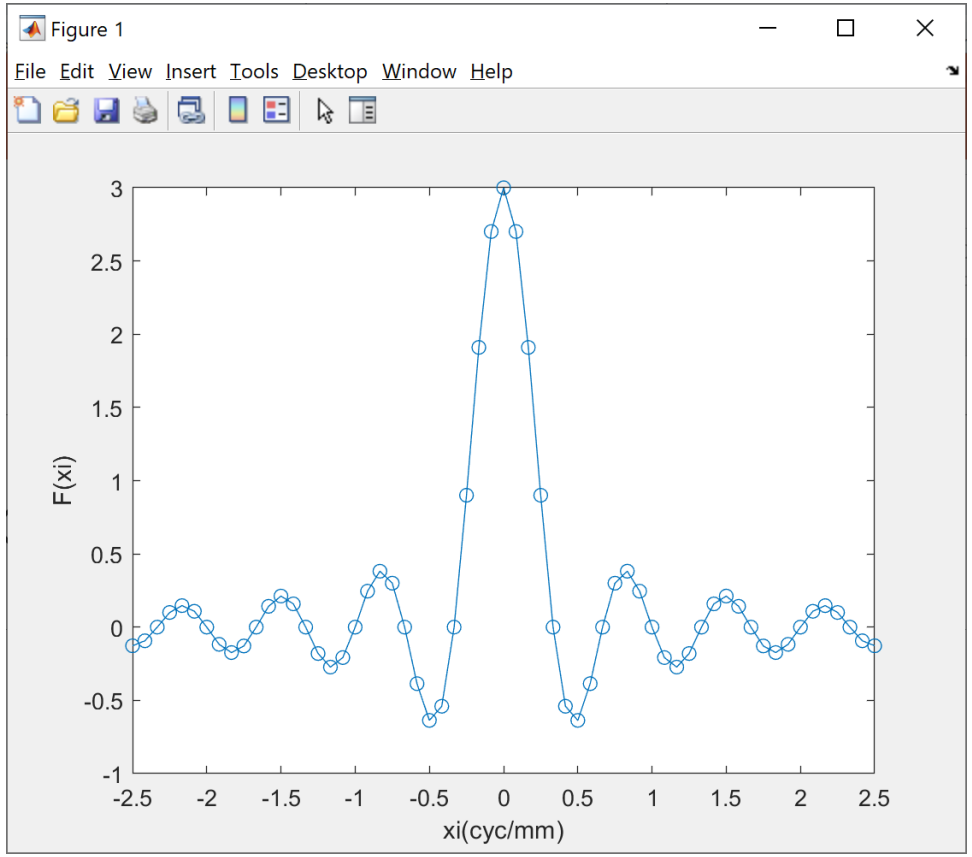
Modeling the Inverse Fourier Transform



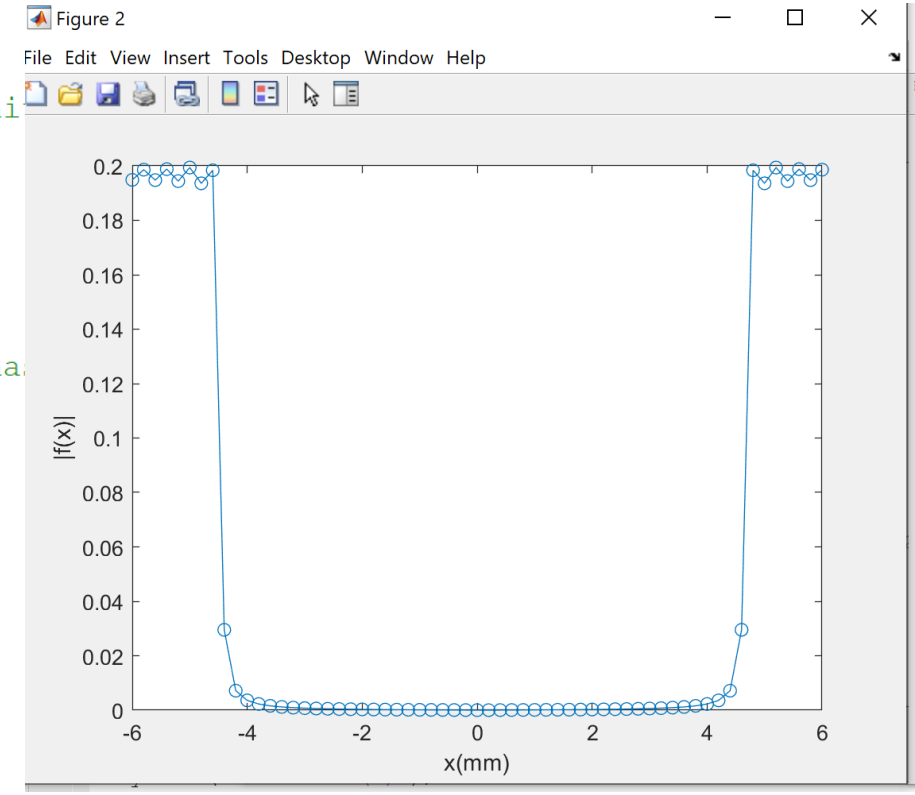
$L_\xi = 5\,cyc/mm$

```
%model inverse transform of 3sinc(3 xi) = rect(x / 3)
b=3.0; %rectangle width (mm)
L_xi=5.0; %vector side length in Fourier space (cyc/mm)
N=60; %number of samples
delta_xi=L_xi/N; %sample interval (cyc/mm)

xi=-L_xi/2:delta_xi:L_xi/2; %coordinate vector
F=b*sinc(b*xi);  %our function in Fourier space

figure(1)
plot(xi,F,'-o'); %plot F(xi) = b sinc(b*xi)
axis([-2.5 2.5 -1 3.0]);
xlabel('xi(cyc/mm)');
ylabel('F(xi)');
```

