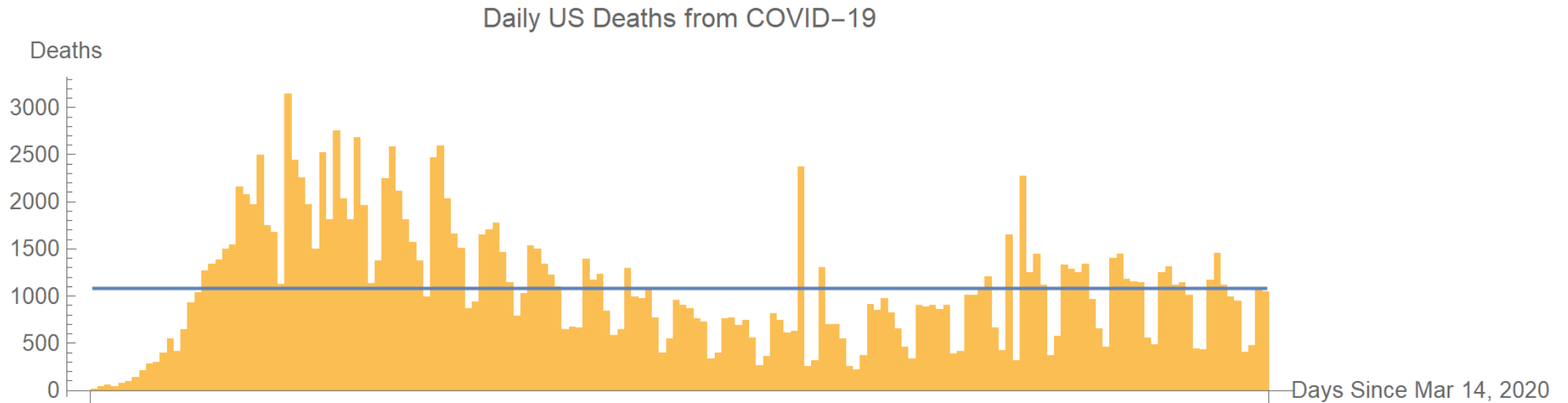


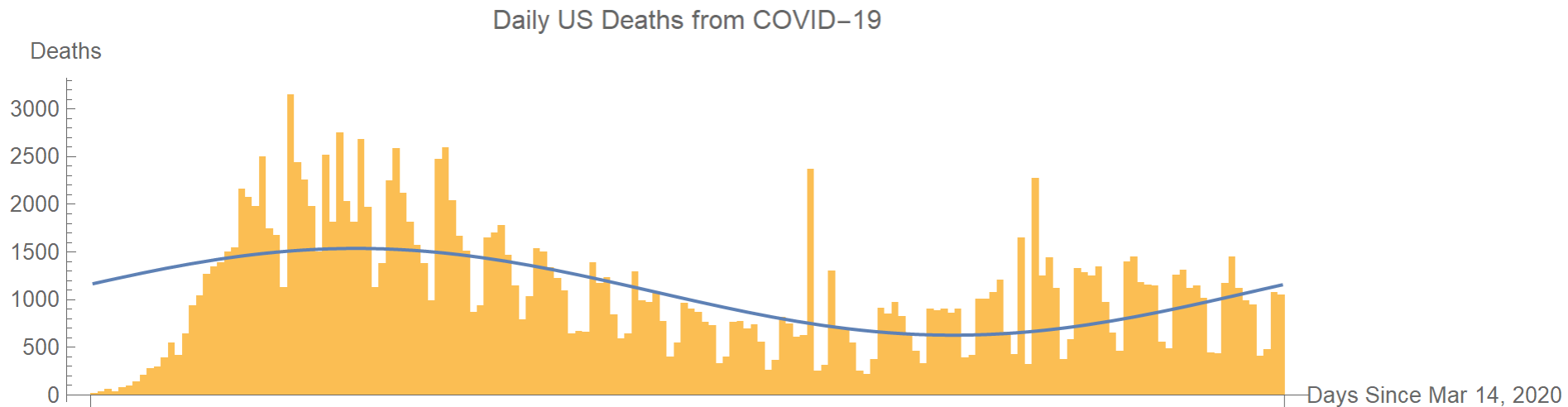
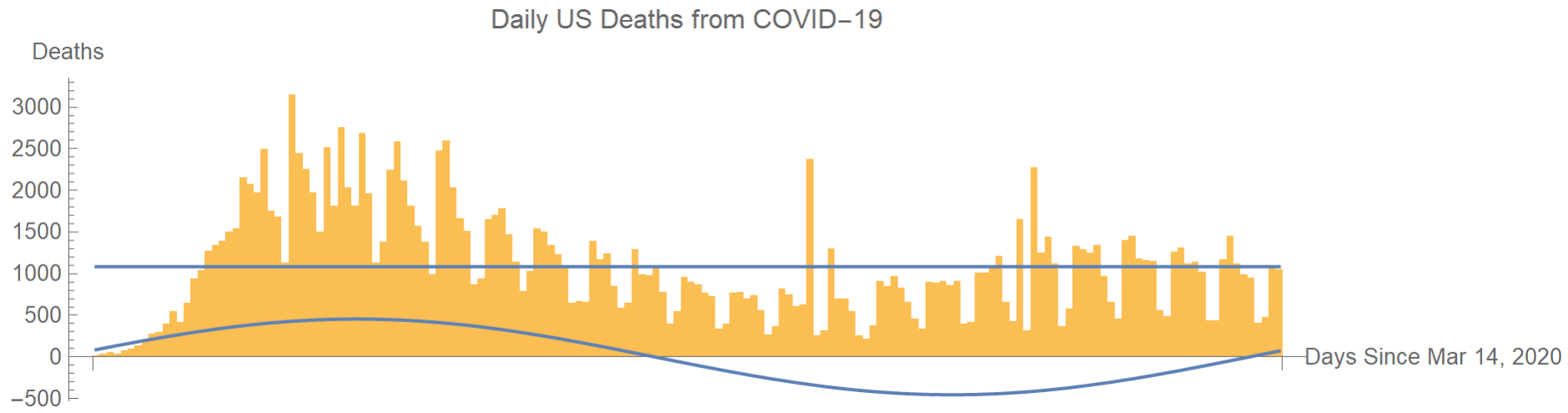
# Covid-19 Data



