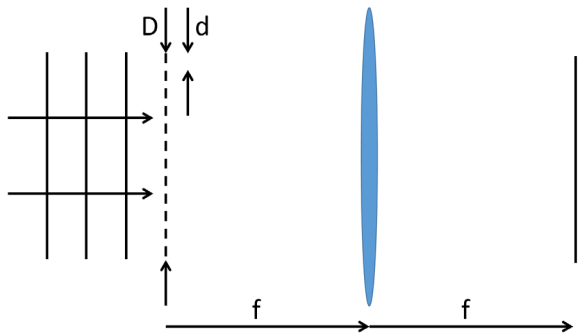
OPTI 512 Review Problem 2

A Ronchi ruling is a glass plate with alternating transparent and opaque strips. The transmission of the ruling is given by

$$t(x,y) = rect\left(\frac{x}{D},\frac{y}{D}\right)\left[rect\left(\frac{2y}{d}\right) ** \frac{1}{d}comb\left(\frac{y}{d}\right)\right],$$

Where D is the width and height of the ruling and d is the period of the ruling pattern.



If the ruling is illuminated with a unit amplitude plane wave and the ruling is located in the front focal plane of a thin lens (you can ignore the effects of the pupil function of the lens), do the following:

(a) Show that the irradiance pattern in the rear focal plane of the lens is

$$I(x_f,y_f,2f) = \left|U(x_f,y_f,2f)\right|^2 = \frac{D^4}{4\lambda^2 f^2} sinc^2\left(\frac{m}{2}\right)\left[\sum_m sinc\left(\frac{Dx_f}{\lambda f}, D\left(\frac{y_f}{\lambda f} - \frac{m}{d}\right)\right)\right]^2$$

(b) The irradiance pattern is often approximated by

$$I(x_f,y_f,2f) \cong \frac{D^4}{4\lambda^2 f^2}\sum_m sinc^2\left(\frac{m}{2}\right) sinc^2\left(\frac{Dx_f}{\lambda f}, D\left(\frac{y_f}{\lambda f} - \frac{m}{d}\right)\right),$$

Which basically says that the overlap between adjacent *sinc* functions is negligible. Plot the approximation of $I(0,y_f,2f)$ for $-10\ mm \le y_f \le 10\ mm$, $D = 1\ mm$, $f = 100\ mm$, $\lambda = 0.5\ \mu m$, and $d = 10\ \mu m$ to convince yourself that this is a valid approximation.

(c) Find a general expression for the location y_f of the peak in the case $m = 1$.

(d) This setup can be used as a spectrometer to determine the wavelength distribution or spectrum of the incident light. The input now is a series of plane waves, each with a different wavelength and amplitude corresponding to its spectral content. The $m = \pm 1$ peaks will disperse into a weighted version of the incident spectrum. If a sensor with $5\ \mu m$ pixels is placed at the rear focal plane of the lens, what is the minimum wavelength difference $\Delta\lambda$ that can be resolved with the setup? Use $f = 100\ mm$ and $d = 10\ \mu m$. Do the peaks move inwards or outwards when going from longer to shorter wavelengths?