpflug imaging
Patricia Rosales PhD, Alberto De Castro MSc, Ignacio Jiménez-Alfaro MD PhD, Susana Marcos PhD

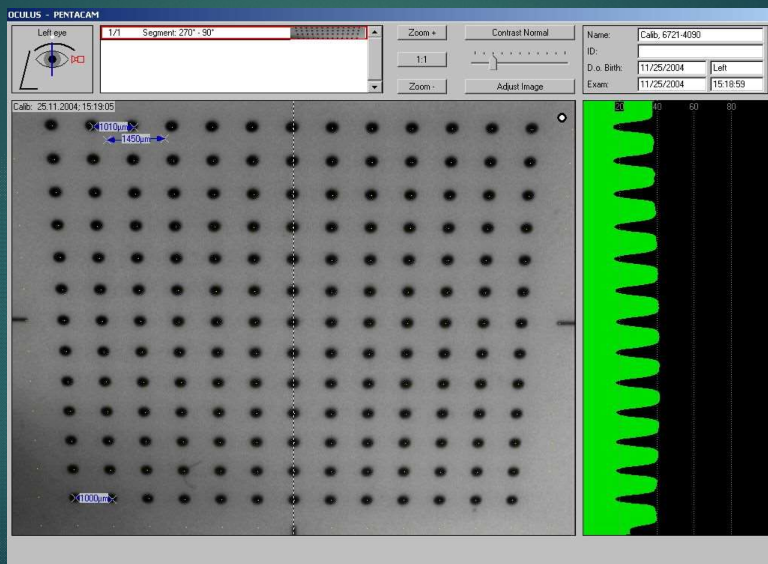
Scheimpflug



Scheimpflug



Keystone Distortion



Corneal Shape

- Axial Power – (sometimes incorrectly called sagittal power) gives a map of corneal curvature in terms of dioptric power. Is related to the equivalent sphere with the same slope at a given point.
- Instantaneous Power – (sometimes called tangential power) gives a map of corneal curvature in terms of dioptric power. Is related to the curvature (2^{nd} derivative) of a point on the cornea.
- Sag or Elevation – gives a map of the surface height at a given point. The height can be relative to a reference surface.

Total Corneal Power

$$\Phi = 1000\left[\frac{0.3771}{7.8} + \frac{1.3374 - 1.3771}{6.5}\right] \text{ Diopters}$$

$$\Phi = 48.346\text{ D} - 6.108\text{ D}$$

$$\Phi = 42.356\text{ D}$$

Find the equivalent power surface based solely on the anterior radius of curvature.

$$\frac{n' - 1}{7.8} = \frac{42.356}{1000}$$

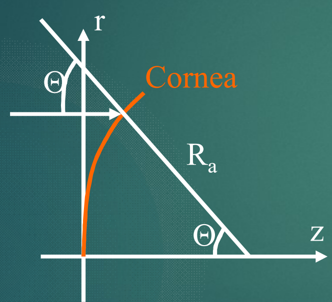
$$n' = 1.3304$$

Keratometric Index of Refraction

- Historically, a keratometric index of refraction n_k was chosen based on available data on the cornea and for mathematical simplicity.
- The index $n_k = 1.3375$, which gives a power of 45 D for a surface with radius 7.5 mm was chosen.
- The purpose of the index is to account for the power of the posterior cornea based solely on the radius of the front surface of the cornea.
- Still used today in keratometry and corneal topography.

Axial Power

Examine each meridian from the axis outward.



$$\sin\Theta = \frac{r}{R_a} \quad \text{and} \quad \tan\Theta = \frac{dz}{dr}$$

$$\frac{dz}{dr} = \frac{\sin\Theta}{\sqrt{1-\sin^2\Theta}}$$

$$\left(\frac{dz}{dr}\right)^2 = \sin^2\Theta\left(1+\left(\frac{dz}{dr}\right)^2\right)$$

