Special Functions

- One application of our 2D special functions is to describe the transmission and shape of apertures. This is a real function that ranges in value from zero to one.
- Another application of our 2D special functions is to describe the irradiance pattern on a detector or screen. More on this in a bit.

The Cylinder cyl() Function

Most optical systems have a circular pupil which can be described by a cyl() function.

$$T(r) = cyl\left(\frac{r}{d}\right)$$

Here, the transmission is roatationally symmetric, so only the radial coordinate r is needed. The variable d can be used to adjust the diameter of the opening.

Square Aperture

Sometimes, you will run into square or rectangular apertures. The transmission function for these are represented by 2D rect() functions. In this example,

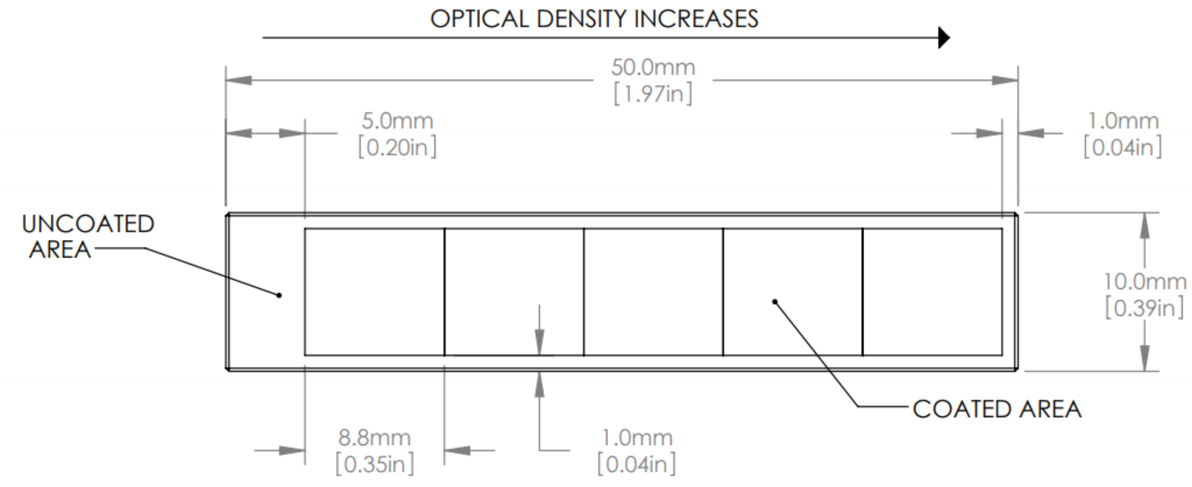
$$T(x,y) = rect\left(\frac{x}{d},\frac{y}{d}\right)$$

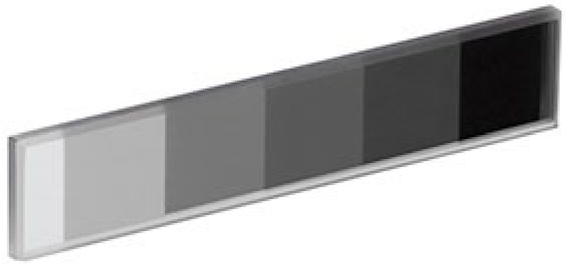
or equivalently

$$T(x,y) = rect\left(\frac{x}{d}\right)rect\left(\frac{y}{d}\right)$$

Here again, the variable d can be adjusted to change the width of the square. Also, the widths don't need to be the same in the x and y directions, so a rectangular aperture can be made.

Thorlabs Stepped Variable Neutral Density Filter





NOTES/SPECIFICATIONS:

1. LENGTH TOLERANCE: +0.00/-0.25mm
2. THICKNESS TOLERANCE: ±0.25mm
3. COATING TOLERANCE: ±0.25mm
4. COATING: FRONT ND, BACK UNCOATED
5. OPTICAL DENSITY RANGE: 0.3, 0.6, 0.8, 1.0, 2.0
6. SPECTRAL RANGE: 240-1200nm
7. SURFACE FLATNESS: 1λ per 25 mm
8. SURFACE QUALITY: 60-40 SCRATCH-DIG
9. WEDGE <3 arcmin
10. OPTICAL DENSITY TOLERANCE ±5%

