OPTI 512 Homework 9 Due: December 9, 2020

1. Prove the following Fourier transform relationship from our derivation of Gaussian beams

$$\mathcal{F}_{1D}\left\{exp\left[-\pi\frac{x^2}{a+ic}\right]\right\} = (a+ic)^{1/2}exp[-\pi(a+ic)\xi^2],$$

where a and c are real, $a \geq 0$ and $a^2 + c^2 < \infty$.

2. The profile of a Gaussian beam emitted from a CO_2 laser with a wavelength $\lambda = 10.6\ \mu m$ is measured at two distances and has widths $w(z_1) = 1.699\ mm$ and $w(z_2) = 3.38\ mm$. The separation between the two measurement locations is $z_2 - z_1 = 10\ cm$. Assume that the distances $z \gg z_o$. Determine:

(a) Using the assumption $z \gg z_o$, approximate the beam width

(b) the beam waist w_o

(c) the Rayleigh range z_o

(d) the measurement locations z_1 and z_2.

(e) the radius of curvature of the beam at z_2.

(f) the angular divergence of the beam.

3. Download the file *phase2.dat* from the course website. You will need Matlab or something equivalent to do this problem. This is a binary file containing 512 x 512 consecutive double values corresponding to a 512 x 512 array of phase values $\Phi(\xi, \eta)$. For this problem, we'll start in Fourier space for simplicity. Do the following:

(a) Use the following Matlab code snippet below to aid in importing the data. You will need to edit the file path accordingly.

```
fid = fopen('C:\Users\jschw\Dropbox\Class\OPTI 512 Linear Systems, Fourier Transforms\Homeworks\phase2.dat','rb');
phase = fread(fid, [512 512], 'double');
imshow(phase,[])
```

Plot the phase pattern $\Phi(\xi, \eta)$. The dimensions of the array from

$$-0.4 \text{ to } +0.4 \ cyc/mm$$

in the horizontal and vertical directions.

(b) Create the complex array $A(\xi, \eta; 0) = \mathcal{F}_{2D}\{U(x, y, 0)\} = exp[i\Phi(\xi, \eta)]$. Now using the Fresnel Transfer function, propagate this pattern a distance $z = 100\ m$, assuming a wavelength of $\lambda = 633\ nm$. This gives $A(\xi, \eta; z)$. Next, inverse transform to get $U(x, y, z) = \mathcal{F}_{2D}^{-1}\{A(\xi, \eta; z)\}$. Plot $|U(x, y, z)|^2$. You may need to adjust the orientation and contrast of this image to see the resultant irradiance pattern.