$$\sin\Theta = \frac{dz/dr}{\sqrt{1+(dz/dr)^2}} = \frac{r}{R_a}$$

$$\Phi_a = \frac{(n_k-1)}{R_a} = \frac{(n_k-1)dz/dr}{r\sqrt{1+(dz/dr)^2}}$$

Axial Power - Example

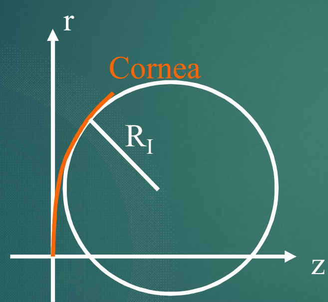
Sphere: $z = R - \sqrt{R^2 - r^2}$

$$\frac{dz}{dr} = \frac{r}{\sqrt{R^2-r^2}}$$

$$\Phi_a = \frac{n_k - 1}{R} = \text{constant}$$

Instantaneous Power

Examine each meridian from the axis outward.



From Calculus:
For a given curve in the r-z plane, there exists a circle of radius R_I which is tangent to the curve at (r,z) and has the same curvature $1 / R_I$ as the curve.

$$\frac{1}{R_I} = \frac{d^2z/dr^2}{\left[1+(dz/dr)^2\right]^{3/2}}$$

$$\Phi_I = \frac{n_k - 1}{R_I} = \frac{(n_k - 1)d^2z/dr^2}{\left[1+(dz/dr)^2\right]^{3/2}}$$

Instantaneous Power - Example

Sphere: $z = R - \sqrt{R^2 - r^2}$

$$\frac{dz}{dr} = \frac{r}{\sqrt{R^2 - r^2}}$$

$$\frac{d^2z}{dr^2} = \frac{R^2}{\left[R^2 - r^2\right]^{3/2}}$$

$$\Phi_i = \frac{n_k - 1}{R} = \text{constant}$$

Axial & Instantaneous Power

Define $f = \frac{dz}{dr}$

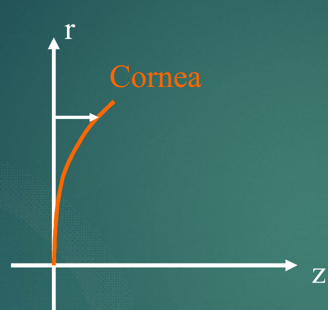
$$r\Phi_a = \frac{(n_k - 1)f}{\sqrt{1+f^2}}$$

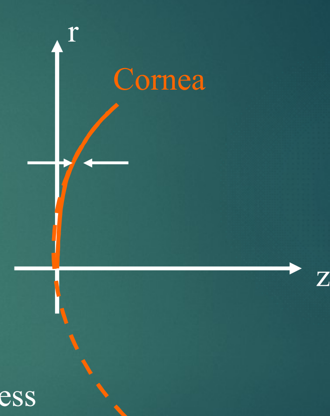
$$\frac{d(r\Phi_a)}{dr} = (n_k - 1)\left[\frac{df}{dr}\frac{1}{\sqrt{1+f^2}} + f\left(\frac{f\ df/dr}{(1+f^2)^{3/2}}\right)\right]$$