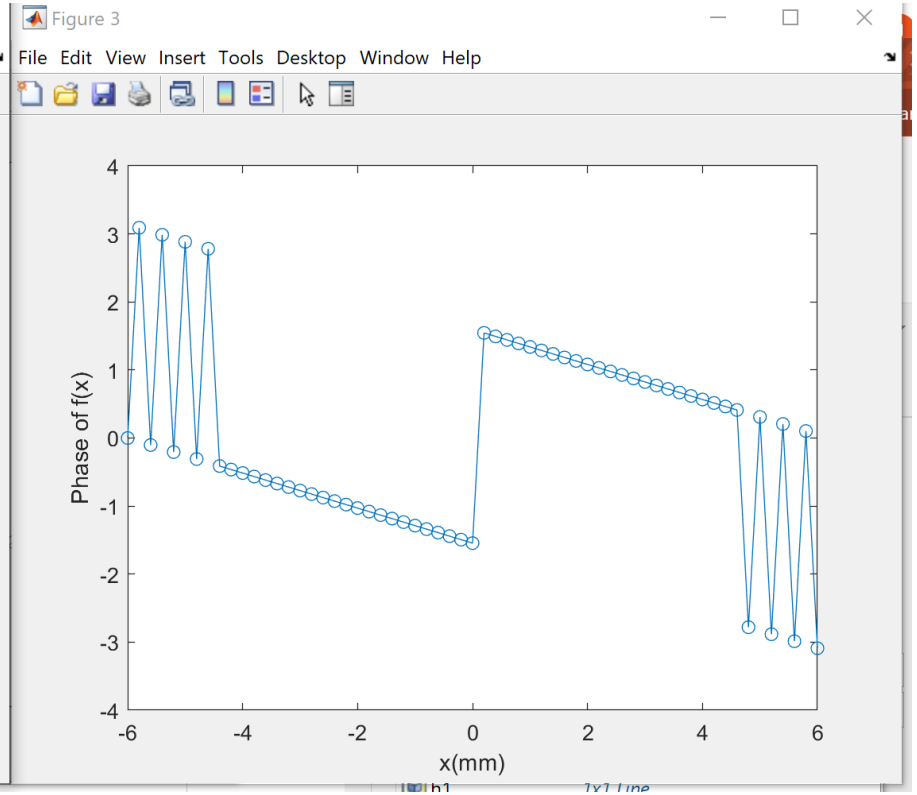
Inverse transform with No Shift

```
Xs=-0.5/xi(1);  %spacing in space domain
xmax=N*Xs/2;  %half width in space domain
x=-xmax:Xs:xmax;
f=ifft(F);
figure(2)
plot(x,abs(f),'-o'); %plot magni
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('|f(x)|');

figure(3)
plot(x,angle(f),'-o'); %plot pha
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('Phase of f(x)');
```

No Shift



Amplitude

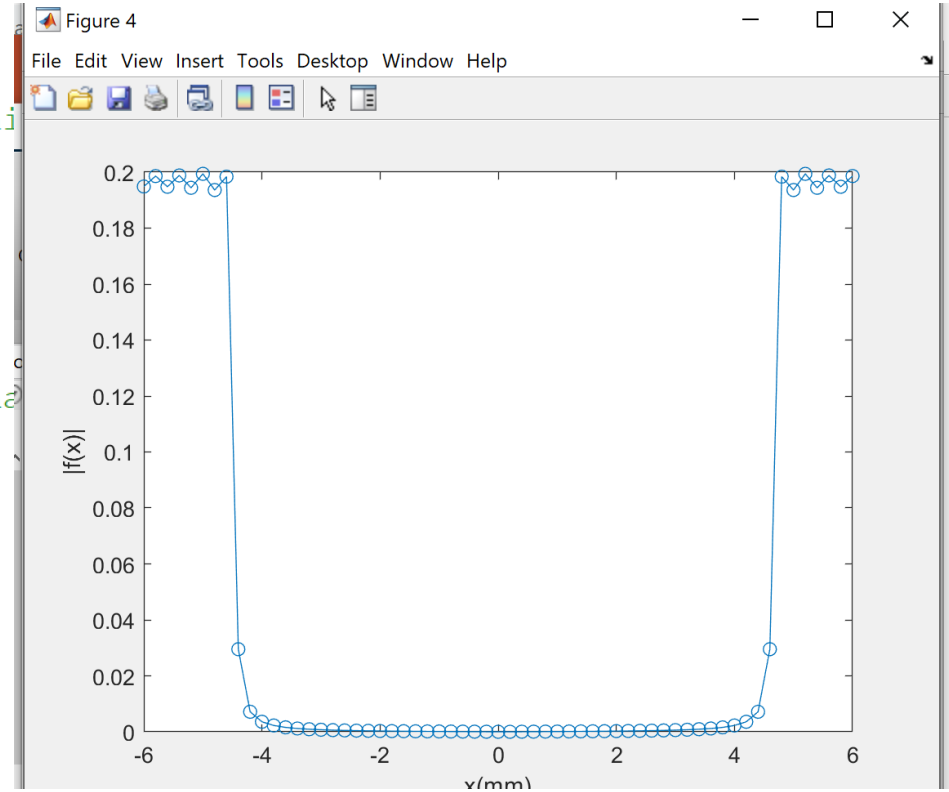


Phase

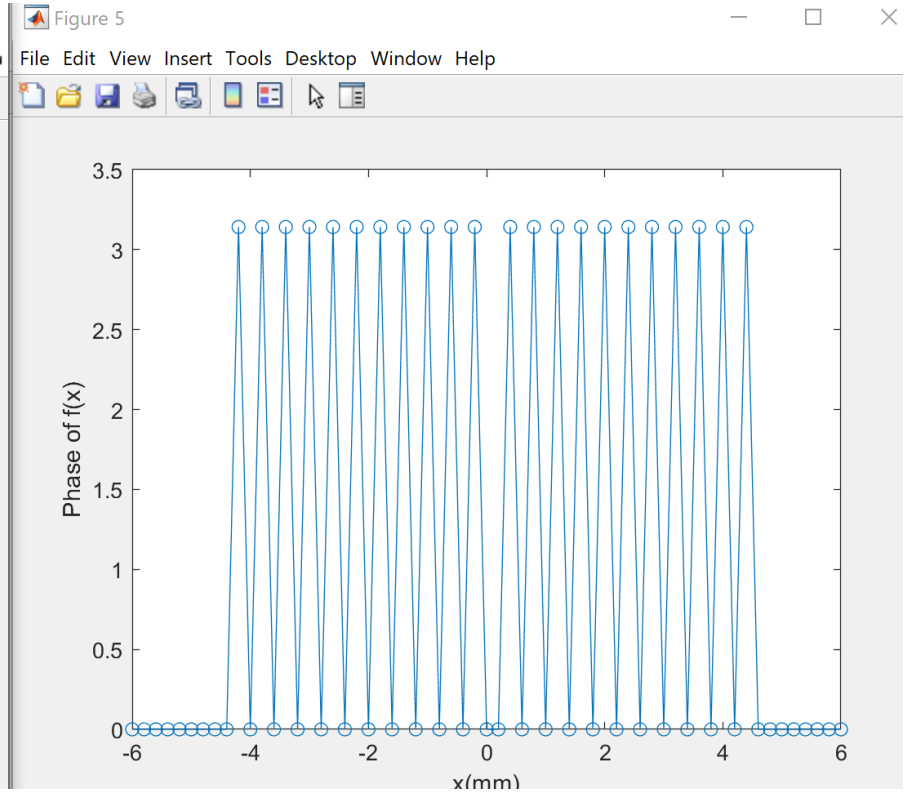
Inverse transform with F(ξ) shifted

```
%Let's shift just F first
f=ifft(ifftshift(F));
figure(4)
plot(x,abs(f),'-o'); %plot magni
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('|f(x)|');

figure(5)
plot(x,angle(f),'-o'); %plot pha
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('Phase of f(x)');
```

