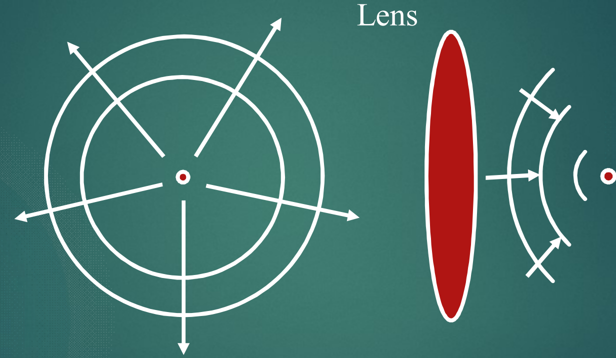
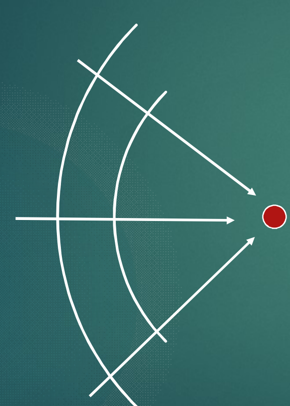
Aberration Theory



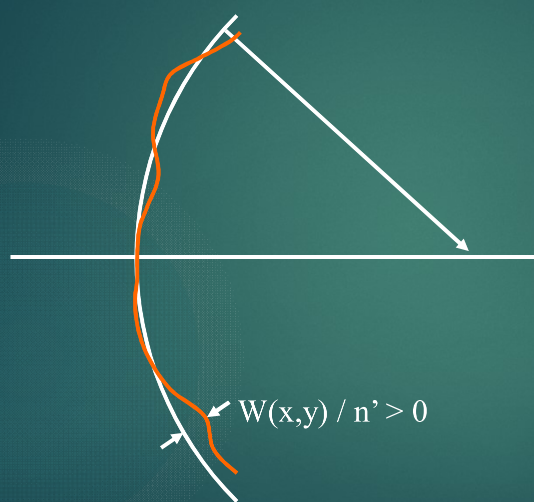
Optical systems convert
the shapes of wavefronts

Aberrations



A perfectly spherical wave will converge to a point. Any deviation from the ideal spherical shape is said to be an aberration.

Wavefront Error

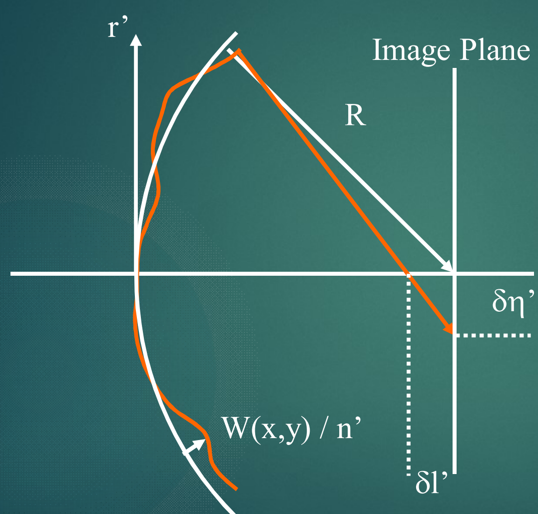


The wavefront error W(x,y) is the difference between a perfect spherical wave and the actual wavefront. It is usually measured in the exit pupil and W(0,0) = 0.

The axis for this system is the Line of Sight.

In general, there is no symmetry in the eye, so W(x,y) can take on any complex shape.