$$\frac{d(r\Phi_a)}{dr} = \Phi_I$$

$$\Phi_a(r) = \frac{1}{r}\int_0^r \Phi_I(r')dr'$$

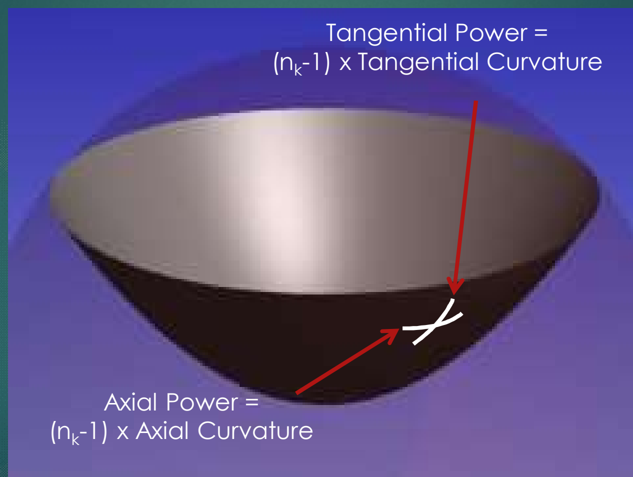
Elevation





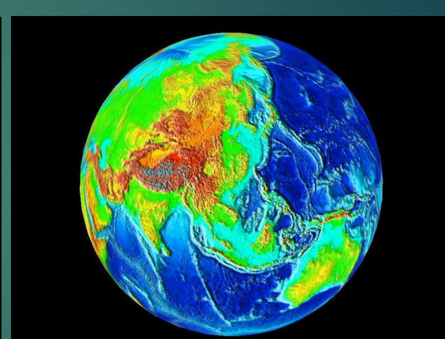
Elevation maps tend to obscure subtle features on the cornea, unless a reference surface is subtracted.