Shift F



Amplitude



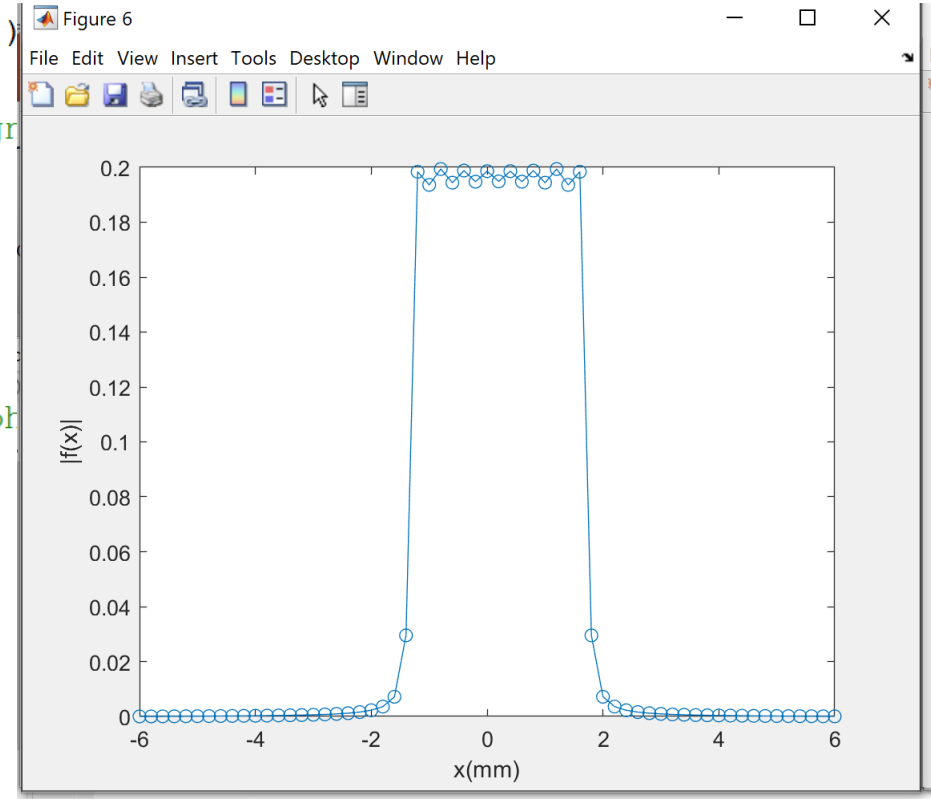
Phase

Inverse transform with F(ξ) and f(x) shifted

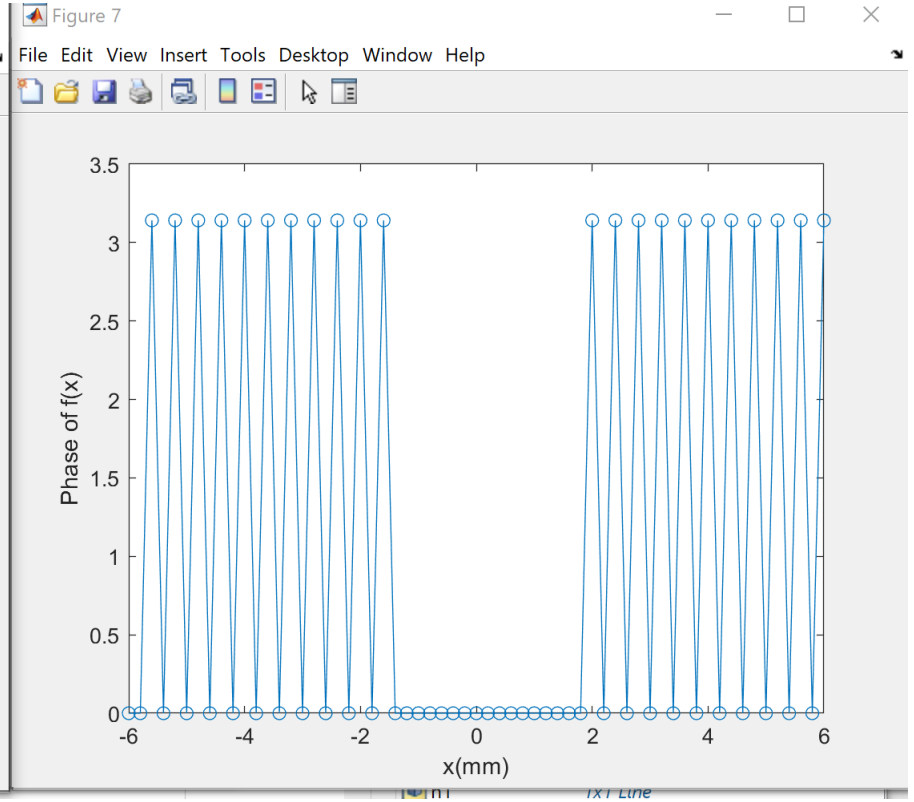
Note: $exp(-i\pi) = -1$

```
%Let's shift both F and f
f=ifftshift(ifft(ifftshift(F)))
figure(6)
plot(x,abs(f),'-o'); %plot magn
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('|f(x)|');

figure(7)
plot(x,angle(f),'-o'); %plot ph
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('Phase of f(x)');
```