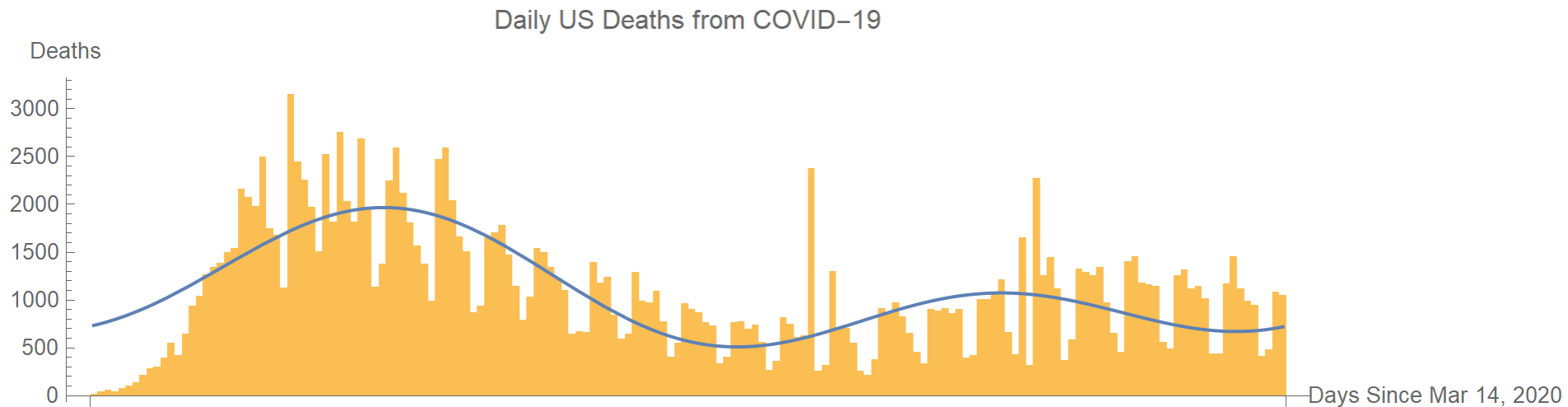
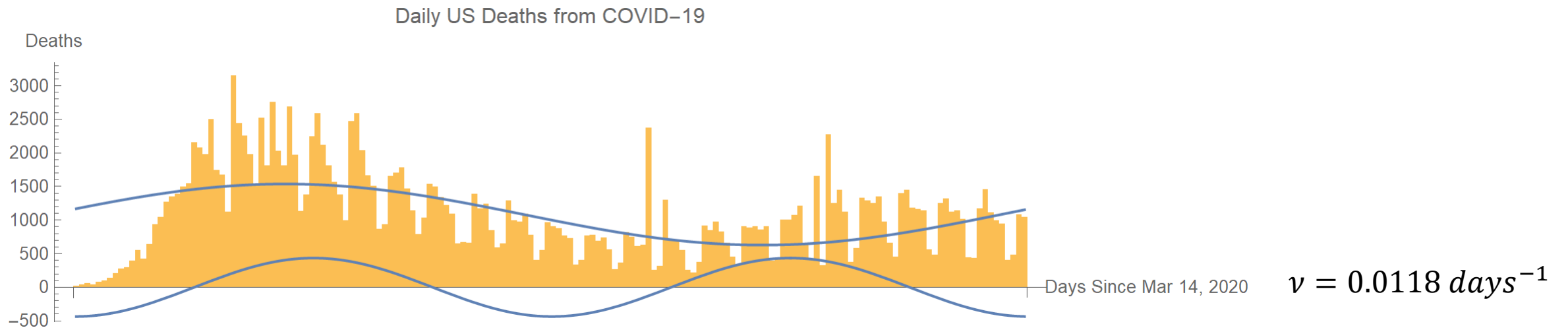
This is the daily death toll in the US from Covid-19 between March 14 and September 2, 2020. On average, there are 1083 deaths per day (blue line). Harmonic analysis is useful for examining variations in this data with respect to the mean value.

This is a time series with units of days. The temporal frequency  $\nu$  of the signal will have units of  $days^{-1}$ .