Differential Geometry



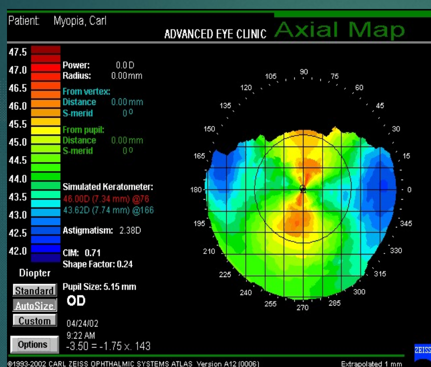
Elevation

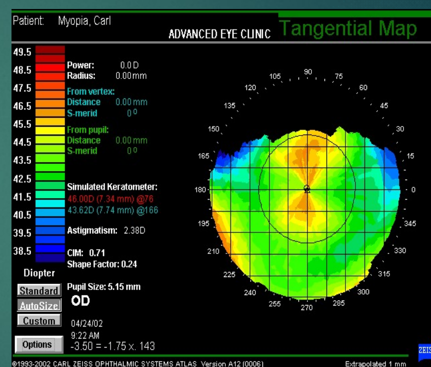
12 mile peak to valley



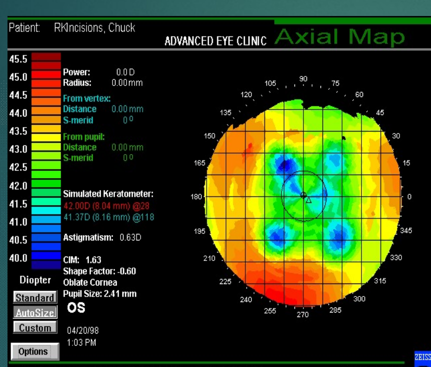
Relative to Sea Level

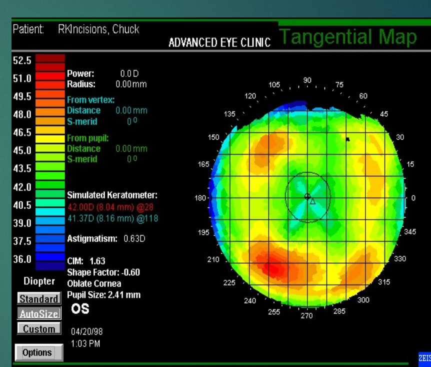
Astigmatism



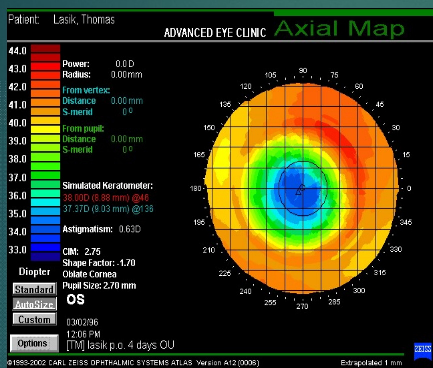


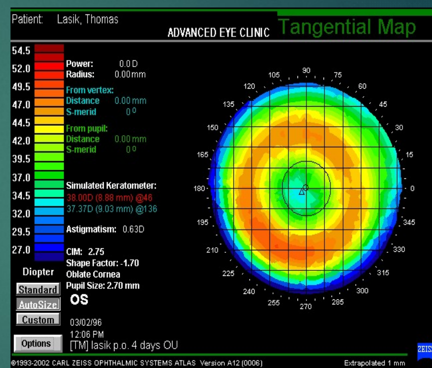
RK Incisions



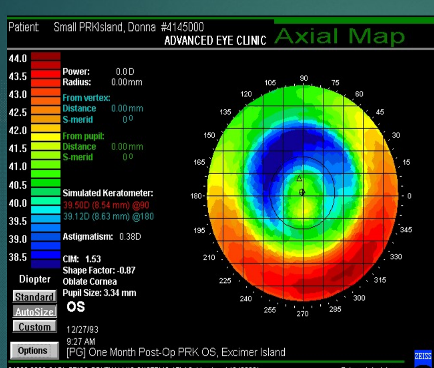


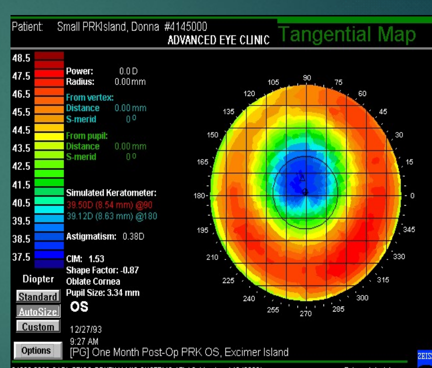
Lasik





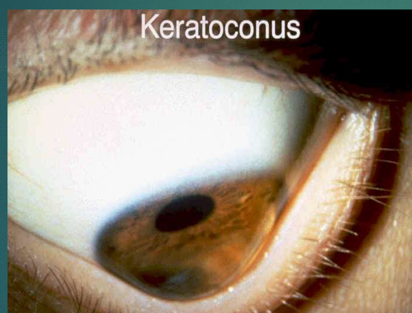
Central Island

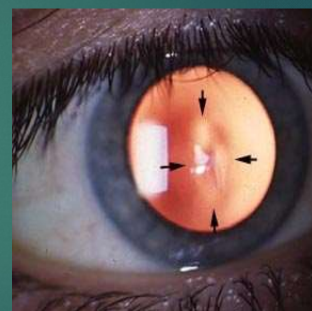




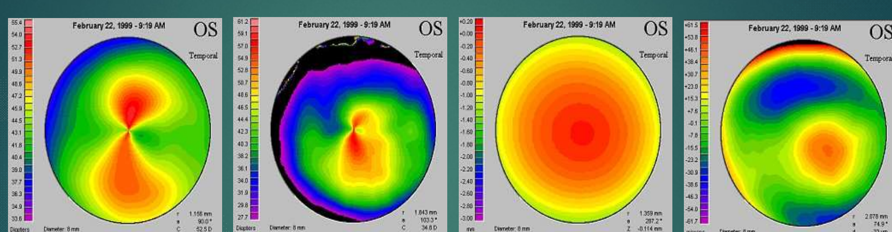
Keratoconus

Localized thin spot in the cornea that progressively bulges.





Map Comparison



Axial Instantaneous Elevation Elevation - Ref

www.euclidsys.com

Zernike Polynomials

- Application of Zernike polynomials has been used to represent both wavefront shape and corneal topography in the eye.
- Would like to recover basic shape information such as radius of curvature, astigmatism and asphericity based on Zernike coefficients.
- For wavefronts, radius of curvature and astigmatism is related to refractive error, and asphericity is related to spherical aberration.
- For corneal topography, radius of curvature and astigmatism is related to keratometry and asphericity is related to corneal eccentricity.

Radii and Astigmatic Axis

Consider the first six terms of a Zernike expansion of a surface, with $\rho = r / r_{max}$

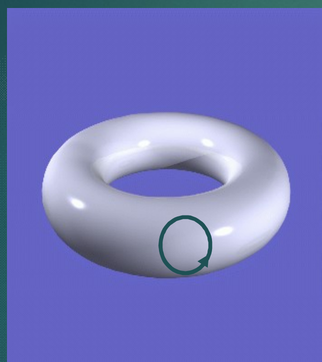
$$f(\rho,\theta) = a_{00} + 2a_{11}\rho\cos\theta + 2a_{1-1}\rho\sin\theta + \sqrt{6}a_{2-2}\rho^2\sin 2\theta + \sqrt{3}a_{20}\left(2\rho^2 - 1\right) + \sqrt{6}a_{22}\rho^2\cos 2\theta$$

