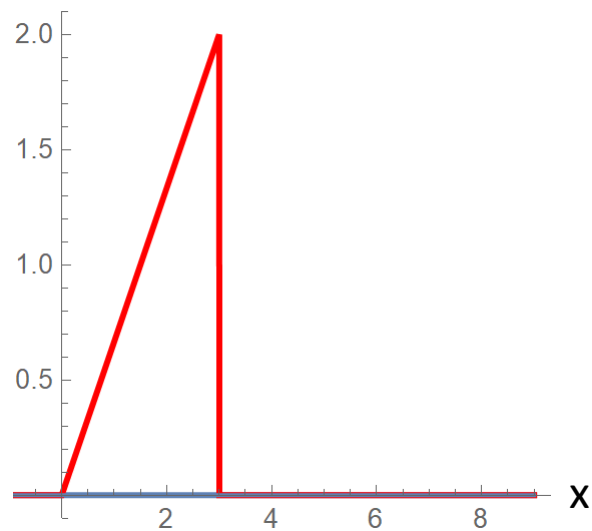


Convolution

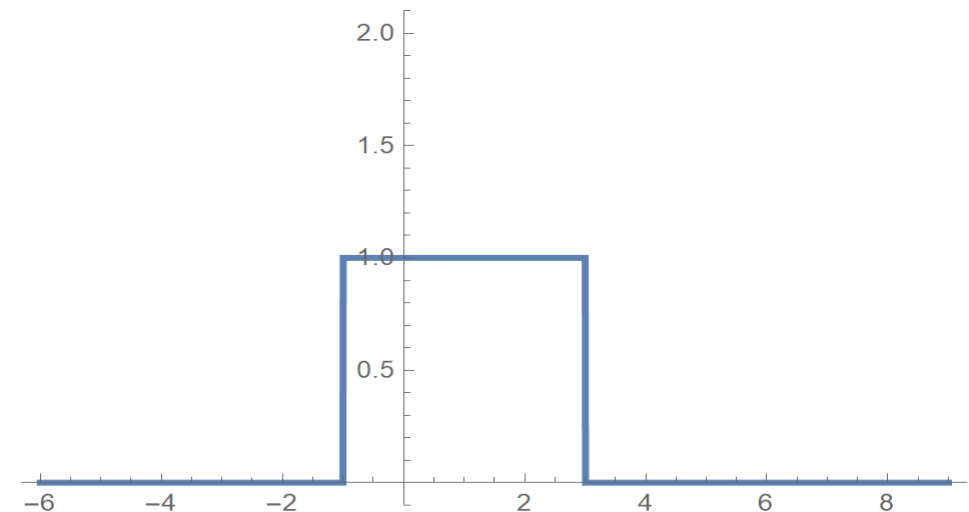
$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\alpha)h(x - \alpha)d\alpha$$