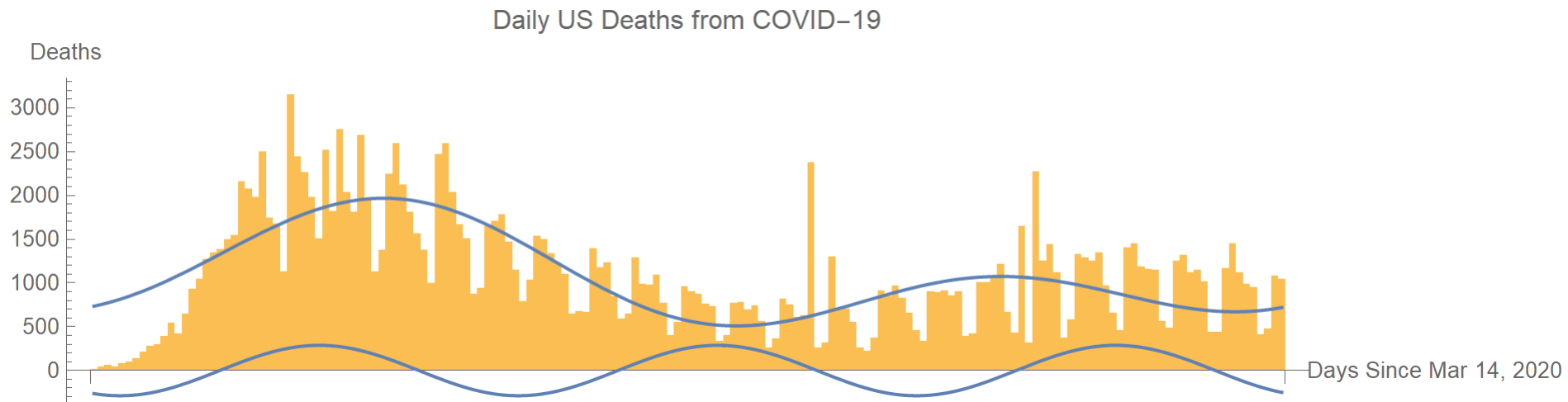
# Largest Sinusoidal Component



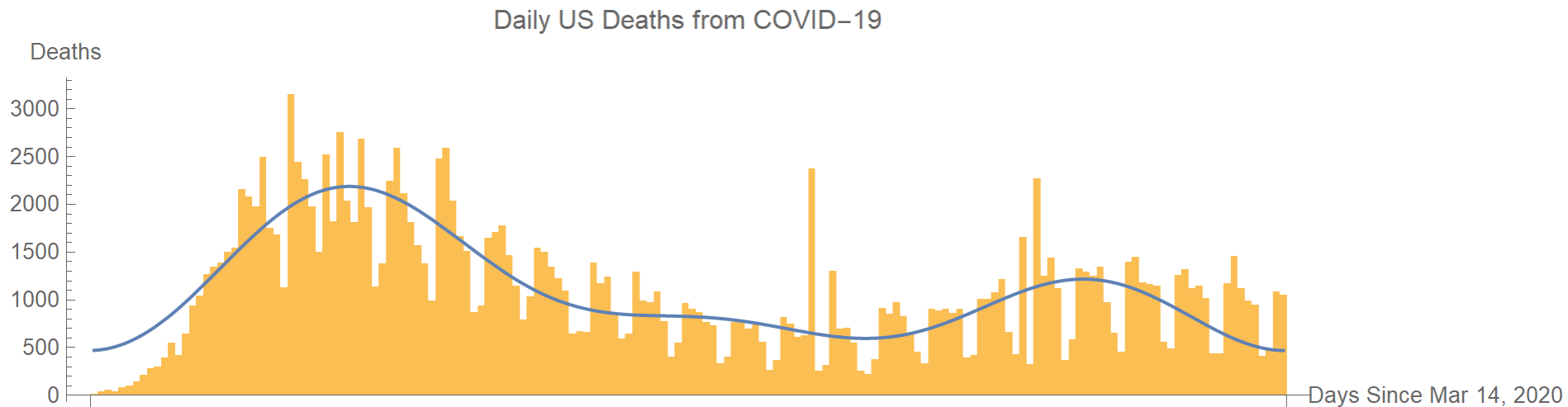
# Largest 2 Sinusoidal Components



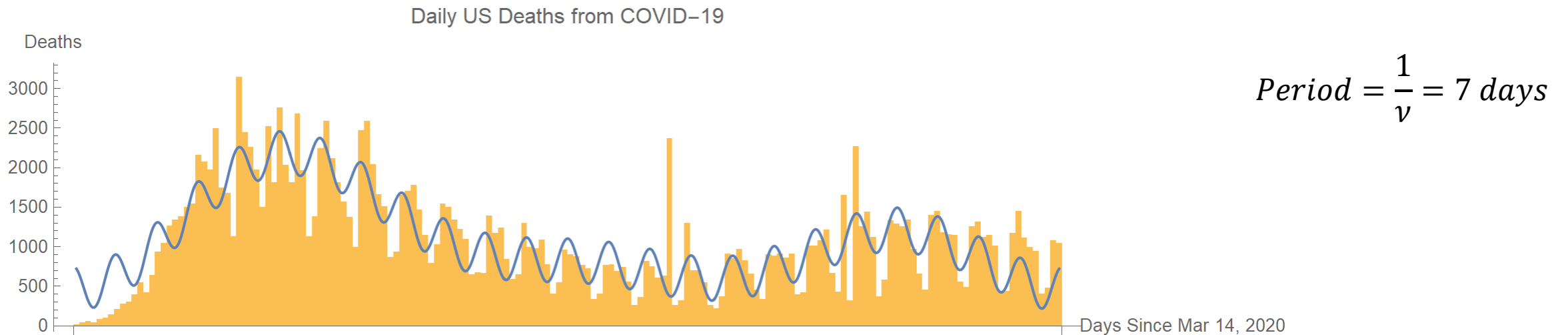
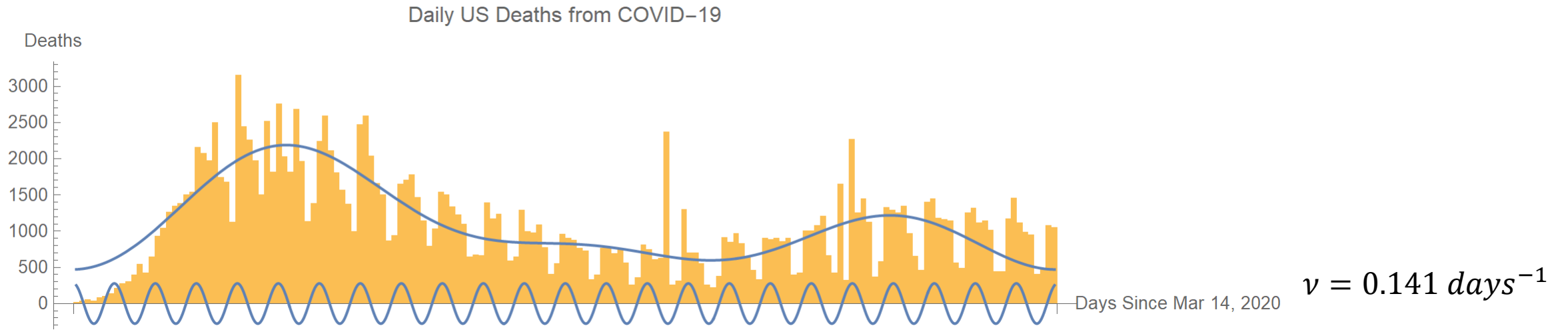
# Largest 3 Sinusoidal Components