Wavefront Error



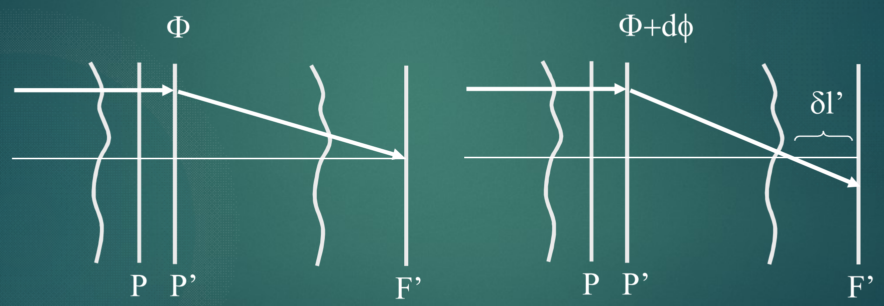
A reference sphere of radius R is taken as the ideal spherical wavefront. R is just the distance from the exit pupil to the image plane.

$\delta\eta'$ is the transverse ray error and represents the distance between where the ideal and the actual rays strike the image plane.

dl' is the longitudinal ray error and represents the distance the actual ray crosses the line of sight from the image plane.

Power Error

In Visual Optics, longitudinal aberrations are usually given in terms of a “power error” as opposed to a distance.



Power Error

The “power error” is related to the slope of the wavefront

$$d\phi = \frac{1}{r}\frac{\partial W(r)}{\partial r}$$

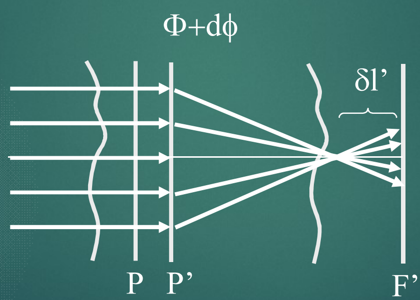
Defocus Example: 2λs of defocus (λ = 0.5 μm) at edge of 6 mm pupil

$$W(r) = \frac{2(0.5\times10^{-3}\ \text{mm})}{(3\ \text{mm})^2} r^2 = 0.0001111\times r^2$$

$$d\phi = 0.0002222\ \text{mm}^{-1} = 0.2222\ \text{Diopters}$$

Power Error

The preceding defocus example shows that $d\phi$ is constant, which means that dl' is the same regardless of where the ray entered the eye.

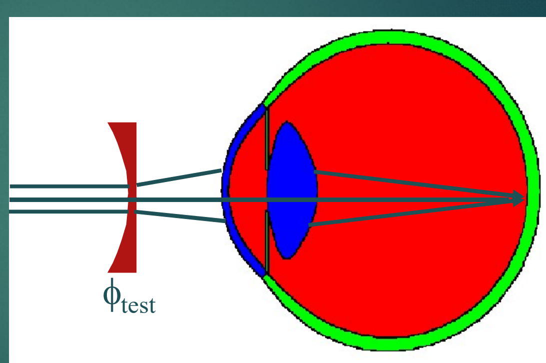


Measuring Defocus in the Eye

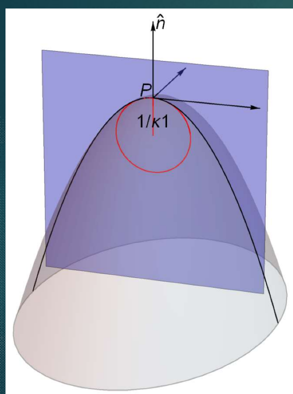
Ideally, put a test lens at the principal plane.

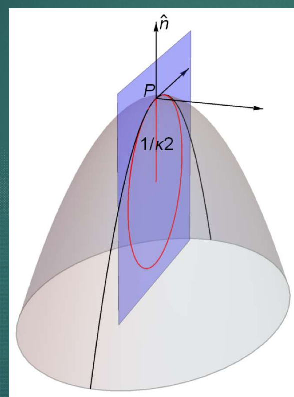
Power changes slightly as lens is moved from the principal plane.

Myopia and hyperopia are defocus, where the prescription is the power of the test lens that cancels $d\phi$.