Example

$$f(x) = \frac{2}{3}x$$



$$h(x) = \text{rect}\left(\frac{x-1}{4}\right)$$

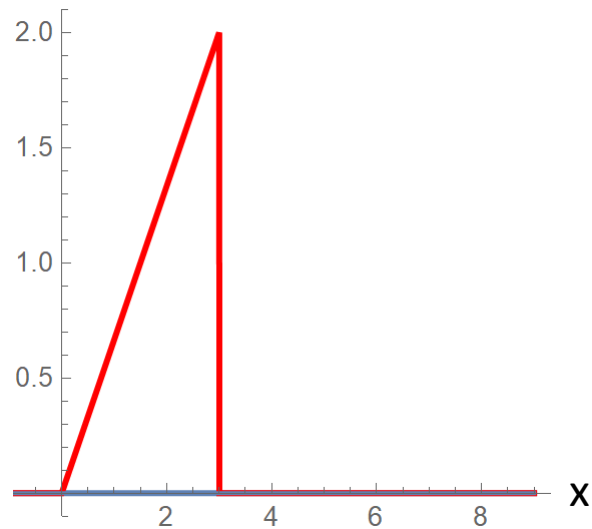


Convolution

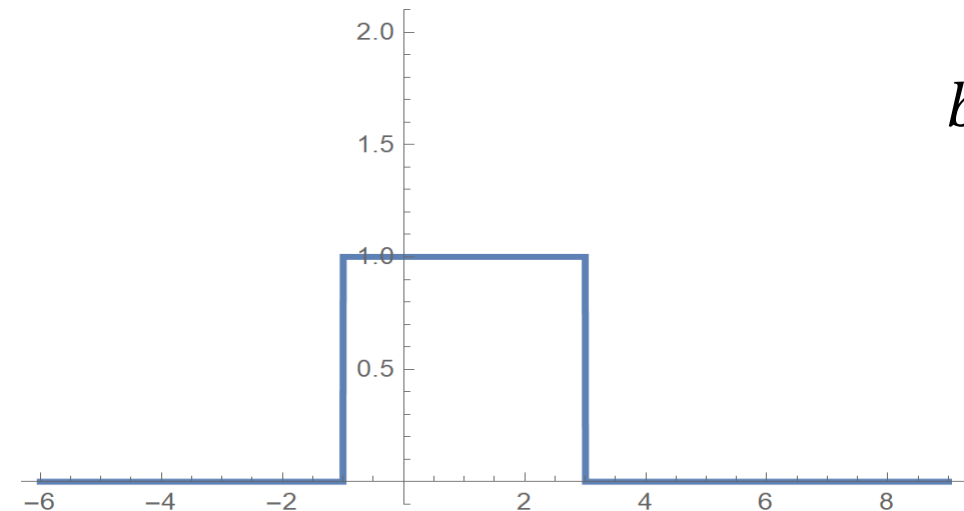
$$g(x) = \int_{-\infty}^{\infty} f(\alpha)h(x - \alpha)d\alpha$$

Example

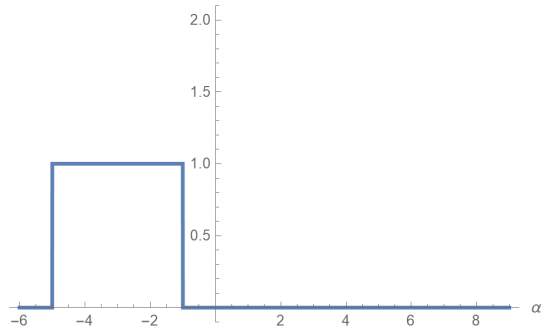
$$f(\alpha) = \frac{2}{3}\alpha$$



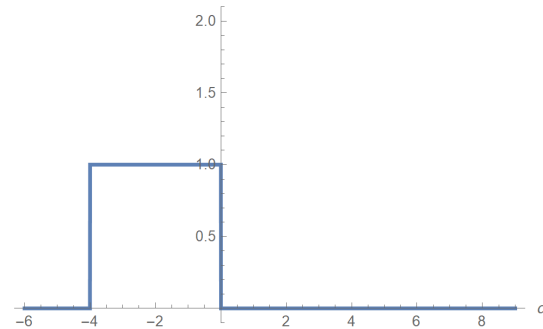
$$h(x - \alpha) = \text{rect}\left(\frac{x - \alpha - 1}{4}\right) = \text{rect}\left(\frac{\alpha - (x - 1)}{-4}\right)$$



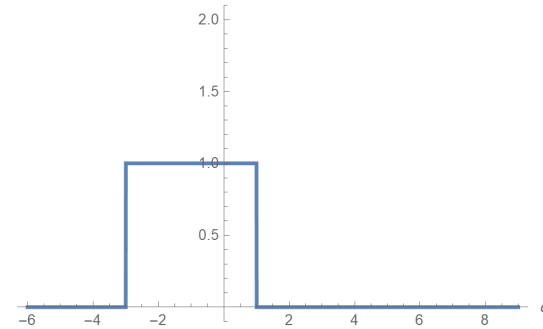
Sliding the Rect



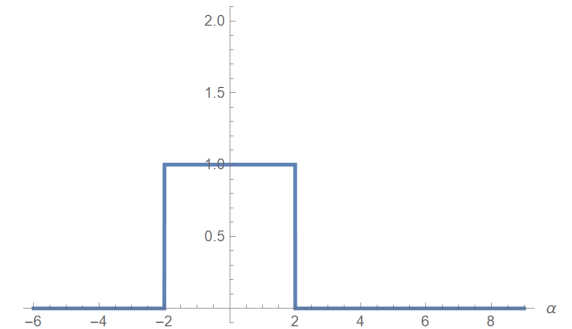
$$\begin{aligned}x &= -2 \\x_o &= -3 \\b &= -4\end{aligned}$$



$$\begin{aligned}x &= -1 \\x_o &= -2 \\b &= -4\end{aligned}$$



$$\begin{aligned}x &= 0 \\x_o &= -1 \\b &= -4\end{aligned}$$



$$\begin{aligned}x &= 1 \\x_o &= 0 \\b &= -4\end{aligned}$$

How is the left edge of the rect related to x ?