$$\nu = 0.0176 \text{ days}^{-1}$$

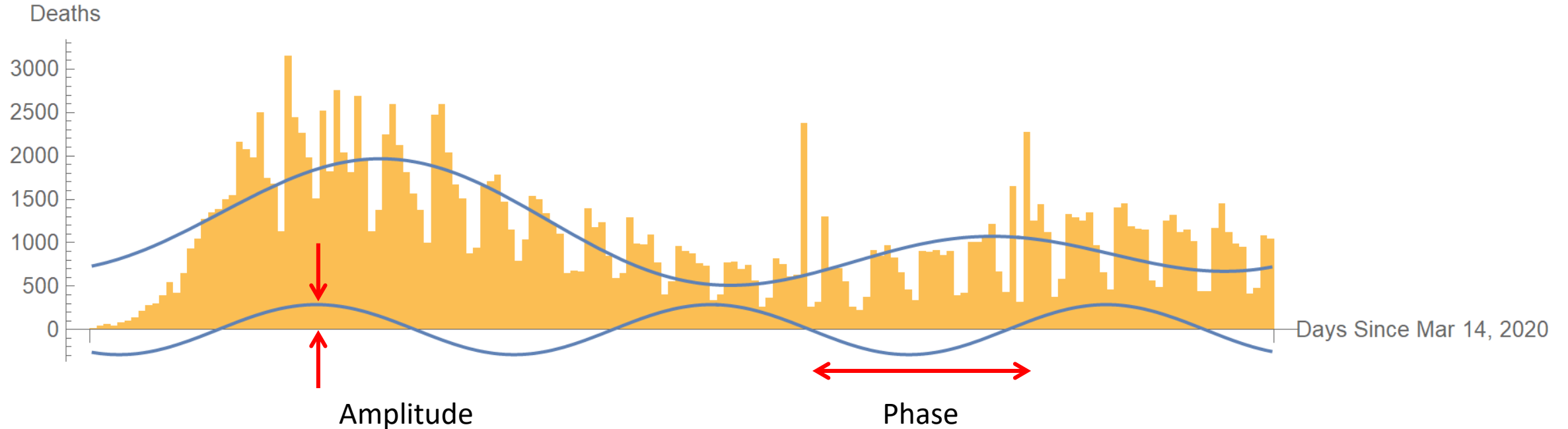


# Largest 4 Sinusoidal Components



# Harmonic Analysis 1D

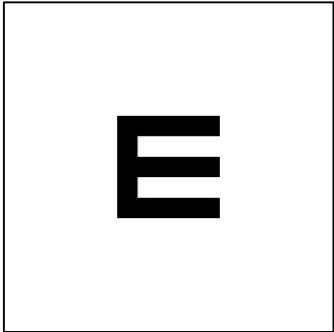
Daily US Deaths from COVID-19



For each sinusoidal component we are adding to approximate the signal, we have control over two items: the amplitude and the phase. The amplitude tells us how high the peaks (and how low the valleys) are. The phase allows us to shift the pattern left or right.