If the z axis is perpendicular to the surface at the origin, then a_{11} and a_{1-1} are both zero. We can also switch from normalized coordinates to regular coordinates such that

$$f(r,\theta) = a_{00} + \sqrt{6}a_{2-2}\frac{r^2}{r_{max}^2}\sin 2\theta + \sqrt{3}a_{20}\left(2\frac{r^2}{r_{max}^2} - 1\right) + \sqrt{6}a_{22}\frac{r^2}{r_{max}^2}\cos 2\theta$$

Axis of Astigmatism

The height of an astigmatic surface will oscillate up and down as it is circumnavigated. The extrema will be along the principal meridia.



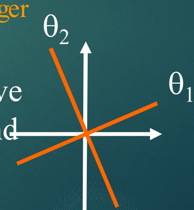
$$\frac{df}{d\theta}(r,\theta) = 2\sqrt{6}a_{2-2}\frac{r^2}{r_{max}^2}\cos 2\theta - 2\sqrt{6}a_{22}\frac{r^2}{r_{max}^2}\sin 2\theta = 0$$

$$a_{2-2}\cos 2\theta = a_{22}\sin 2\theta$$

$$\tan 2\theta = \frac{a_{2-2}}{a_{22}}$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{a_{2-2}}{a_{22}}\right) + \frac{m\pi}{2} \qquad m \text{ integer}$$

Two values of m will give θs that lie between 0° and 180°



Radii of Curvature

Along θ_1

$$f(r,\theta_1) = a_{00} + \sqrt{6}a_{2-2}\frac{r^2}{r_{max}^2}\sin 2\theta_1 + \sqrt{3}a_{20}\left(2\frac{r^2}{r_{max}^2} - 1\right) + \sqrt{6}a_{22}\frac{r^2}{r_{max}^2}\cos 2\theta_1$$

$$f(r,\theta_1) = \underbrace{a_{00} - \sqrt{3}a_{20}}_{\text{Constant Offset}} + \underbrace{r^2\left[\frac{\sqrt{6}}{r_{max}^2}\left(a_{2-2}\sin 2\theta_1 + a_{22}\cos 2\theta_1\right) + \frac{2\sqrt{3}}{r_{max}^2}a_{20}\right]}_{\text{Parabola}\ \frac{r^2}{2R_1}}$$

$$R_1 = \frac{r_{max}^2}{2}\left[\sqrt{6}\left(a_{2-2}\sin 2\theta_1 + a_{22}\cos 2\theta_1\right) + 2\sqrt{3}a_{20}\right]^{-1}$$

A similar expression holds for θ_2

Average Conic Constant

Equation of a conic

$$z = \frac{1}{K+1}\left[R - \sqrt{R^2 - (K+1)r^2}\right] \cong \left[\frac{r^2}{2R} + \frac{(K+1)r^4}{8R^3} + \ldots\right]$$

Find expansion terms that go as ρ^4

$$K = \frac{8R^3}{r_{max}^4}\left[6\sqrt{5}a_{40} - 30\sqrt{7}a_{60} + 270a_{80} + \cdots\right] - 1$$