Differential Geometry





- Every point on a continuous surface has two Principal Curvatures.
- These curvatures represent the maximum and minimum curvature through this point and
- The principal curvatures are always along orthogonal axes.
- Calculated from the Fundamental Forms)

Fundamental Forms

First Fundamental Form

$$E = 1 + \left(\frac{\partial f}{\partial x}\right)^2 \qquad F = \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right) \qquad G = 1 + \left(\frac{\partial f}{\partial y}\right)^2$$

Second Fundamental Form

$$L = \frac{\partial^2 f / \partial x^2}{\left[EG - F^2\right]^{1/2}} \qquad M = \frac{\partial^2 f / \partial x \partial y}{\left[EG - F^2\right]^{1/2}} \qquad N = \frac{\partial^2 f / \partial y^2}{\left[EG - F^2\right]^{1/2}}$$

Curvatures

Mean Curvature

$$H = \frac{EN + GL + 2FM}{2(EG - F^2)} = \frac{1}{2}(\kappa 1 + \kappa 2)$$

Gaussian Curvature

$$K = \frac{LN - M^2}{EG - F^2} = \kappa 1 \kappa 2$$

Principal Curvatures

$$\kappa 1 = H + \sqrt{H^2 - K}$$

$$\kappa 2 = H - \sqrt{H^2 - K}$$

Axial Astigmatism

Since the eye is not rotationally symmetric, astigmatism can appear on-axis. This astigmatism is primarily due to the ocular surfaces having a toric or biconic shape.

$$z = R_x - \sqrt{\left(R_x - R_y + \sqrt{R_y^2 - y^2}\right)^2 - x^2}$$

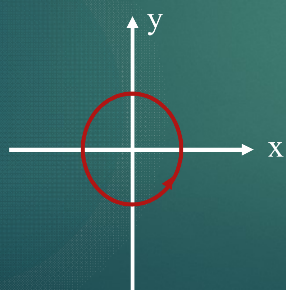
Principal Curvatures are $1/R_x$ and $1/R_y$ at the origin

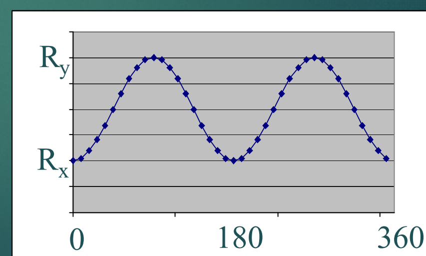
Biconic Surfaces

Computationally similar to the toric surface, but more versatile since the biconic surface can add asphericity.

$$z = \frac{x^2/R_x + y^2/R_y}{1 + \sqrt{1 - (1+K_x)\frac{x^2}{R_x^2} - (1+K_y)\frac{y^2}{R_y^2}}}$$

Principal Curvatures are $1/R_x$ and $1/R_y$ at the origin





Astigmatic Surfaces

- The axes with the maximum and minimum radii of curvature are called the Principal Meridia (any axis is called a meridian).
- There is a steep meridian corresponding to the minimum radius of curvature.
- There is a flat meridian corresponding to the maximum radius of curvature.
- The principal meridia are always perpendicular to each other.

Astigmatic Surfaces

In general, and in the eye, the principal meridia do not lie along the x and y axis, but are rotated through some angle θ_o.

$$z = \frac{r^2\cos^2(\theta-\theta_o)/R_x + r^2\sin^2(\theta-\theta_o)/R_y}{1+\sqrt{1-(1+K_x)\frac{r^2\cos^2(\theta-\theta_o)}{R_x^2}-(1+K_y)\frac{r^2\sin^2(\theta-\theta_o)}{R_y^2}}}$$

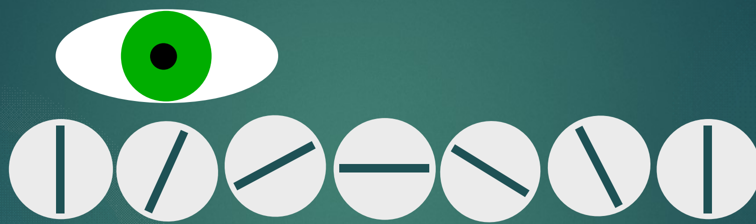
Astigmatic Power Error

$$W(r) = Ax^2 + By^2 = Ar^2\cos^2\theta + Br^2\sin^2\theta$$

$$d\phi = \frac{1}{r}\frac{\partial W(r)}{\partial r} = 2A\cos^2\theta + 2B\sin^2\theta$$

For a given value of θ, $d\phi$ is a constant. Axial astigmatism can be thought of as defocus that depends on meridian.

