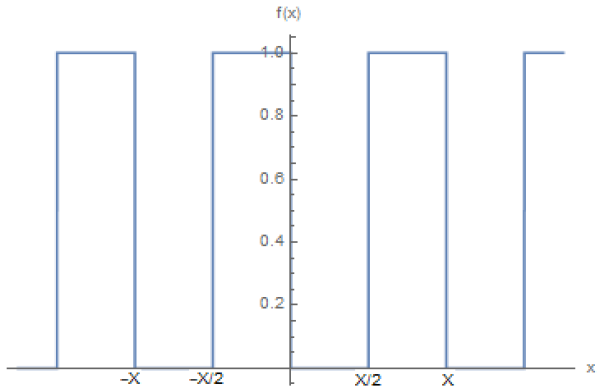
OPTI 512 Review Problem 1

Let's look at another variation on the Fresnel Zone Plate. Consider the periodic function $f(x)$ shown below. The function $f(x) = 1$ for $-X/2 < x \leq 0$ and $f(x) = 0$ for $0 < x \leq X/2$ and then repeats on intervals of X.



(a) Find the complex Fourier series coefficient a_m representing $f(x)$.

$$a_m = \frac{1}{X}\int_{-X/2}^{X/2} f(x)exp(-i2\pi m\xi_o x)dx$$

$$a_m = \frac{1}{X}\int_{-X/2}^{0} exp(-i2\pi m\xi_o x)dx$$

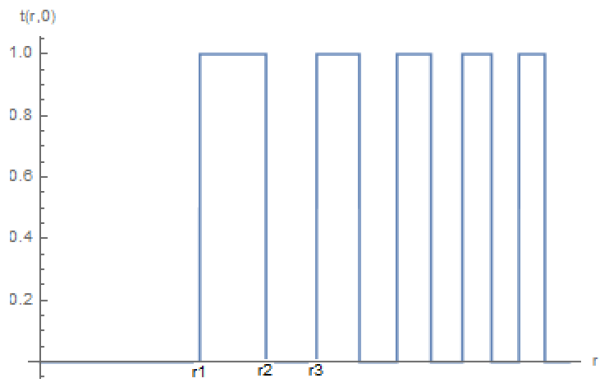
$$a_m = \frac{1}{X}\frac{exp(-i2\pi m\xi_o 0) - exp\big(-i2\pi m\xi_o(-X/2)\big)}{-i2\pi m\xi_o}$$

Remember that $\xi_o = 1/X$.

$$a_m = \frac{1 - exp(i\pi m)}{-i2\pi m} = \frac{exp(i\pi\, m/2)}{-i2\pi m}\big(exp(-i\pi\, m/2) - exp(i\pi\, m/2)\big)$$

$$a_m = \frac{1}{2}exp(i\pi\, m/2)sinc\left(\frac{m}{2}\right)$$

(b) The transmission of the zone plate is now given by $t(r,\theta) = f(r^2)$ and is rotationally symmetric. Plot $t(r,0)$ for the first five transparent zones.



(c) Find a general expression for the values r_j which is the radial position where the jth transition between the opaque and transmissive regions occur.

The transitions occur whenever $x = r_j^2 = jX/2$, *so* $r_j = \sqrt{jX/2}$.

(d) Calculate the diffraction efficiency η_m.

$$\eta_m = |a_m|^2 = \frac{1}{4} sinc^2\left(\frac{m}{2}\right)$$

(e) How does this pattern differ from the example Fresnel Zone Plate we did in class (notes 144-149)? How is this zone plate identical to the class example?

This pattern is the compliment to the one done in class. Here the center zone is opaque and then the rings start to alternate. However, even with this change, the focusing properties and diffraction efficiency for all of the orders are exactly the same as the class example.

Know how to do the phase only version of this problem, much like the phase reversal zone plate on pages 149-150 of the notes.