Spherical Aberration

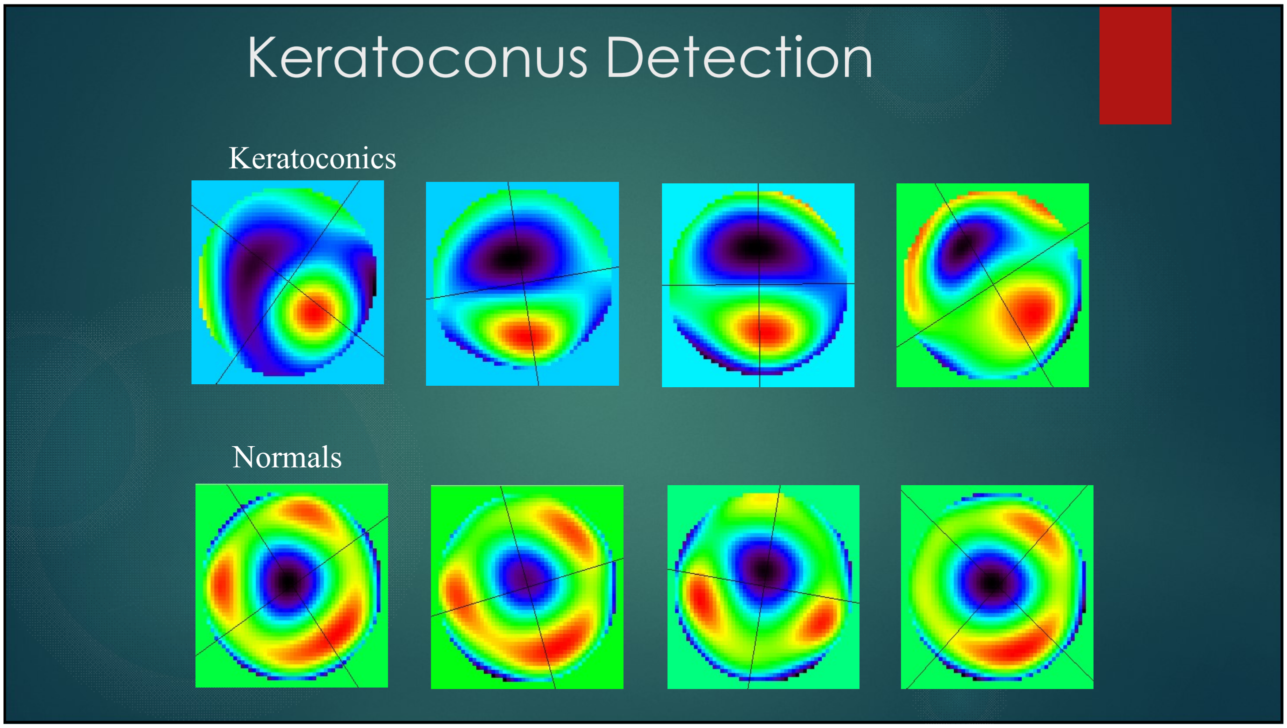
$$W(\rho,\theta) = a_{40}\sqrt{5}\left(6\rho^4 - 6\rho^2 + 1\right)$$

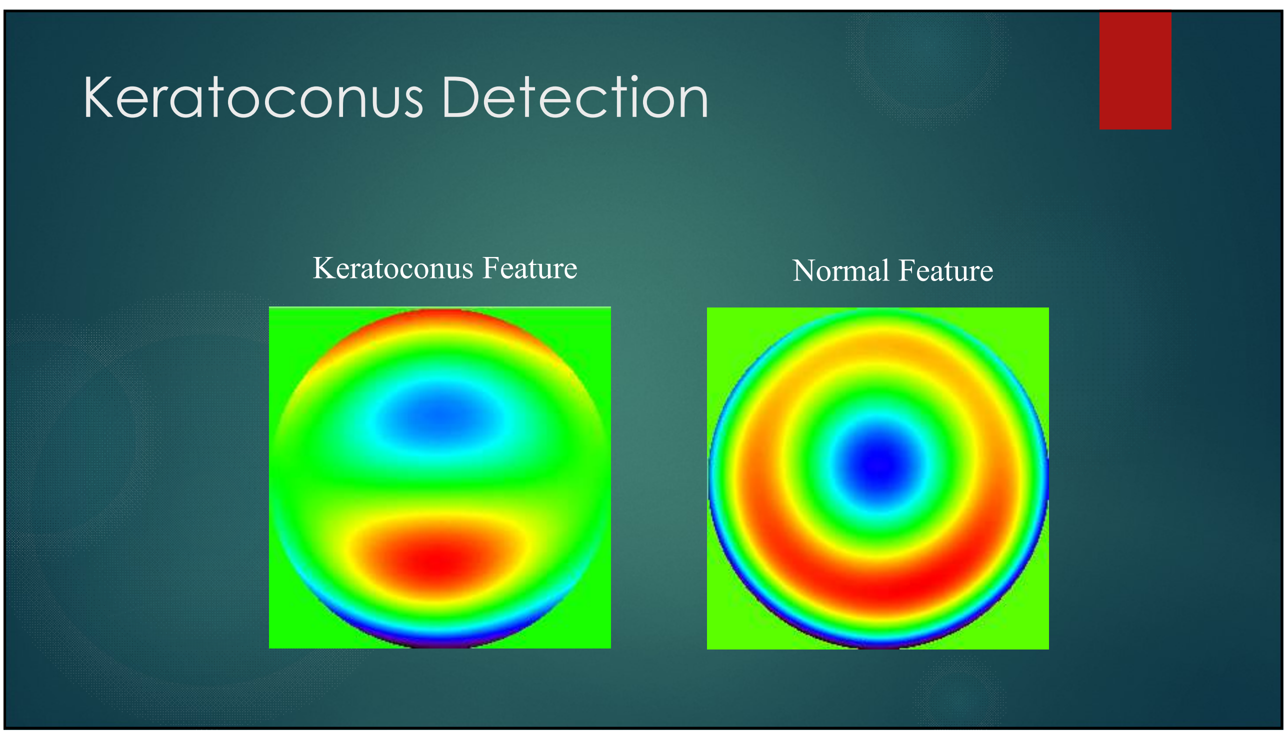
$$d\phi = \frac{-1000}{r}\frac{dW(r,\theta)}{dr} \quad \text{(in Diopters for W in mm)}$$