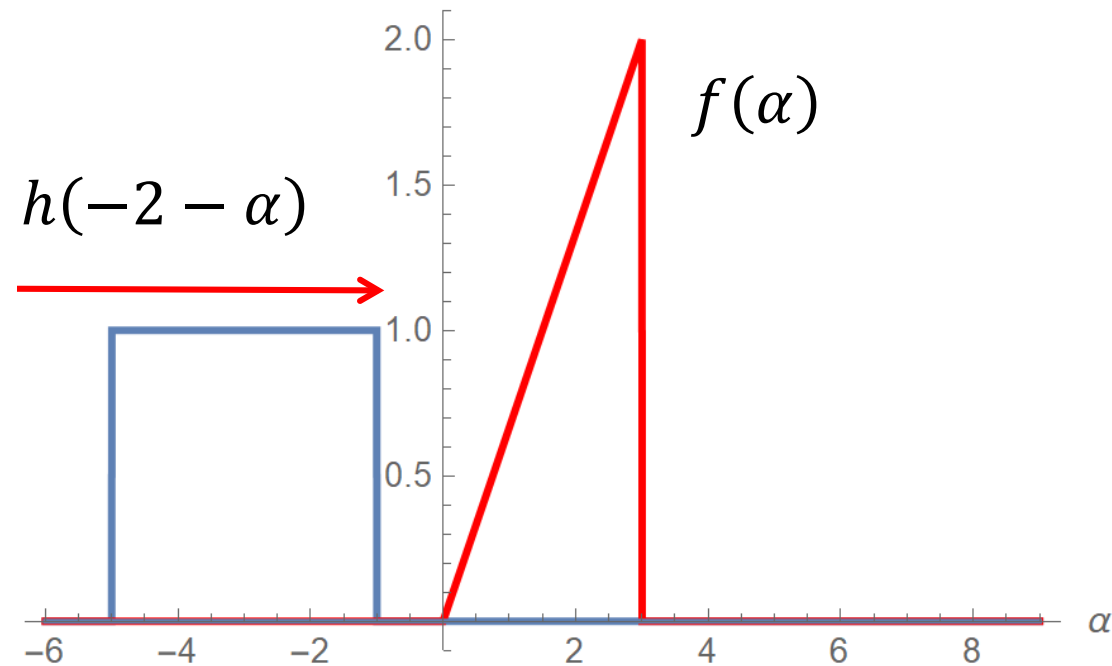
$$\textit{Left Edge} = x - 3$$

How is the right edge of the rect related to x ?