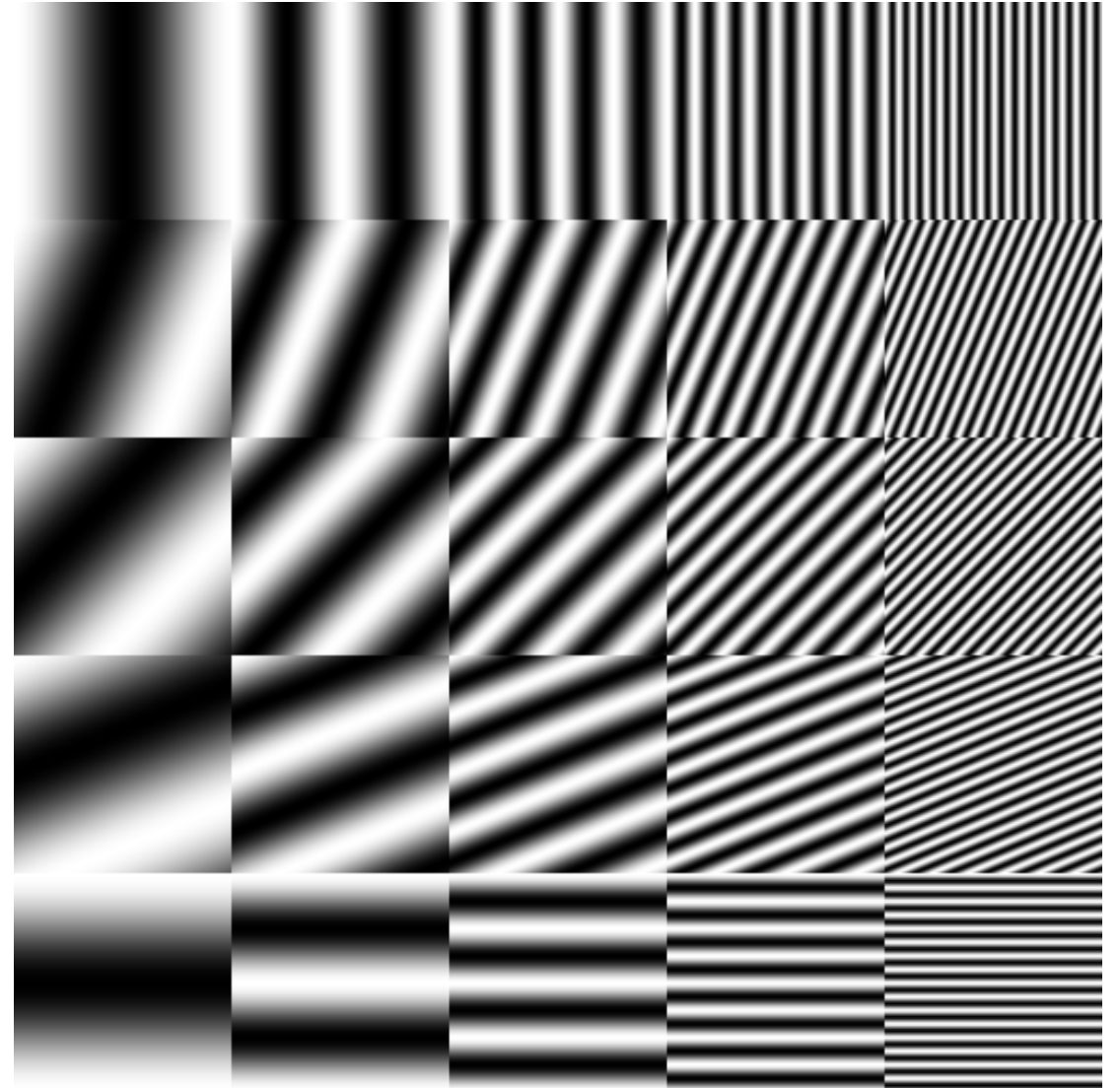
# Harmonic Analysis 2D

In 2D, you want to represent some 2D pattern like the letter E here as a combination of sines and cosines.



Now you have three things that can be varied:

- The amplitude, which controls how “white” the peaks are and how “black” the troughs are.
- The phase, which just shifts the pattern.
- The orientation, which is the angle the pattern makes with the x-axis.



# Period & Spatial Frequency

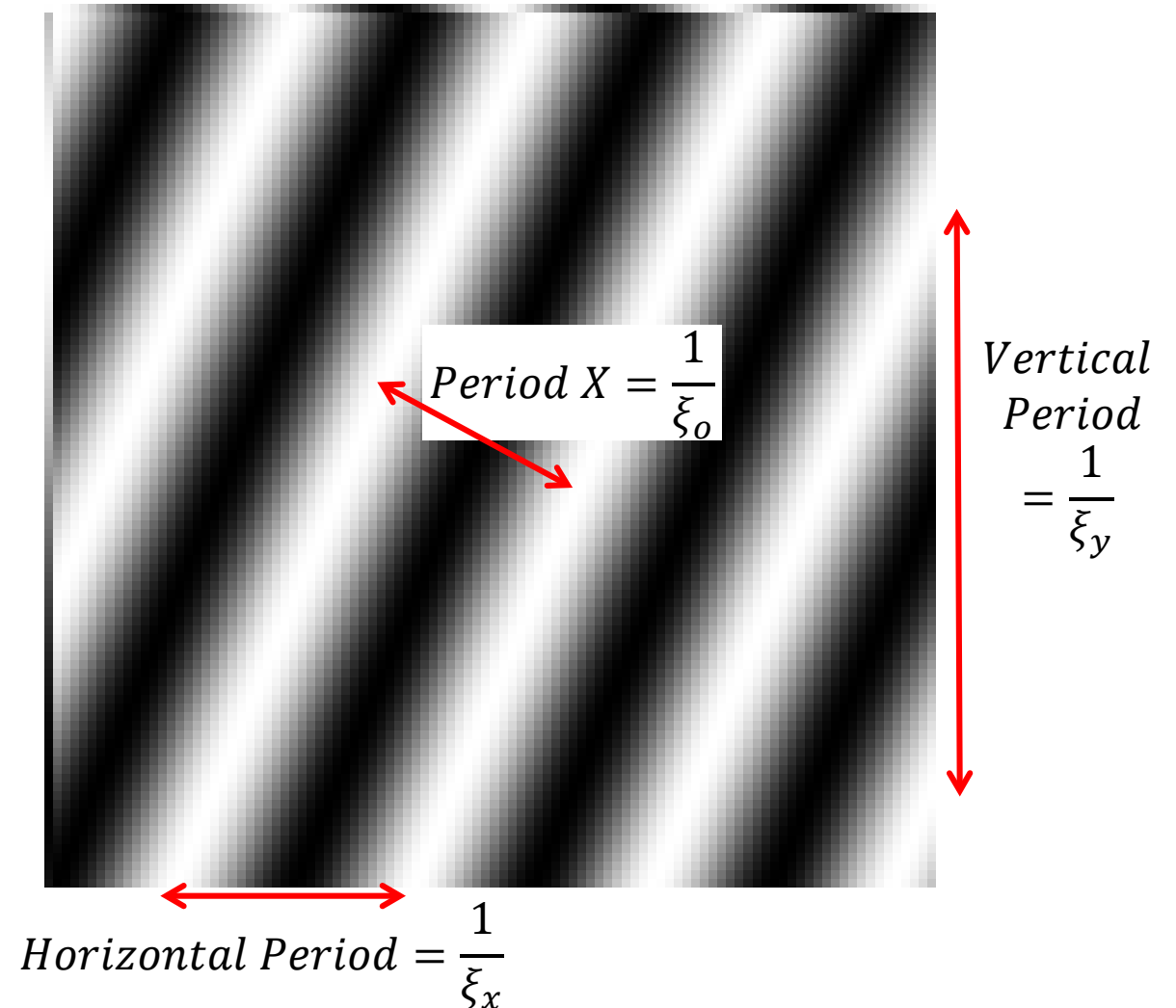
The *period*  $X$  is just the distance for one complete cycle of the sinusoid.