Axial Astigmatism



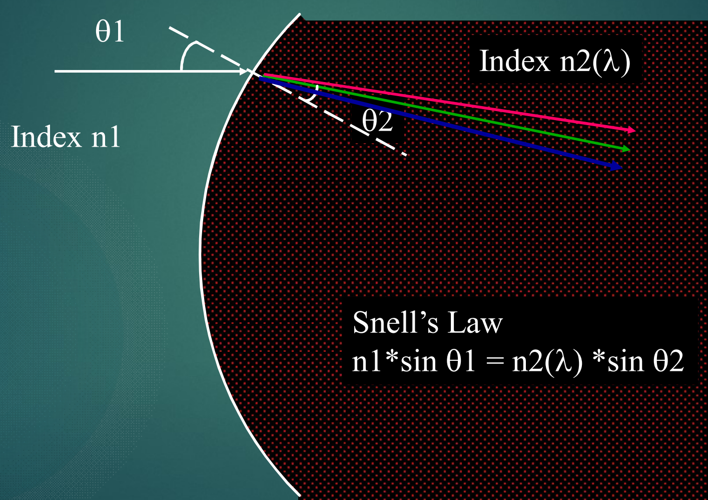
If a refraction is performed through a series of slits rotated at various angles, the refractive error will oscillate between a minimum and maximum value in the presence of axial astigmatism.

Jackson Crossed Cylinder

Crossed cylinders have a power +D along one meridian and a power -D along the orthogonal meridian.

