OPTI 512 Final Date: December 11-16, 2020

1. Suppose the electric field at the plane $z = 0$ is given by

$$U(x, y, 0) = exp\left(i2\pi\left(\frac{\beta}{\lambda}\right)y\right) + exp\left(-i2\pi\left(\frac{\beta}{\lambda}\right)y\right).$$

Do the following:

a) Calculate the angular spectrum $A(\xi, \eta; 0)$ of this field.

b) Given the approximate Fresnel transfer function

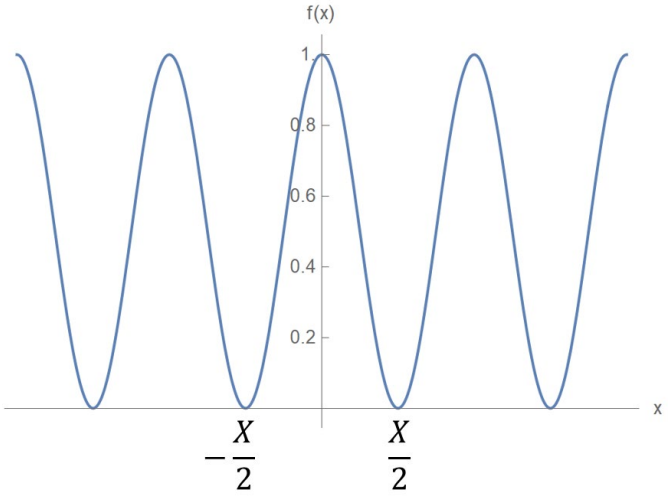
$$H(\xi, \eta) = exp(ikz)exp[-i\pi\lambda z(\xi^2 + \eta^2)],$$

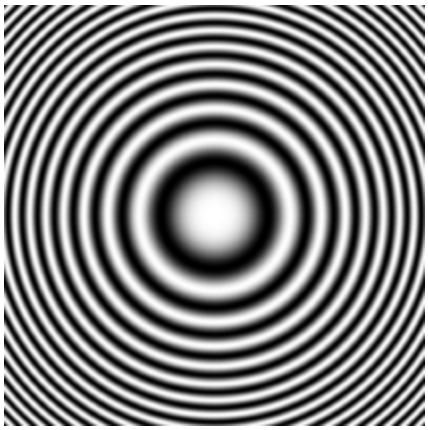
calculate the angular spectrum $A(\xi, \eta; z)$ at a plane some distance z away.

c) Calculate the field on this remote plane $U(x, y, z)$.

d) Plot the irradiance pattern $|U(x, y, z)|^2$. What is the separation between the peaks of the pattern?

2. A Sinusoidal Zone Plate has a periodic pattern $f(x)$ shown below. The peak transmission of the pattern is 1.0. The amplitude profile of the Sinusoidal Zone Plate is given by $f(r^2)$.





a) Calculate the *complex* Fourier series coefficients a_m of the function

$f(x) = \frac{1}{2} + \frac{1}{2}cos(2\pi\xi_0 x)$, where $\xi_o = 1/X$.

b) What is the diffraction efficiency η_m for the Sinusoidal Zone Plate?

c) How many diffraction orders have non-zero diffraction efficiency? What are the values of these diffraction efficiencies?

3. A circular pupil of diameter D is placed against an ideal thin lens with focal length f. Calculate the following:

a) What is the field $U(x, y, f)$ in the rear focal plane of the lens? Note: you can ignore the size of the lens aperture here.

b) What is the irradiance $I(x, y, f)$ in the rear focal plane of the lens?

c) Plot a radial profile through the irradiance pattern.

d) For what value of the radial coordinate r does the 1st zero occur in the irradiance pattern?

e) How does this pattern change if the diameter D is reduced?

4. Given the function $f(x) = sinc(100x)$, calculate the following:

a) Write an expression for the sampled version $f_s(x)$ of this function with sample spacing equal to X_s.

b) Calculate $F_s(\xi) = \mathcal{F}\{f_s(x)\}$. What is the Nyquist frequency N_ξ?

c) What should the sample spacing X_s be so that aliasing does not occur?