Shift F & f



Amplitude



Phase

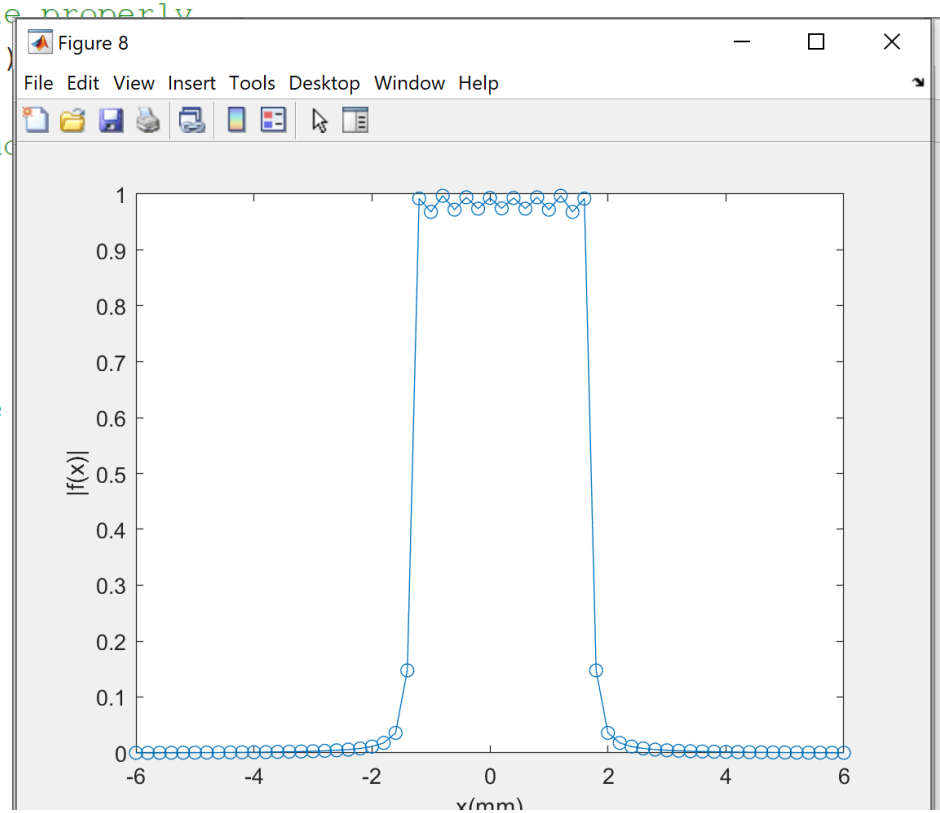
Inverse transform, F(ξ) and f(x) shifted, scaled

Note: L_ξ is the width of the array in Fourier space.

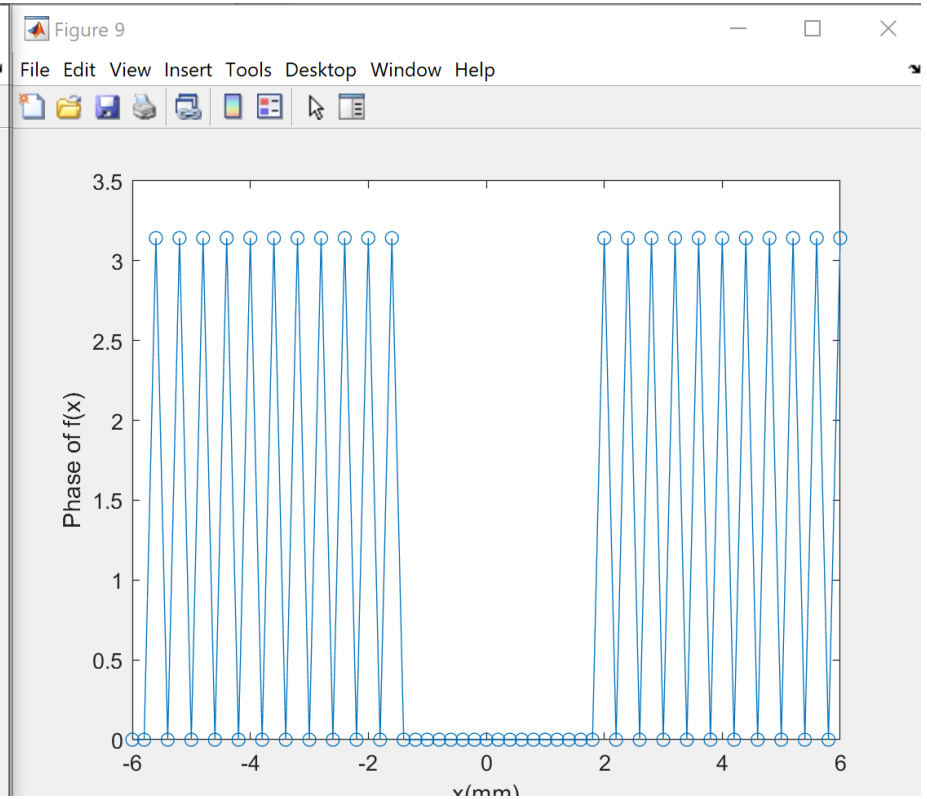
Note: $exp(-i\pi) = -1$

```
%Let's shift both F and f and scale properly
f=L_xi*ifftshift(ifft(ifftshift(F)))
figure(8)
plot(x,abs(f),'-o'); %plot magnitud
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('|f(x)|');

figure(9)
plot(x,angle(f),'-o'); %plot phase
%axis([-6 6 -0.5 1.5]);
xlabel('x(mm)');
ylabel('Phase of f(x)');
```