The spatial frequency  $\xi_o$  is just the reciprocal of this period  $\xi_o = 1/X$ .

Sometimes, we refer to the horizontal spatial frequency  $\xi_x$  and the vertical spatial frequency  $\xi_y$ , which are just the reciprocals of the periods along the x- and y-axis, respectively.

Use a little trig to convince yourself that

$$\xi_o = \sqrt{\xi_x^2 + \xi_y^2}$$





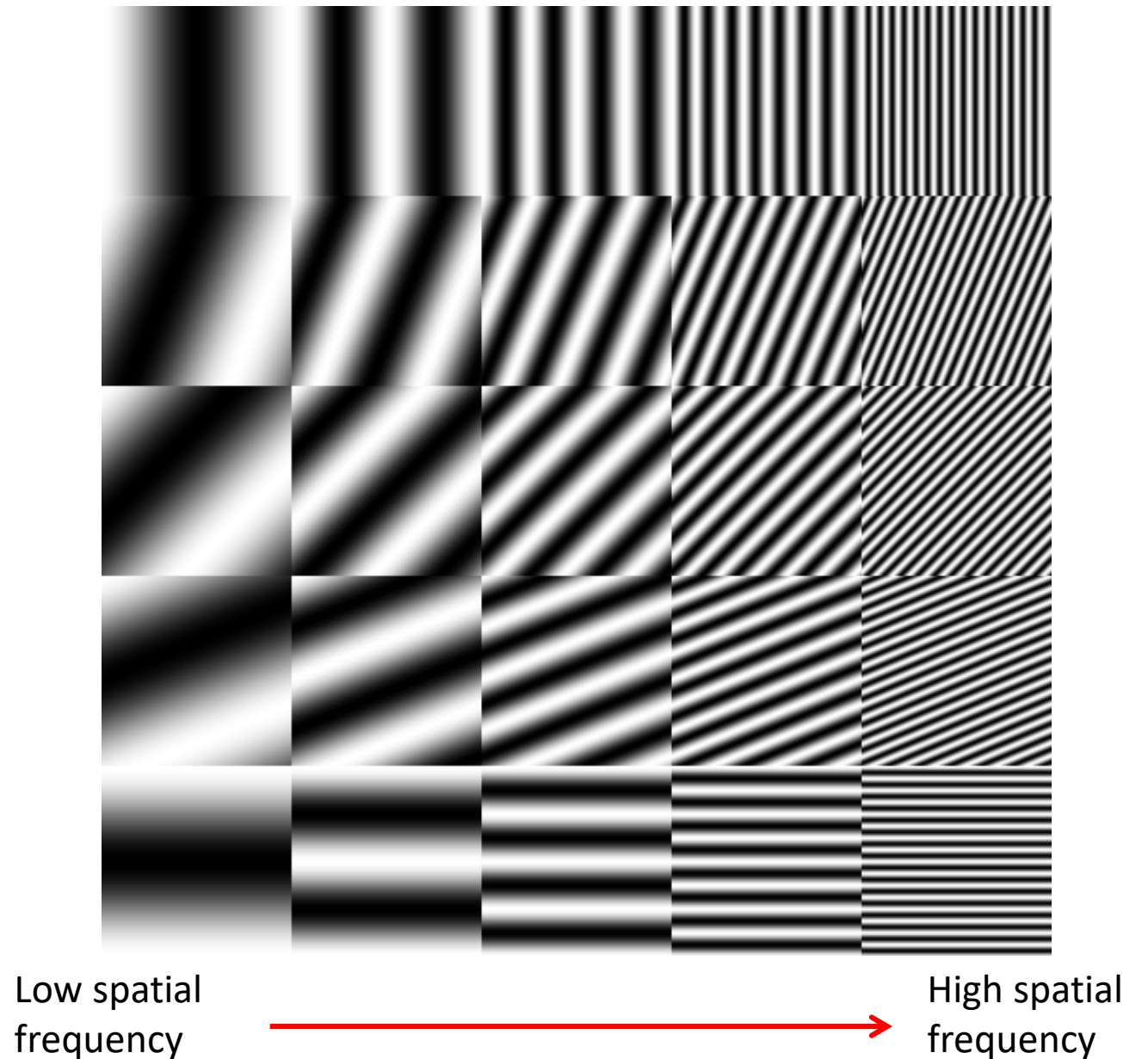
# Spatial Frequency

Each column has the same spatial frequency  $\xi_0$ , but the orientation of the pattern rotates.