$$\textit{Right Edge} = x + 1$$

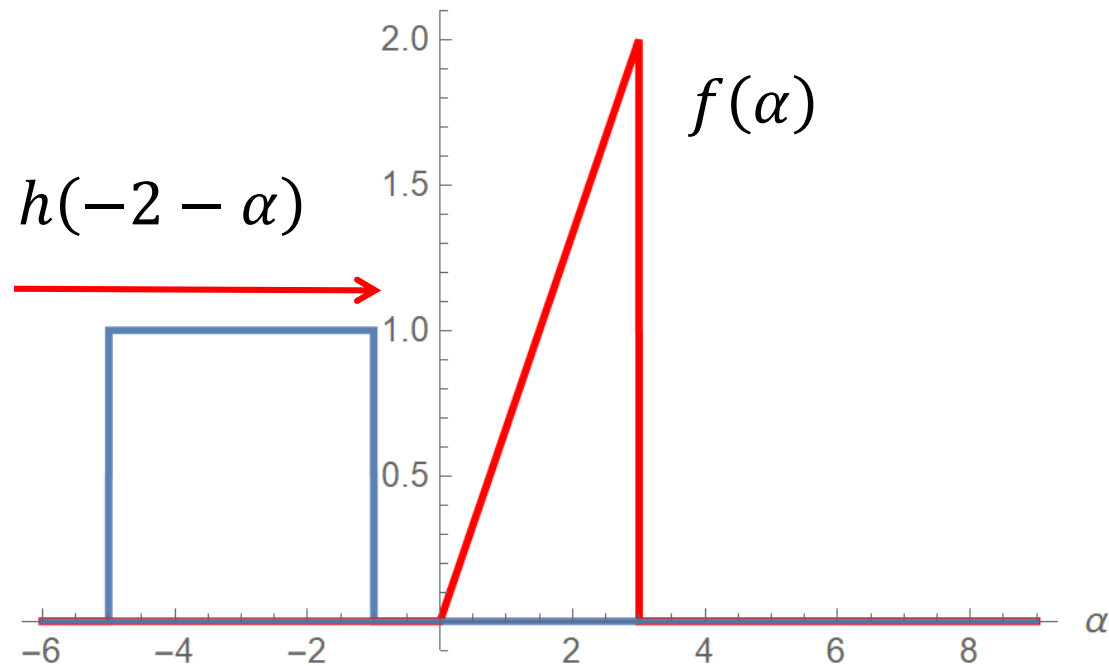
Both Functions

$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\alpha)h(x - \alpha)d\alpha$$



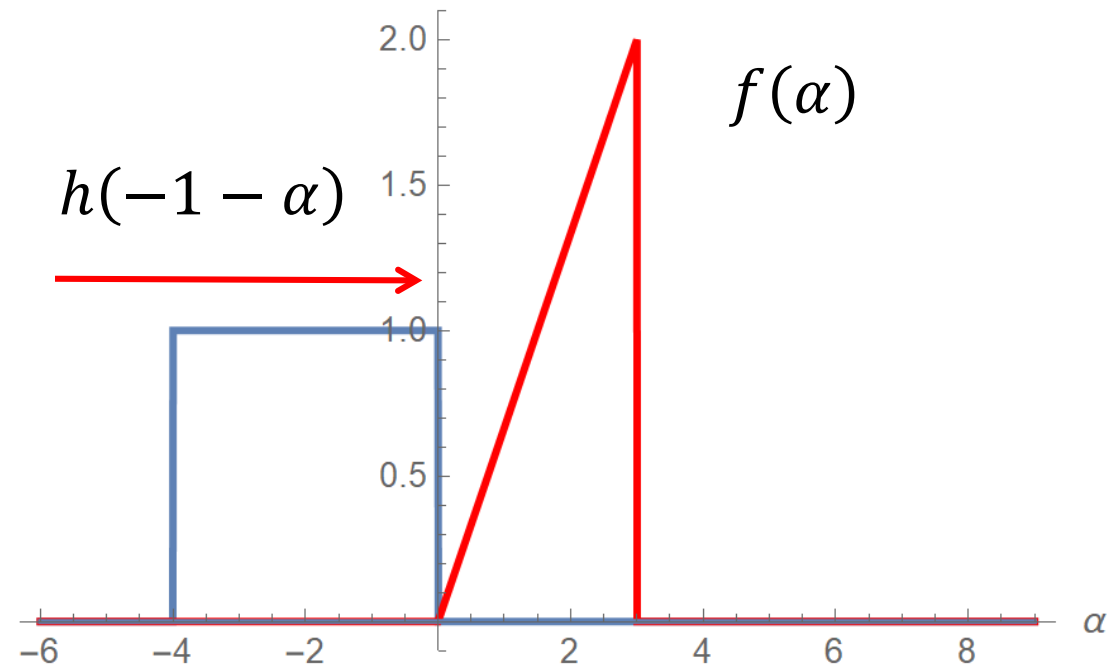
Both Functions

$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\alpha)h(x - \alpha)d\alpha$$