FOR INFORMATION ONLY
NOT FOR MANUFACTURING PURPOSES

DRAWING PROJECTION			THORLABS www.thorlabs.com	
	NAME	DATE	STEP ND FILTER, 10 x 50 mm OPTICAL DENSITY: 0.3 - 2.0	
DRAWN	SR	05/JAN/11		
APPROVAL	DD	15/FEB/11	MATERIAL UV FUSED SILICA	REV C
COPYRIGHT © 2010 BY THORLABS			ITEM # NDL-10S-2	APPROX WEIGHT 0.02 kg
VALUES IN PARENTHESIS ARE CALCULATED AND MAY CONTAIN ROUNDOFF ERRORS				

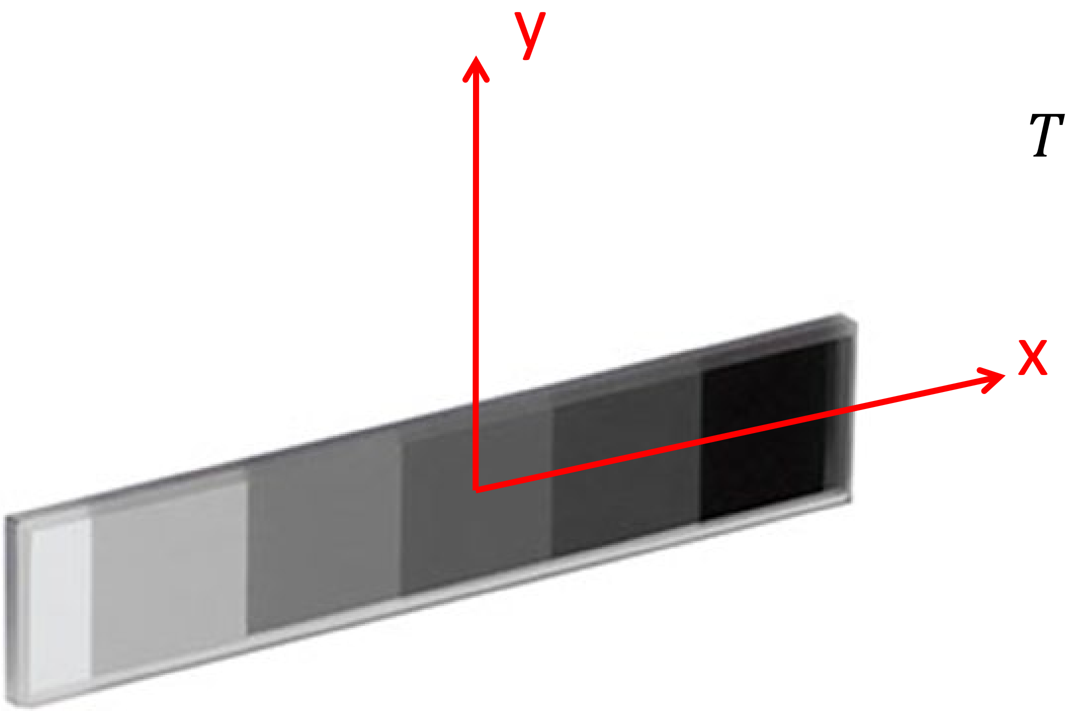
Optical Density (OD)

- This is a logarithmic description of the transmittance of a filter.
- 0D = 0 means 100% transmission, whereas higher ODs mean lower transmission.

$$Transmission = 10^{-OD}$$

- For the example filter, the ODs are 0.3, 0.6, 0.8, 1.0, 2.0
- These correspond to transmissions of 0.5, 0.25, 0.16, 0.10, 0.01
- Size of each square region is 8.8 mm.