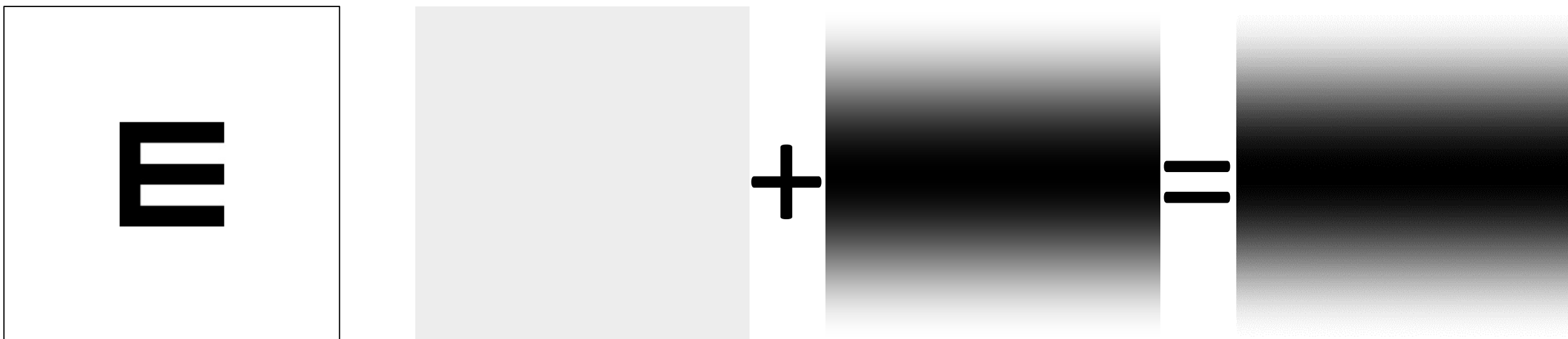
Each row has the same orientation, but increasing spatial frequency.

Low spatial frequency means a large period.

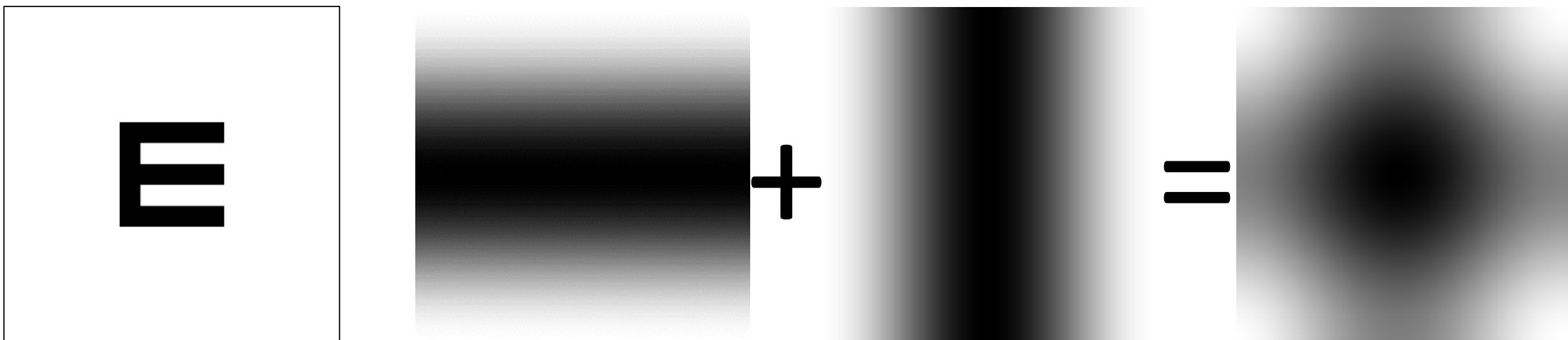
High spatial frequency means a small period.



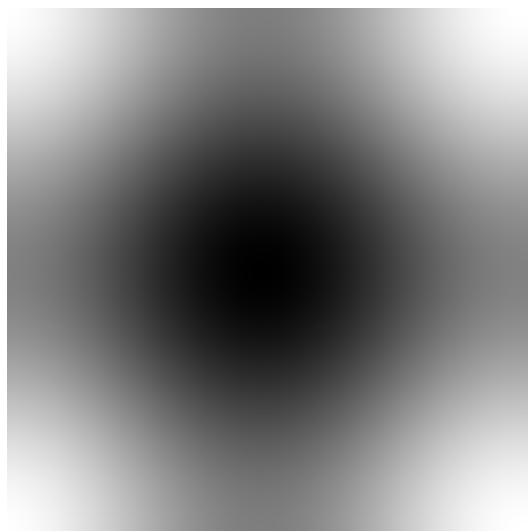
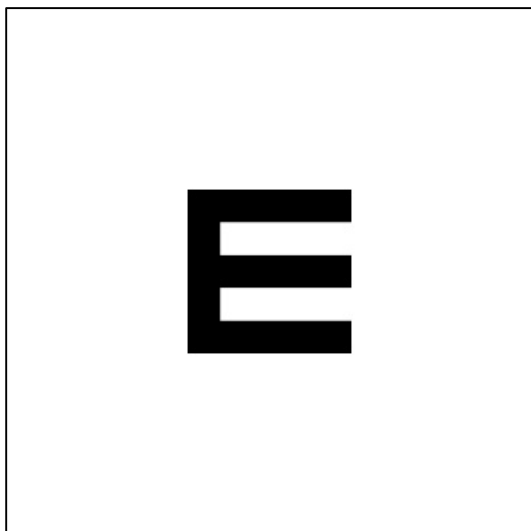
# Harmonic Analysis 2D