Shift F & f and scale



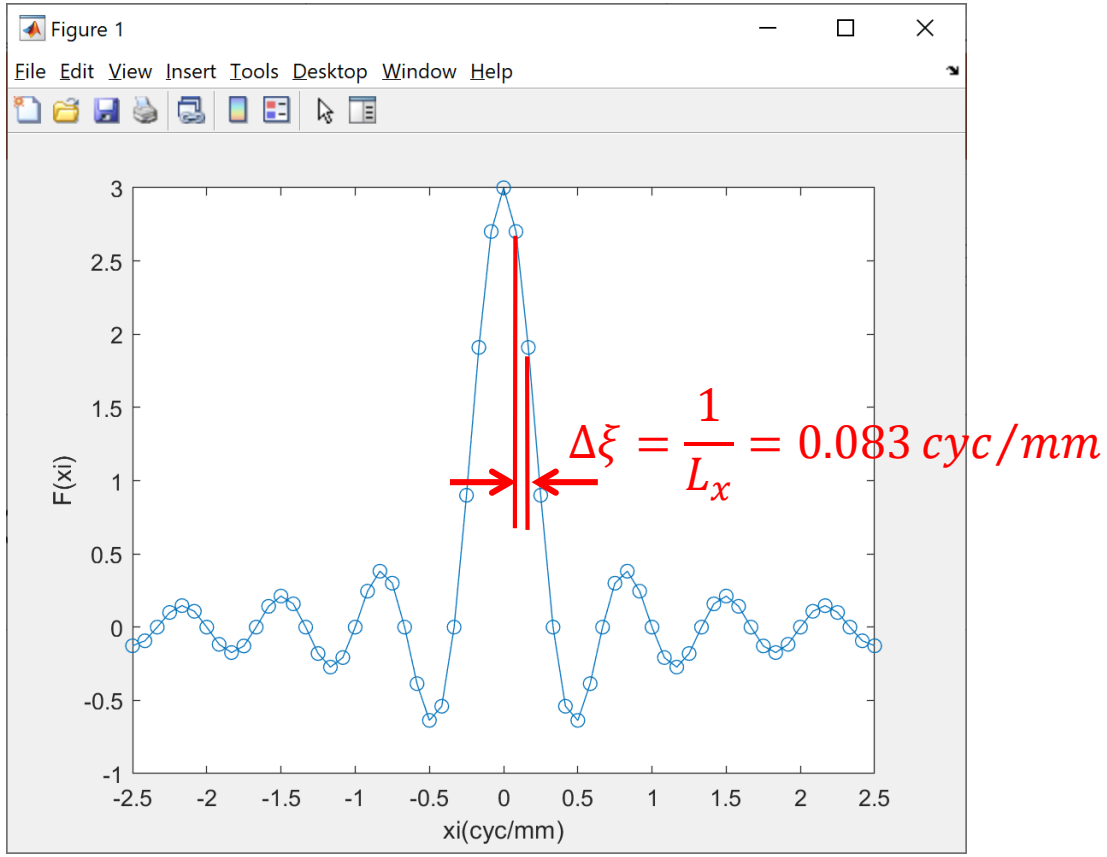
Amplitude



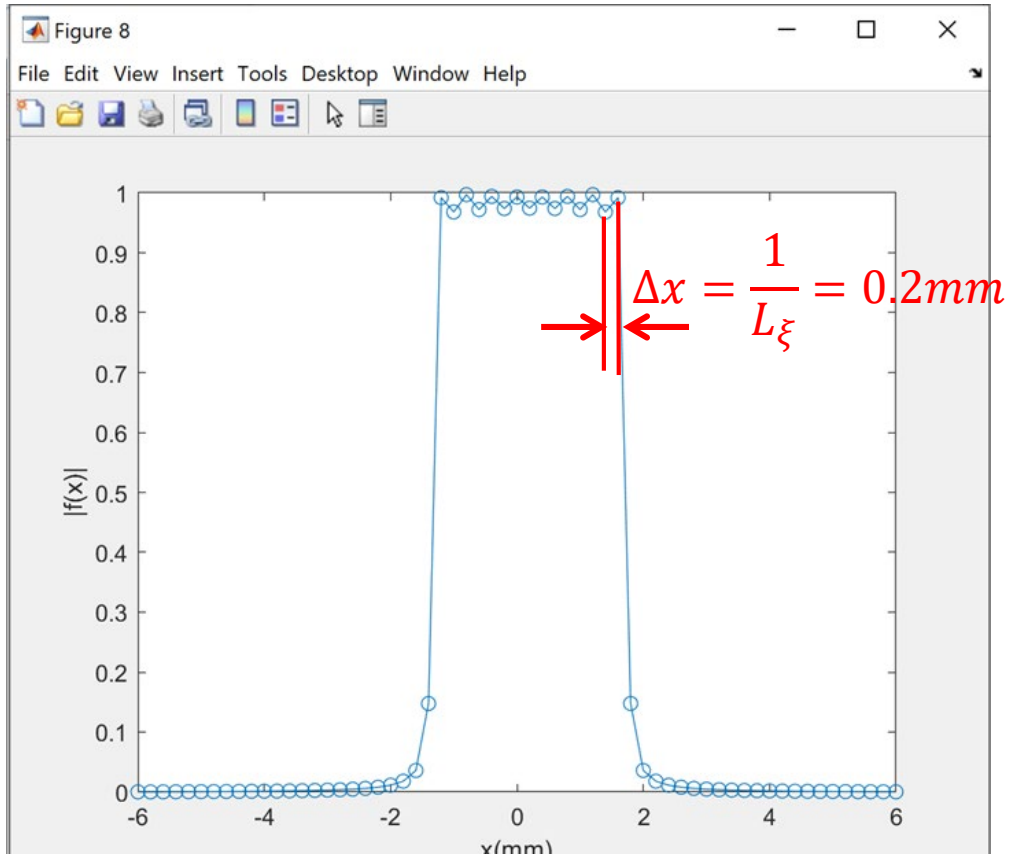
Phase

Relating Dimensions in DFTs

$$N = Number\ of\ Elements\ in\ Array = 60$$



$$L_\xi = N\Delta\xi = 5\ cyc/mm$$

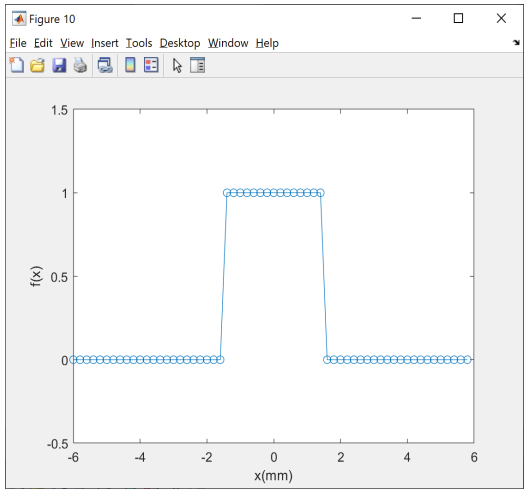


$$L_x = N\Delta x = 12\text{mm}$$

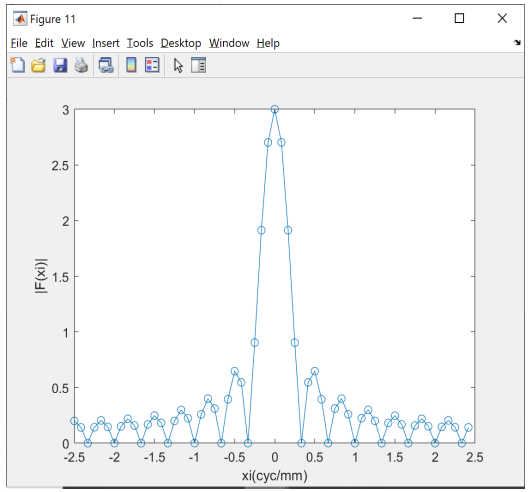
Scaling Comparison

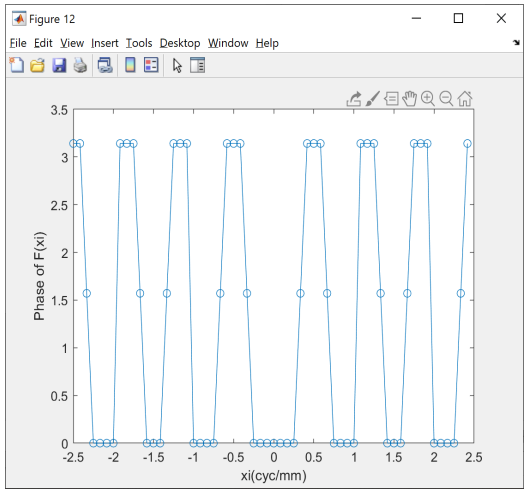
Forward Transform

F=Xs*fftshift(fft(fftshift(f)));