Transmission Function



$$T(x,y) = 0.50rect\left(\frac{x+17.6}{8.8}\right)rect\left(\frac{y}{8.8}\right) +$$
$$0.25rect\left(\frac{x+8.8}{8.8}\right)rect\left(\frac{y}{8.8}\right) +$$
$$0.16rect\left(\frac{x}{8.8}\right)rect\left(\frac{y}{8.8}\right) +$$
$$0.10rect\left(\frac{x-8.8}{8.8}\right)rect\left(\frac{y}{8.8}\right) +$$
$$0.01rect\left(\frac{x-17.6}{8.8}\right)rect\left(\frac{y}{8.8}\right)$$

Wave Propagation

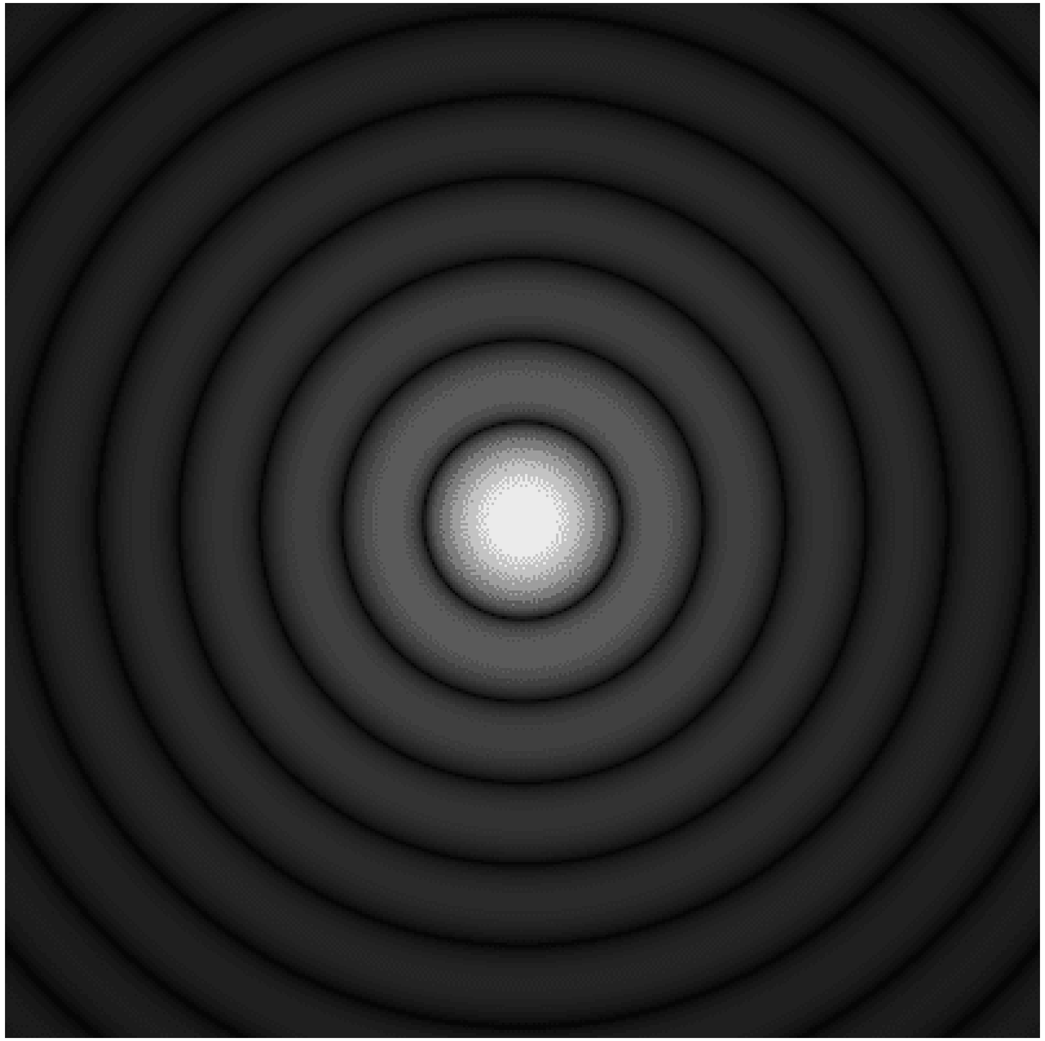
- Waves propagating through space are represented by complex functions.

$$E(x, y; z = z_o) = A(x, y)exp\big(i\phi(x, y)\big)$$

where $A(x, y)$ is the amplitude and $\phi(x, y)$. Think about this as being a complex number at each point (x, y).