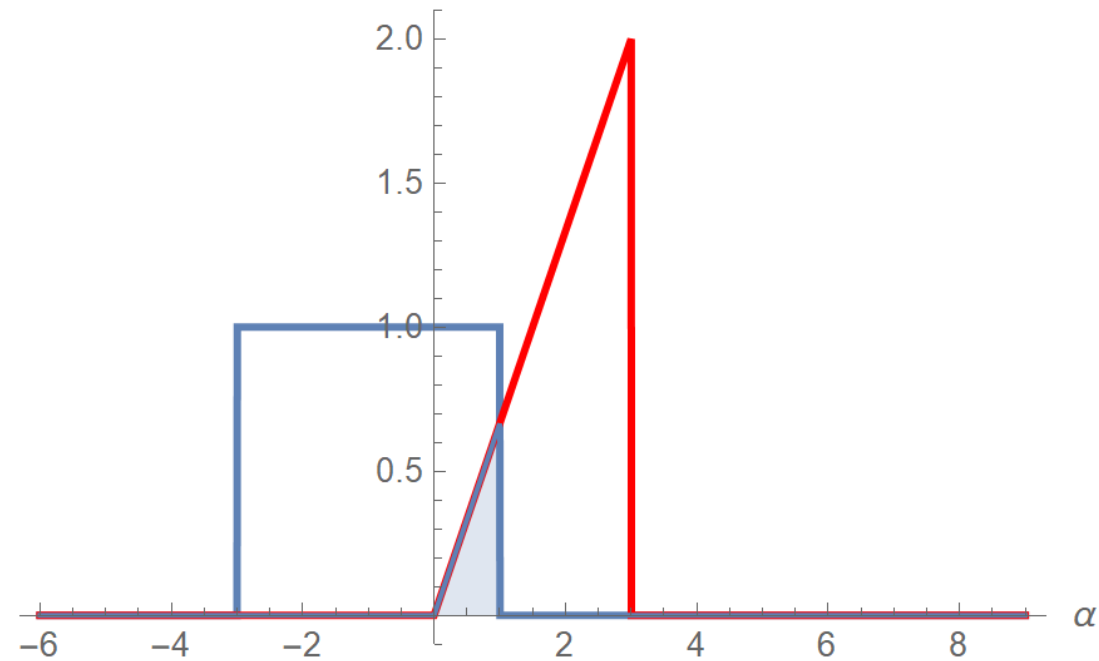
Critical Points

$$g(x \leq -1) = \int_{-\infty}^{\infty} 0 \, d\alpha = 0$$



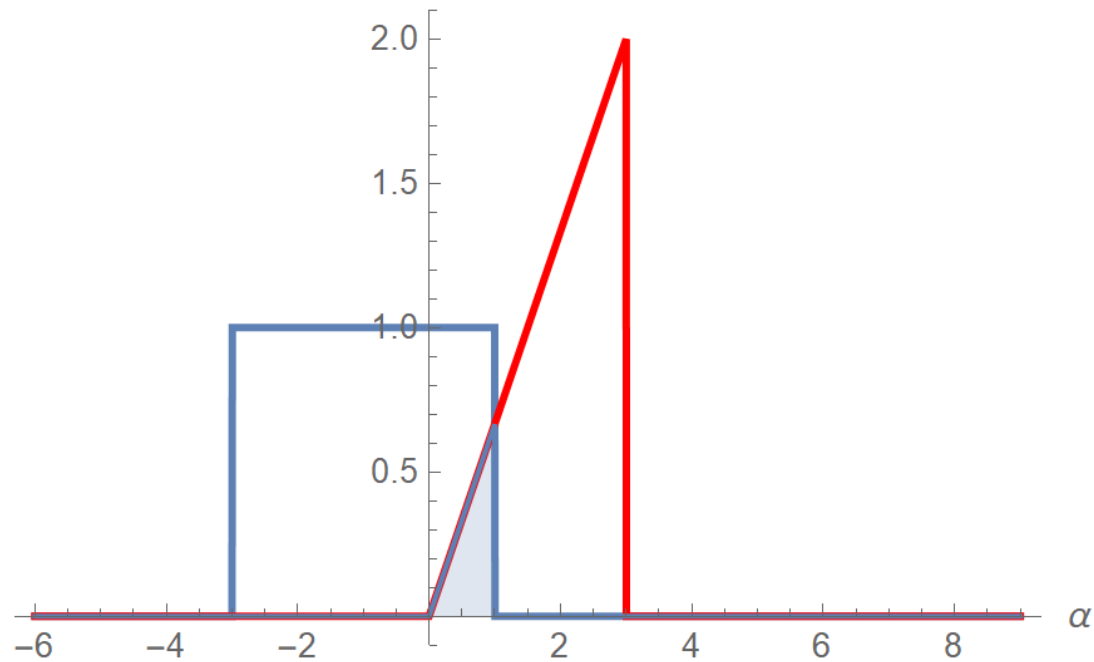
Critical Points

$$g(-1 < x \leq 2) = \int_0^{x+1} \frac{2}{3} \alpha \cdot 1 \, d\alpha = \frac{(x+1)^2}{3}$$