Spherical Equivalent = Average power

Chromatic Aberration



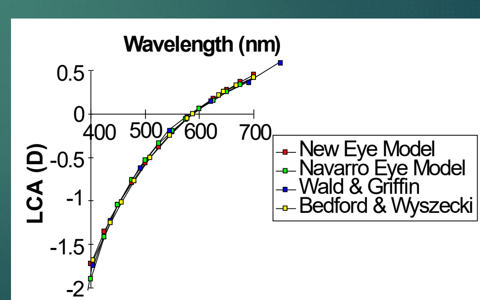
Chromatic Aberration

Chromatic Aberration is wavelength dependent defocus

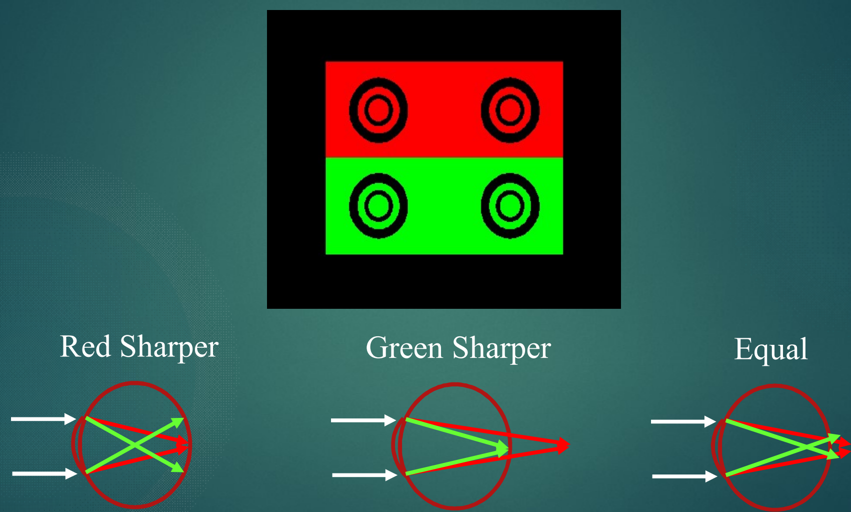
$$W(r) = A(\lambda) r^2$$

$$d\phi(\lambda) = 2A(\lambda)$$

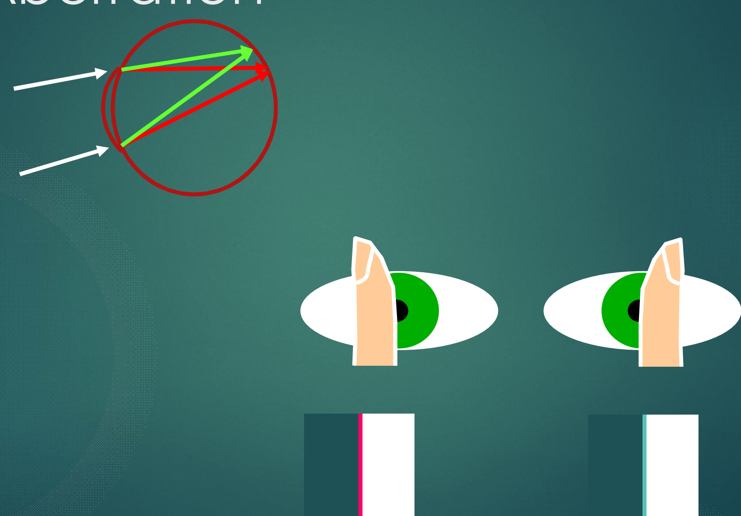
For a given λ, dϕ is constant



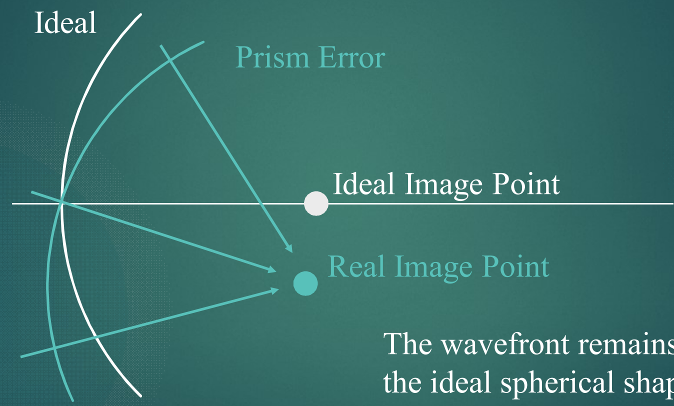
Duochrome (Bichrome) Test



Transverse Chromatic Aberration



Wavefront Tilt or Prism

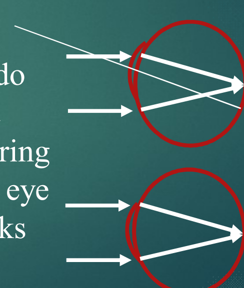


The wavefront remains
the ideal spherical shape, but it is
tilted relative to the ideal wavefront.

Wavefront Tilt

Strabismus or “Lazy Eye” causes wavefront tilt

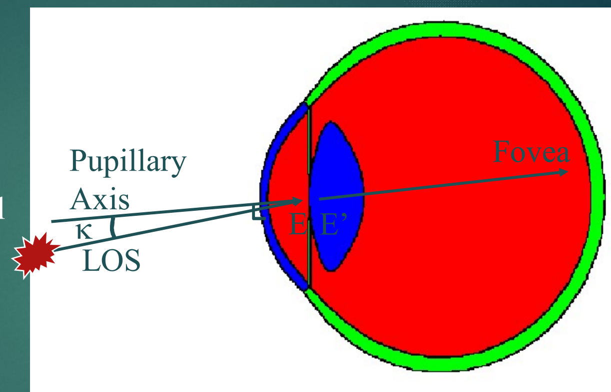
The brain sees independent images that do not fuse in the brain. The brain does not like this discrepancy, so it will start ignoring one image. If this is not fixed early, one eye will become “blind”, even though it works fine from an optical standpoint.