- Typically, we are interested in the wave on a plane $z = z_o$. (e.g. pupil plane or image plane).
- Detectors can only “see” a real function. Furthermore, detectors measure energy or irradiance, which is proportional to $|A(x, y)|^2$.
- We can’t “see” phase. Many areas of optics come up with clever ways to modified the wave so that $\phi(x, y)$ can be recorded.

Airy Pattern

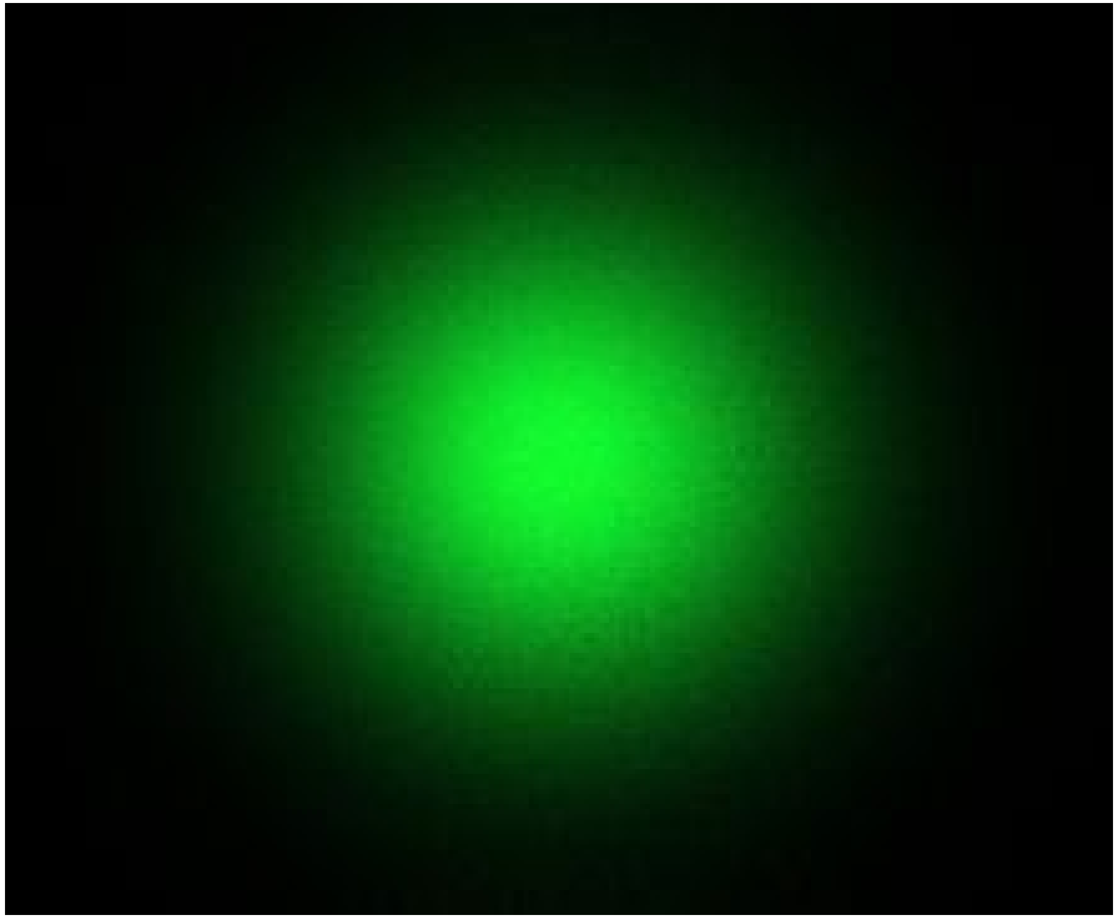


An optical system with a circular aperture and no aberrations will focus light to the Airy pattern (not a true point) due to diffraction. The irradiance pattern

$$|A(r)|^2 \propto somb^2\left(\frac{r}{d}\right)$$

where d will depend on the wavelength, the focal length of the system, and the diameter of the circular aperture.

Gaussian Beams



A laser operating in the TEM_{00} mode will have a Gaussian shape as it propagates. The irradiance pattern

$$|A(r)|^2 \propto Gaus^2\left(\frac{r}{d}\right)$$

where d will depend on the distance the beam has propagated.