Critical Points

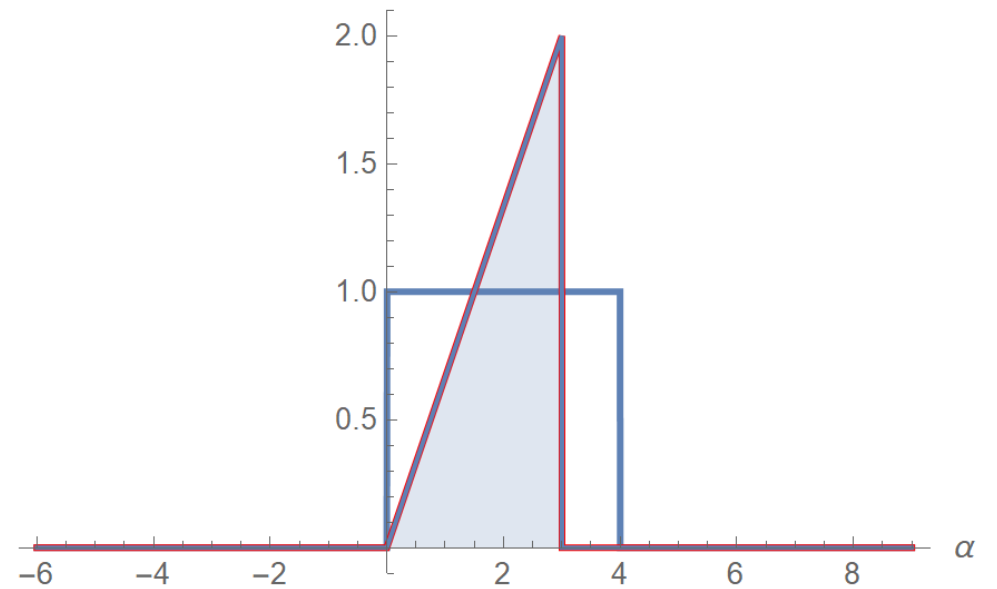
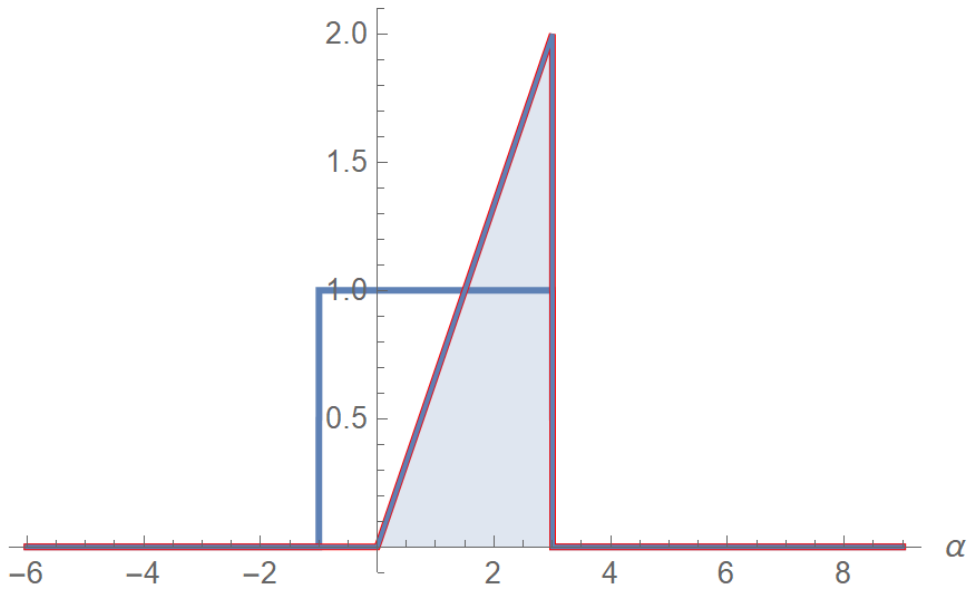
$$g(-1 < x \leq 2) = \int_0^{x+1} \frac{2}{3} \alpha \cdot 1 \, d\alpha = \frac{(x+1)^2}{3}$$



