OPTI 512 Homework 8 Due: November 23, 2020

1. Given the function $f(x) = sinc^2(100x)$, calculate the following:

   a) Write an expression for the sampled version $f_s(x)$ of this function with sample spacing equal to X_s.

   b) Calculate $F_s(\xi) = \mathcal{F}\{f_s(x)\}$. What is the Nyquist frequency N_ξ?

   c) Plot $F_s(\xi)$ in the range $-300 \leq \xi \leq 300$ for the cases where the sampling frequency $1/X_s = 2N_\xi$, $1/X_s = 1.5N_\xi$, and $1/X_s = N_\xi$.

   d) The function $G(\xi)$ is used to recover the central spectrum of $F_s(\xi)$, where

   $$G(\xi) = F_s(\xi) rect\left(\frac{\xi}{200}\right).$$

   Write equivalent expressions for $G(\xi)$ for the three sampling cases $1/X_s = 2N_\xi$, $1/X_s = 1.5N_\xi$, and $1/X_s = N_\xi$. Note, this just means write a function that has the same shape as $G(\xi)$ which will be *rect()* and *tri()* functions.

   e) Calculate and plot $g(x) = \mathcal{F}^{-1}\{G(\xi)\}$ for the three sampling cases. How do these compare to the original function $f(x)$?

2. An interferometer is used to test the shape of an optical surface. The interference pattern in the camera sensor of the system is given by

   $$I(x,y) = \frac{1}{2} + \frac{1}{2} cos(2\pi r^4)$$

   where $r^2 = x^2 + y^2$, and $0 \leq r \leq 3mm$. Do the following:

   a) The interference pattern has a bright fringe whenever $2\pi r^4 = 2m\pi, m\ integer$. Two adjacent fringes occur when $2\pi r_1^4 = 2m_o\pi$ and $2\pi r_2^4 = 2(m_o - 1)\pi$. Calculate the value of m_o when $r_1 = 3mm$.

b) Calculate the value of r_2 based on the results from part (a). What is the distance between these two fringes $r_1 - r_2$? In this case, this will be the smallest period of the fringe pattern.

c) What is the Nyquist frequency N_ξ in cyc/mm associated with the interference pattern?

d) What sample spacing X_s is needed to avoid aliasing?

e) Given this sample spacing, how many pixels N are required to cover the full width of the interference pattern?

f) Are the values of N and X_s achievable with current camera sensor technologies?

3. Suppose the electric field at the plane $z = 0$ is given by

$$U(x, y, 0) = exp\left(i2\pi\left(\frac{\beta}{\lambda}\right)y\right) + exp\left(-i2\pi\left(\frac{\beta}{\lambda}\right)y\right).$$

Do the following:

a) Calculate the angular spectrum $A(\xi, \eta; 0)$ of this field.

b) Given the transfer function $H(\xi, \eta) = exp\left(i2\pi\sqrt{1/\lambda^2 - \xi^2 - \eta^2} \cdot z\right)$, calculate the angular spectrum $A(\xi, \eta; z)$ at a plane some distance z away.

c) Calculate the field on this remote plane $U(x, y, z)$.

d) Plot the irradiance pattern $|U(x, y, z)|^2$. What is the separation between the peaks of the pattern?