$$W(r,\theta) = a_{40}\sqrt{5}\left(6(r/r_{max})^4 - 6(r/r_{max})^2 + 1\right)$$

$$d\phi = \frac{-1000}{r}a_{40}\sqrt{5}\left(24\frac{r^3}{r_{max}^4} - 12\frac{r}{r_{max}^2}\right)$$

$$d\phi = \frac{-24000a_{40}\sqrt{5}}{r_{max}^4}r^2 + \frac{12000a_{40}\sqrt{5}}{r_{max}^2}$$


Keratoconus Detection
Keratoconics
Normals


Keratoconus Detection
Keratoconus Feature
Normal Feature

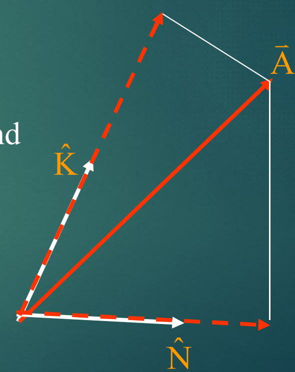
Keratoconus Detection

$\hat{K}$ = unit vector of average higher order coefficients of Keratoconus Patients

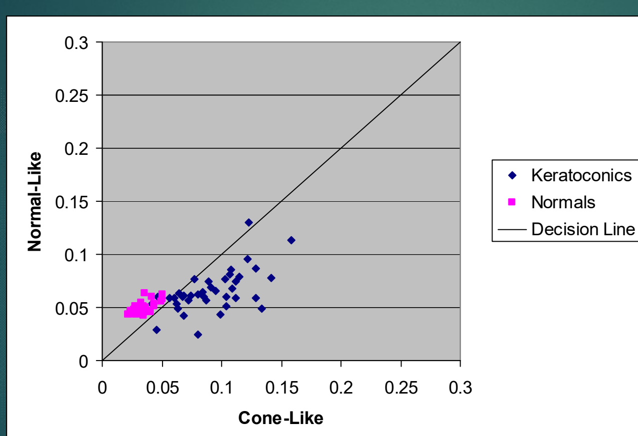
$\hat{N}$ = unit vector of average higher order coefficients of Normal Patients

$\vec{A} = < a_{3,-3}, a_{3,-1}, a_{3,1}, a_{3,3}, a_{4,-4}, \ldots >$

The dot product of the feature vectors and the vector under test gives a measure of how much the test vector looks like the feature vector.



Keratoconus Detection



35/40 (87%) Cones Correctly Classified

40/40 (100%) Normals Correctly Classified

Misclassified Cones