Critical Points

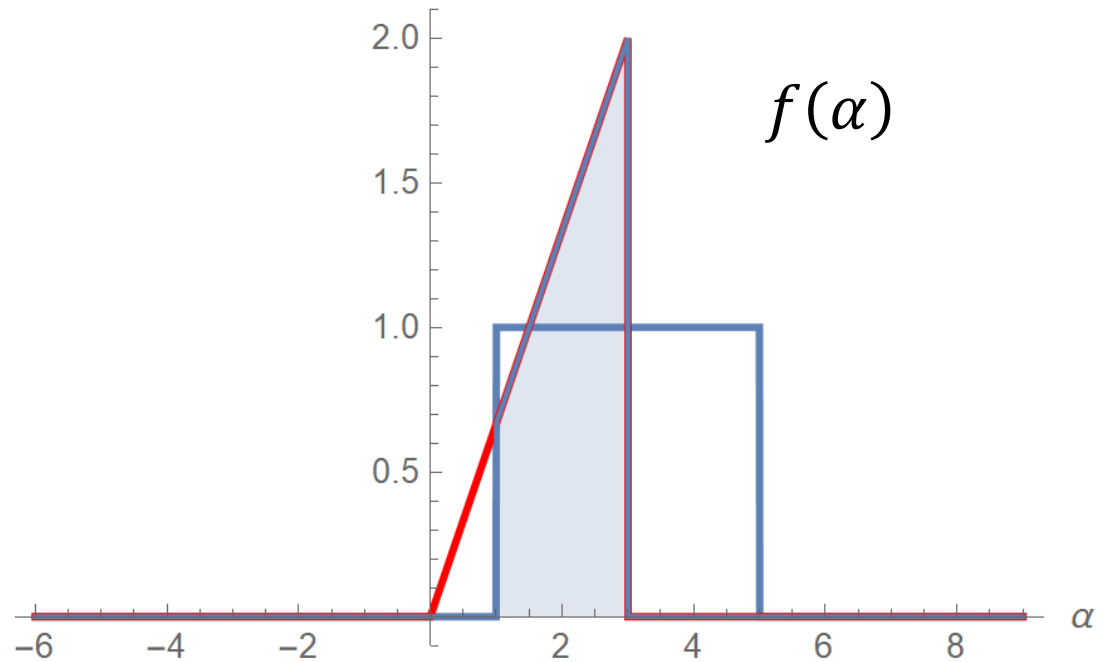
$$g(x = 2) = \int_0^3 \frac{2}{3} \alpha \cdot 1 d\alpha = \frac{(2 + 1)^2}{3} = 3$$

$$g(x = 3) = \int_0^3 \frac{2}{3} \alpha \cdot 1 d\alpha = \frac{(2 + 1)^2}{3} = 3$$