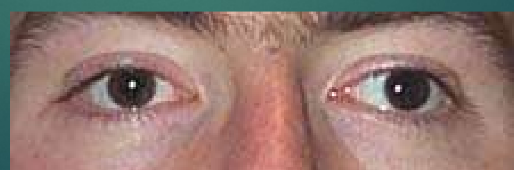


Hirschberg Test

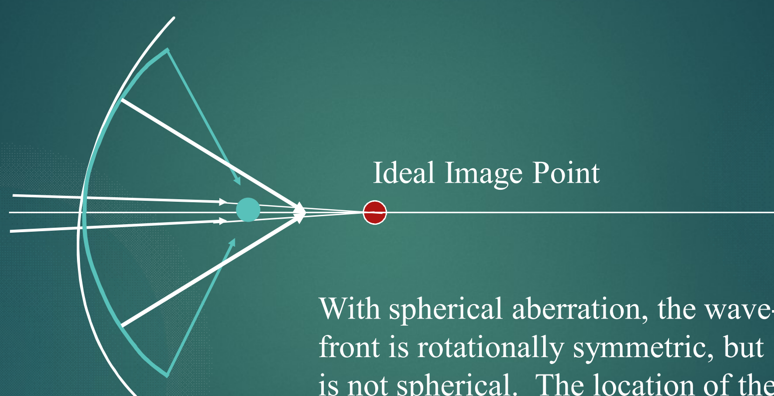
The subject fixates on a point source. The reflection of the source should be slightly nasal of the pupil center.

Asymmetry between the eyes suggests ocular misalignment



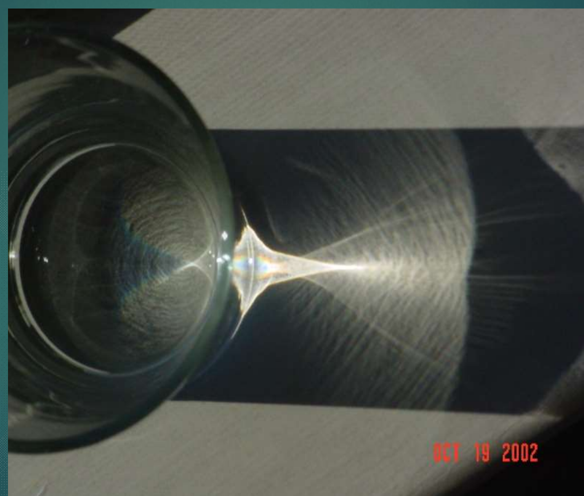


Spherical Aberration



With spherical aberration, the wave-front is rotationally symmetric, but is not spherical. The location of the focus varies with the distance from the centerline.

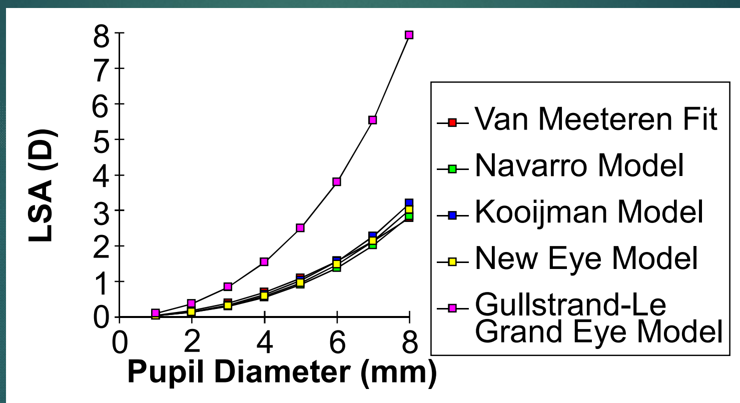
Spherical Aberration



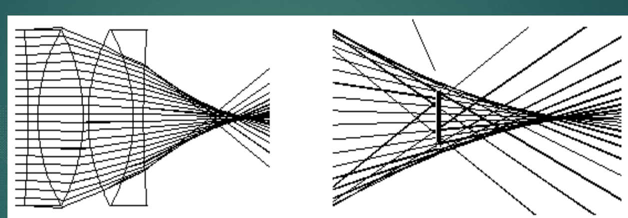
Spherical Aberration

If a refraction is performed through a series of annular rings, the refractive error will vary with the size of the average ring diameter, when spherical aberration is present.