$\mathfrak{F}\{\ \}$

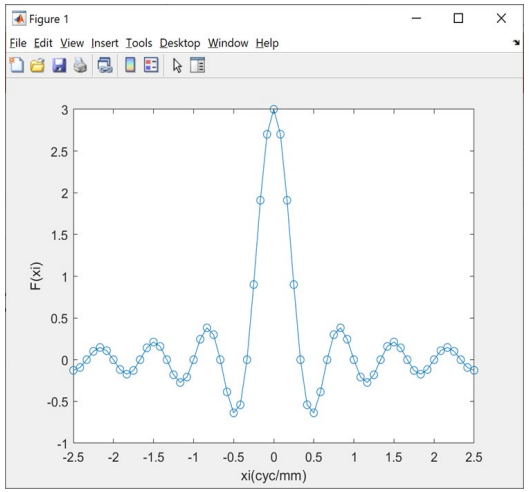




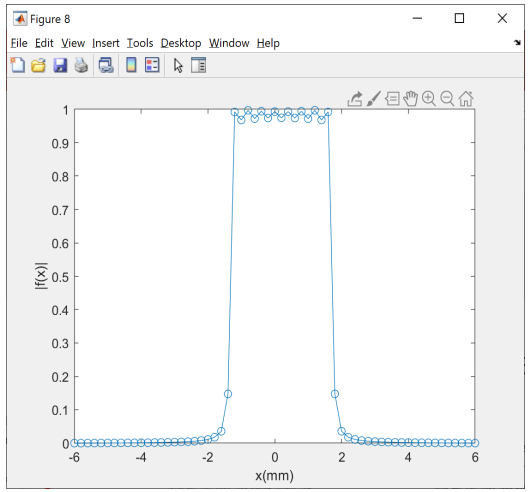
Inverse Transform

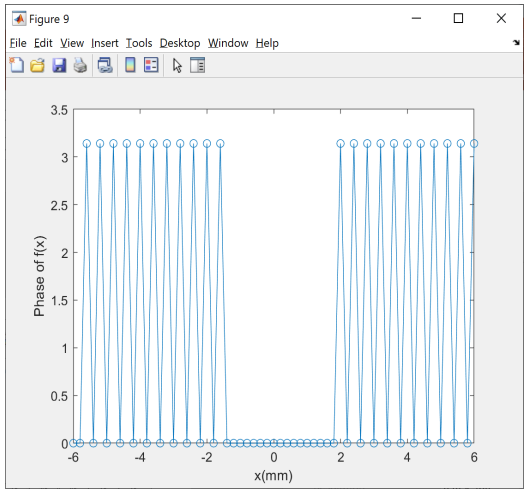
f=L_xi*ifftshift(ifft(ifftshift(F)));