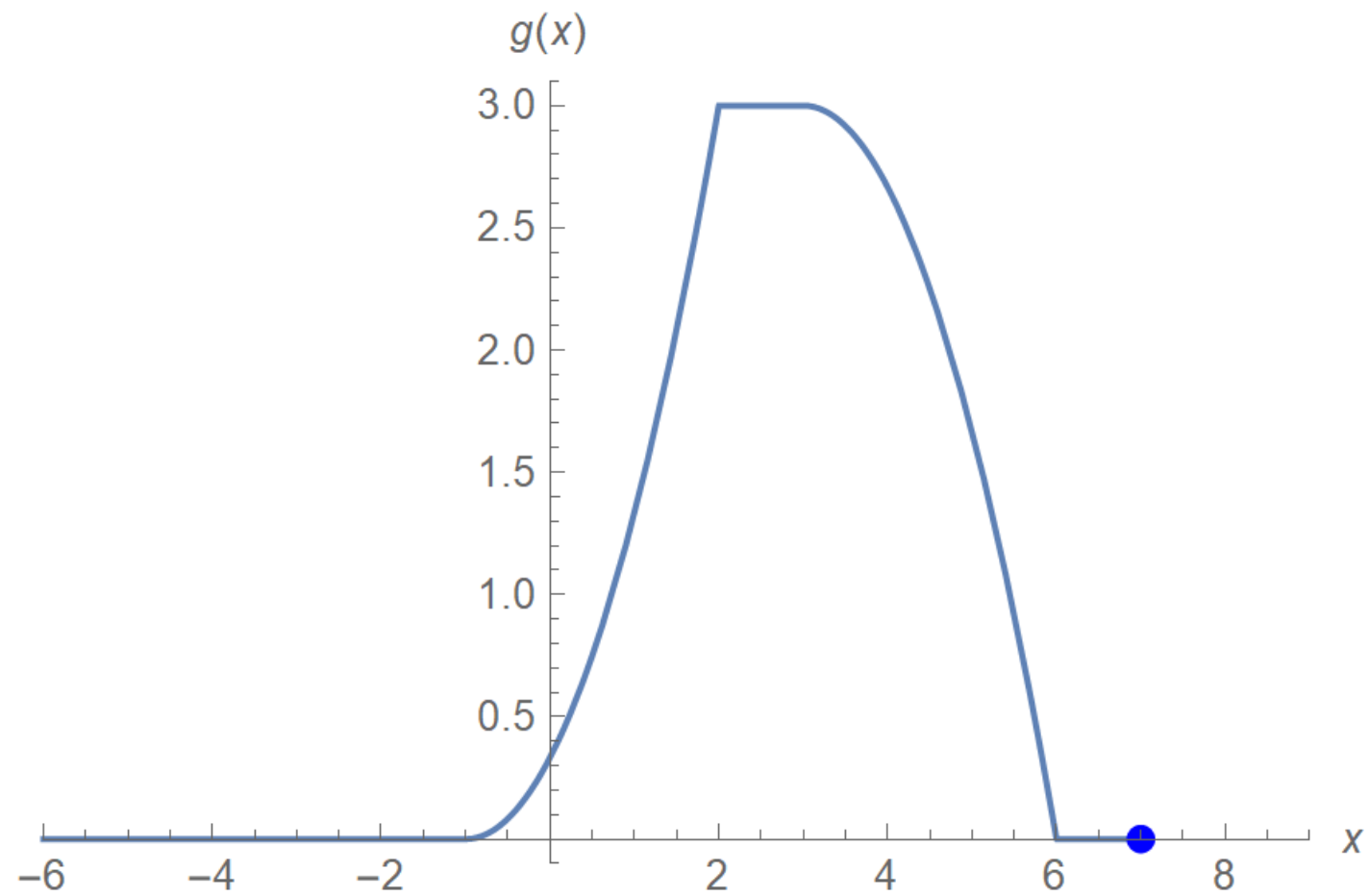


Critical Points

$$g(3 < x \leq 6) = \int_{x-3}^3 \frac{2}{3} \alpha \cdot 1 d\alpha = 3 - \frac{(x-3)^2}{3}$$



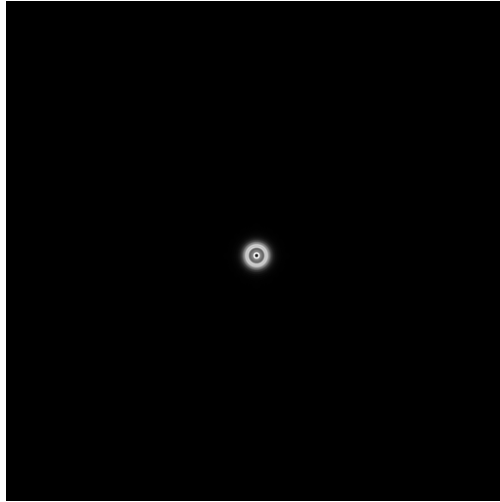
Result



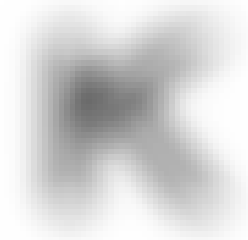
Convolution

K

*



=



Correlation

Input image



Template



Correlation



Normalized cross correlation