Longitudinal Spherical Aberration

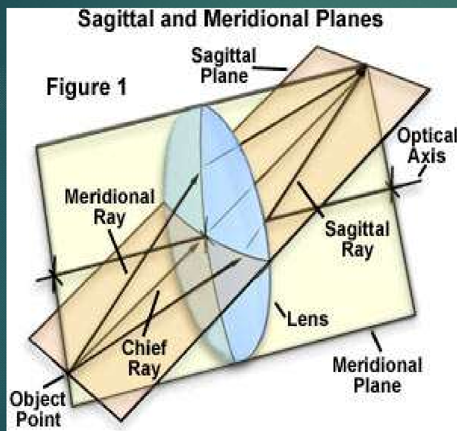


Night Myopia



There is a myopic shift of the smallest point that is formed on the retina as the pupil dilates.

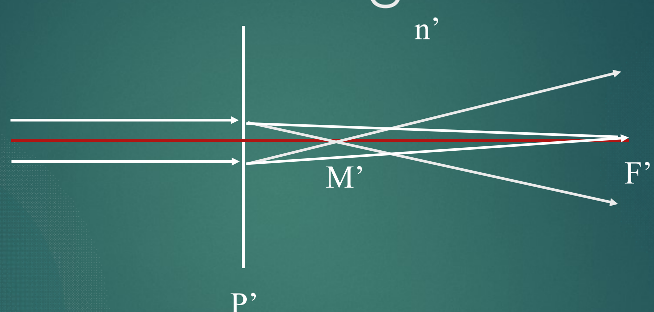
Oblique Astigmatism



"Off-axis" astigmatism Δ given as the difference between the meridional and sagittal foci

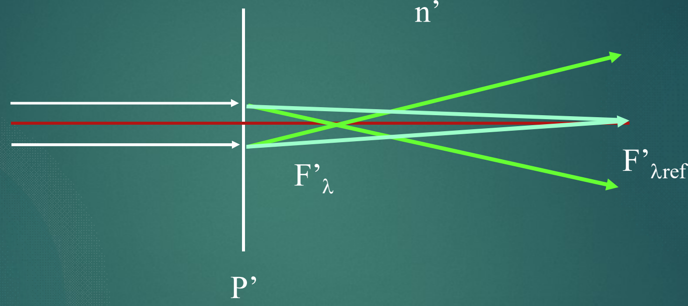
$\Delta = \alpha^{1.5} \times 10^{-2}$ Diopters
where α is the field angle in degrees

Aberrations From Raytracing Defocus and Axial Astigmatism



$$d\phi = \frac{n'}{P'F'} - \frac{n'}{P'M'}$$

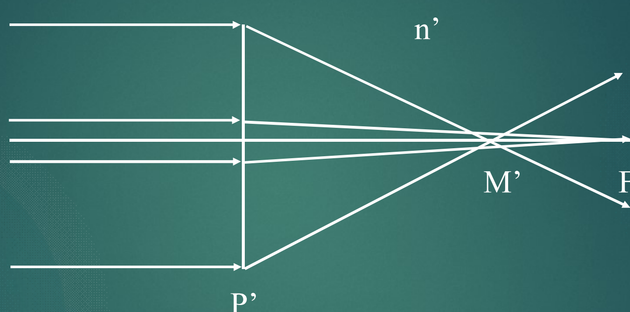
Aberrations From Raytracing
Chromatic Aberration



$$d\phi = \frac{n'}{P'F'_{\lambda ref}} - \frac{n'}{P'F'_{\lambda}}$$

λ_{ref} is typically 587.6 nm

Aberrations From Raytracing
Spherical Aberration



$$d\phi = \frac{n'}{P'M'} - \frac{n'}{P'F'}$$