$$\Delta x = \frac{1}{L_\xi} \Longrightarrow L_\xi = \frac{1}{X_s}$$

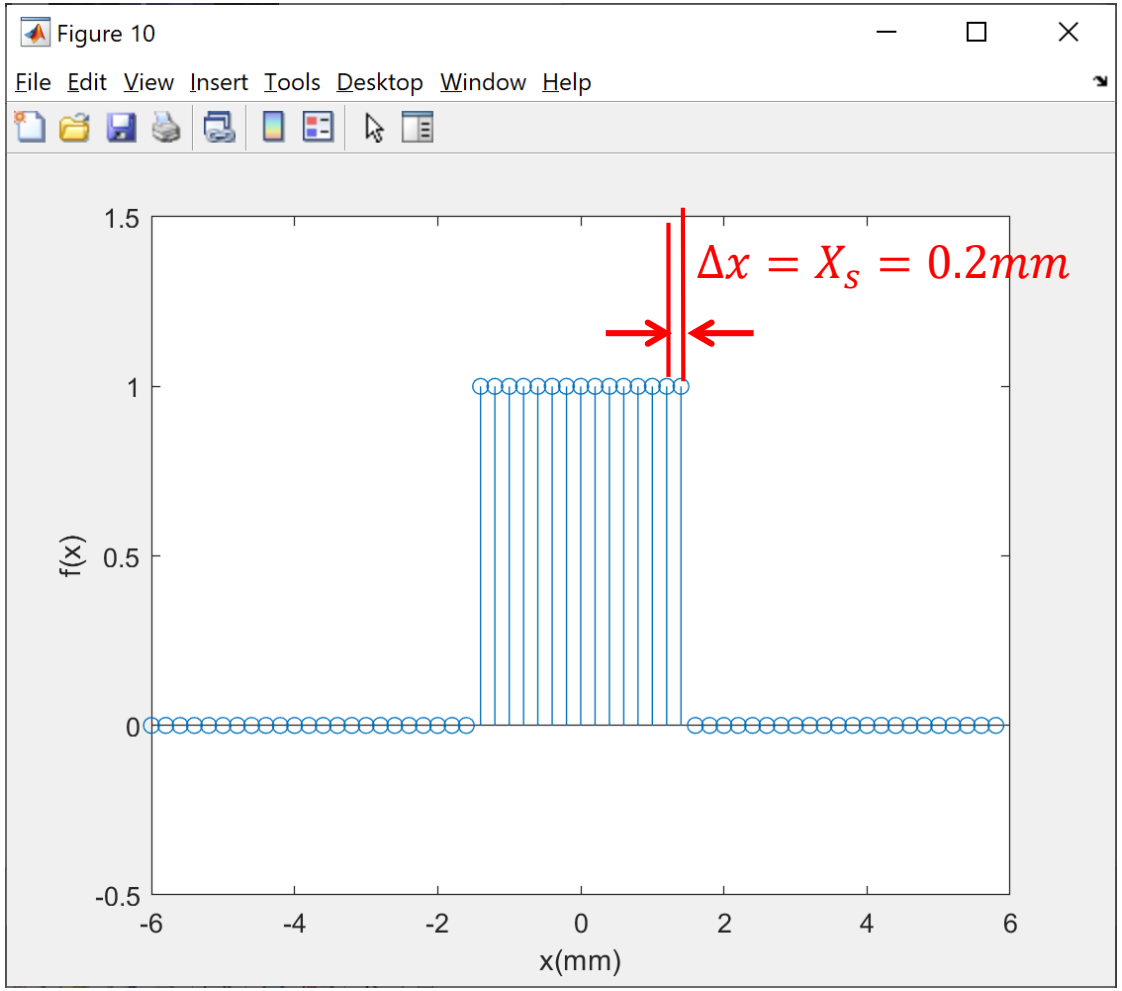


$\mathfrak{F}^{-1}\{\ \}$





Sampling Theorem



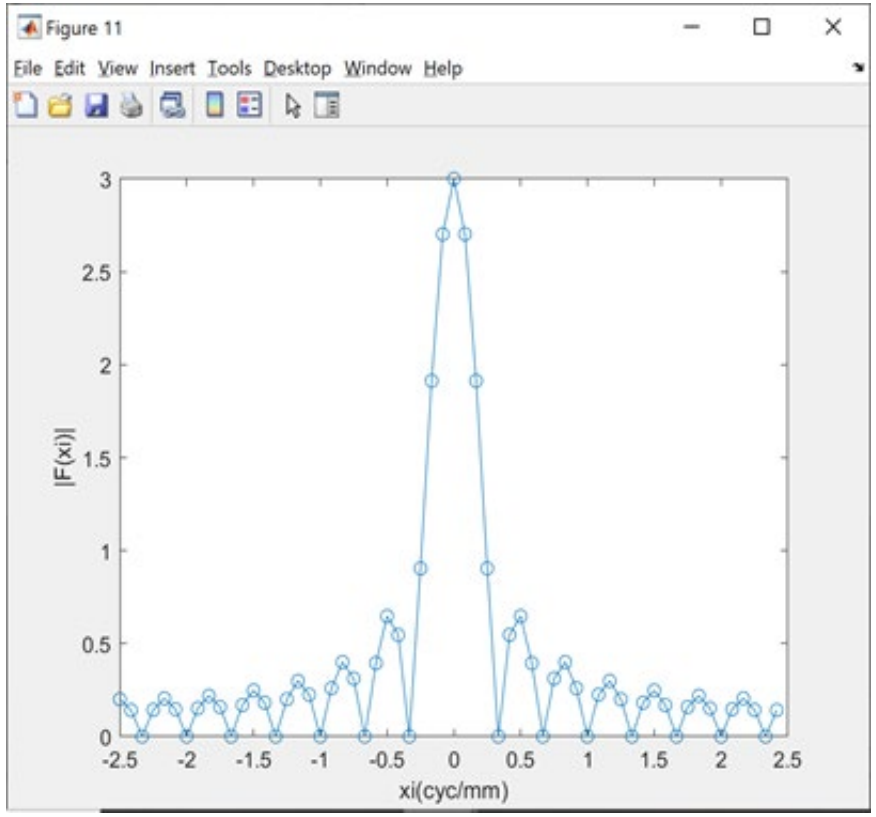
Δx = the spacing between the points in the array.

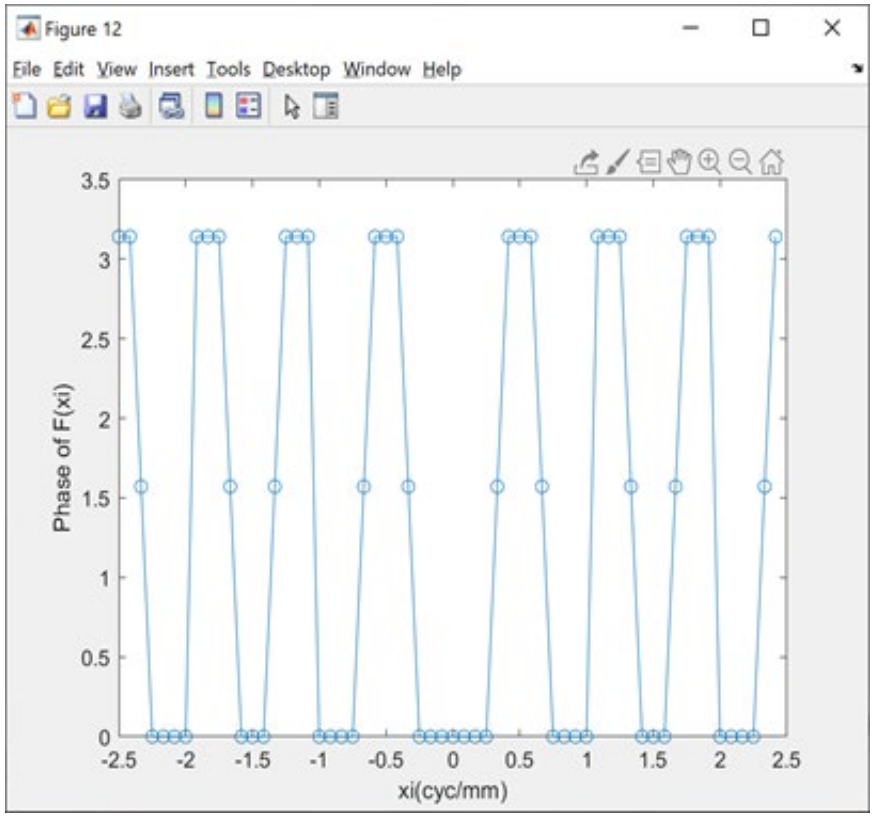
X_s = the spacing between the samples of the function.

In this case, the two values are the same.

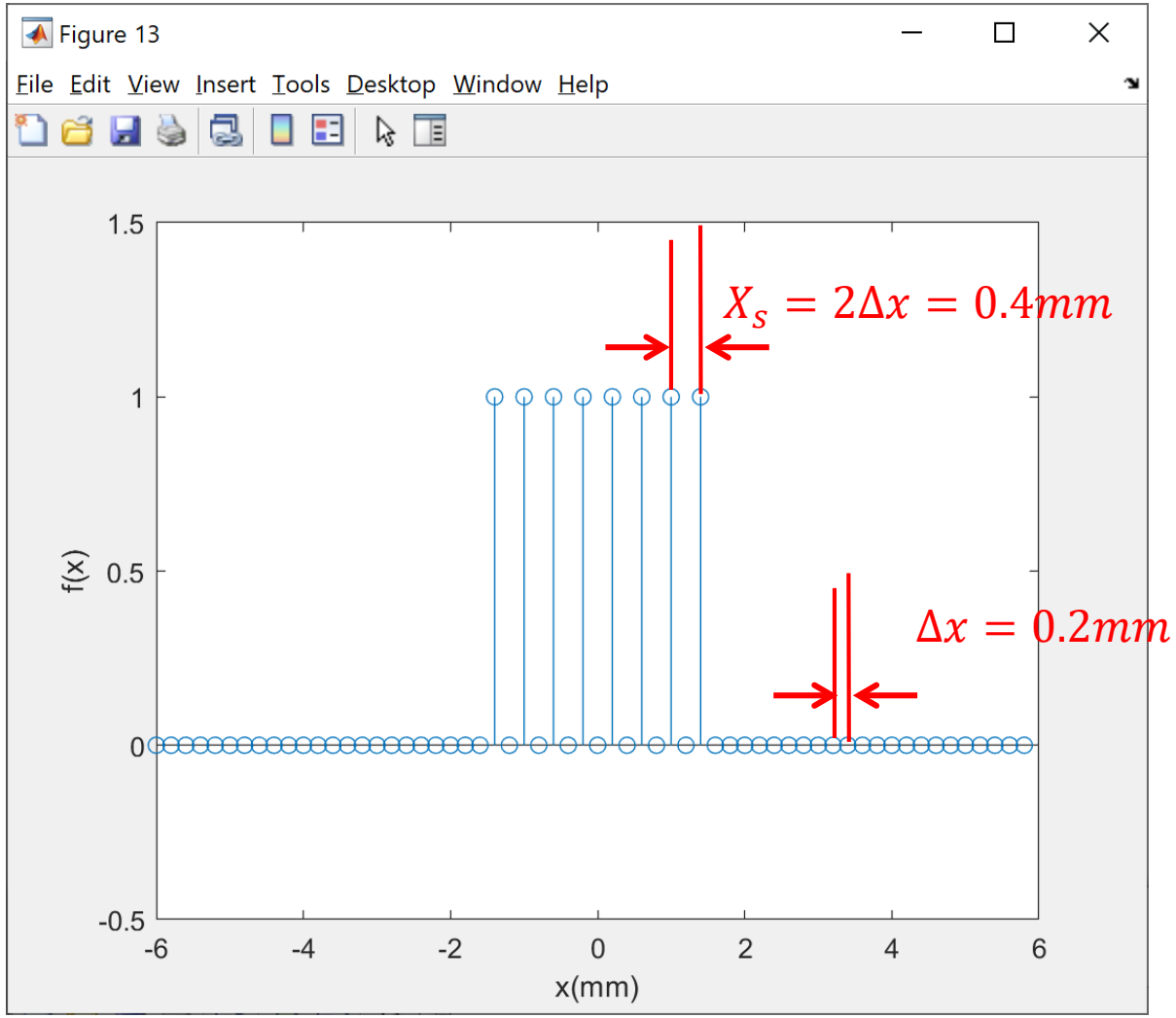
Sampling Theorem

Fourier transforming gives our approximation to the *sinc* function. We are sampling right at twice the Nyquist frequency so these patterns “repeat” outside of the array and the edges of the patterns just touch.





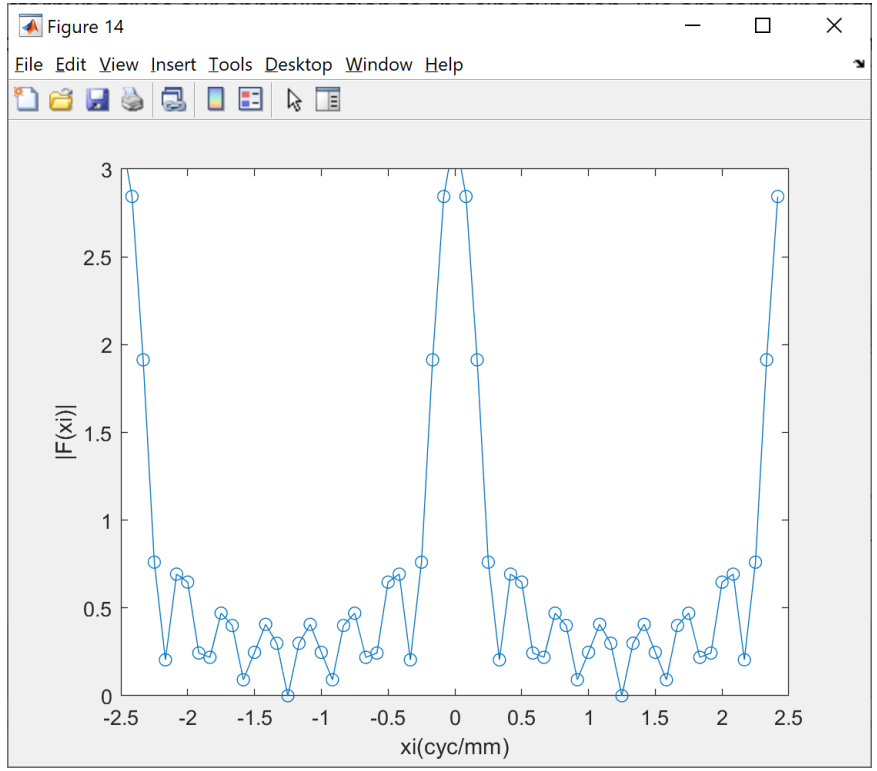
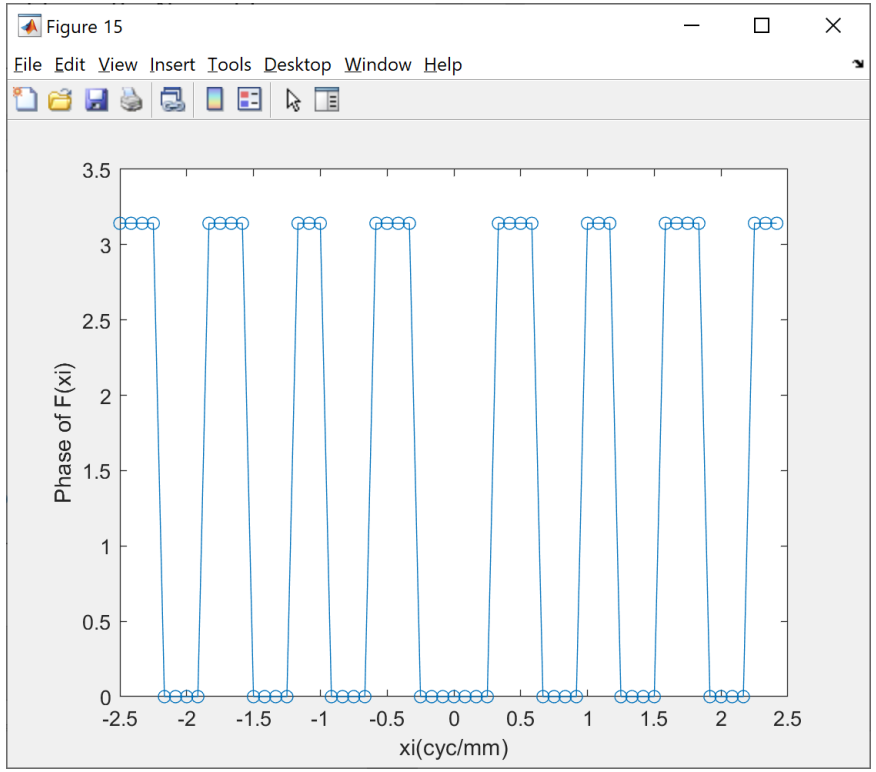
Sampling Theorem



Δx = the spacing between the points in the array.

X_s = the spacing between the samples of the function.

In this example, X_s is double Δx.

Sampling Theorem

Fourier transforming gives our approximation to the sinc function. We are sampling right at the Nyquist frequency so these patterns now “repeat” inside the array and the edges of the patterns overlap.