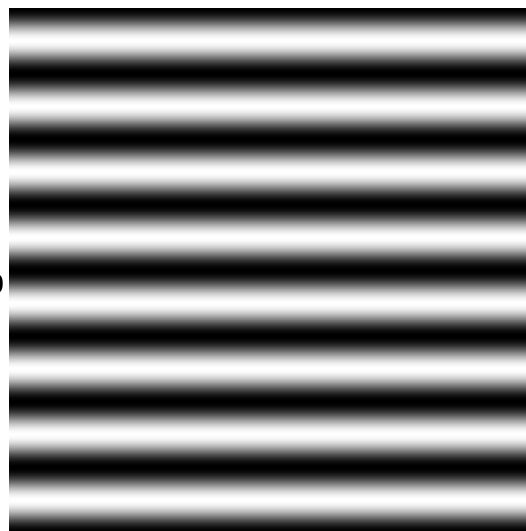
# Harmonic Analysis 2D



# Harmonic Analysis 2D



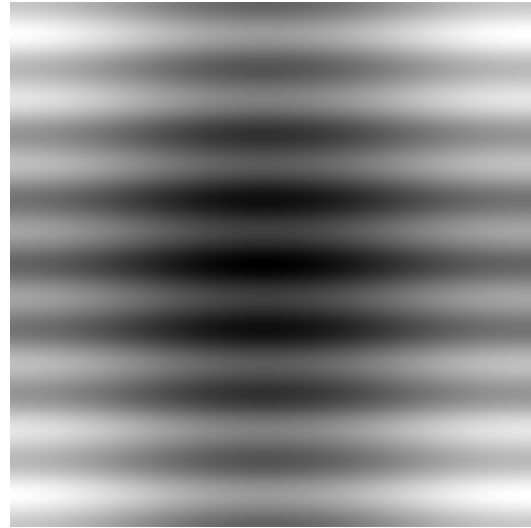
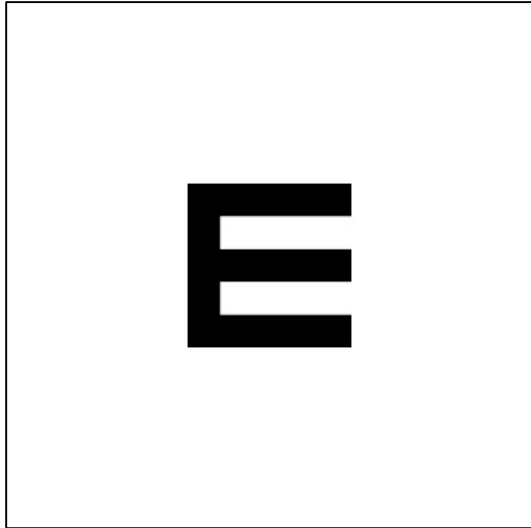
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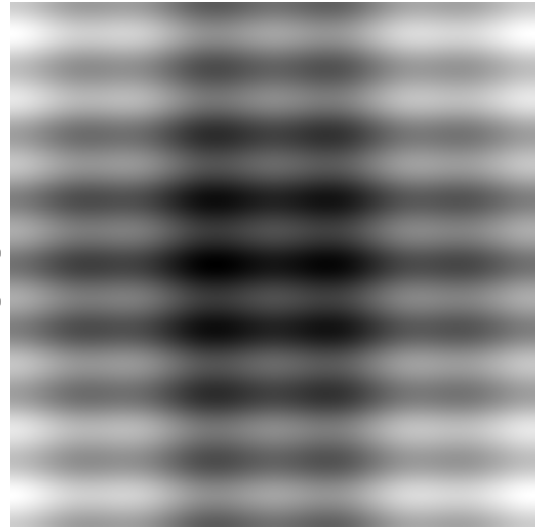
# Harmonic Analysis 2D



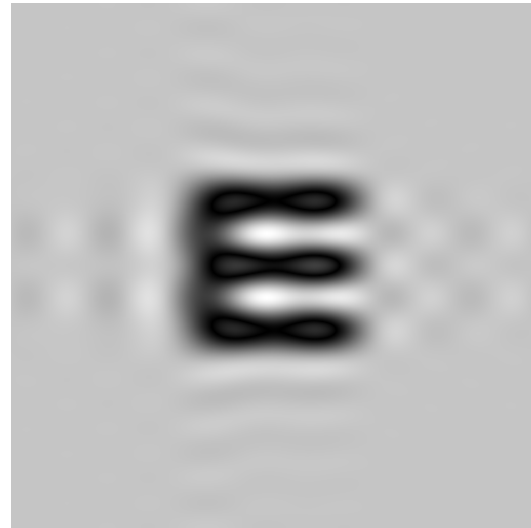
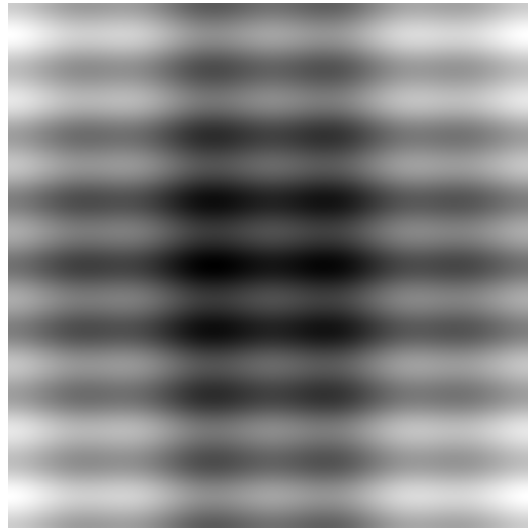
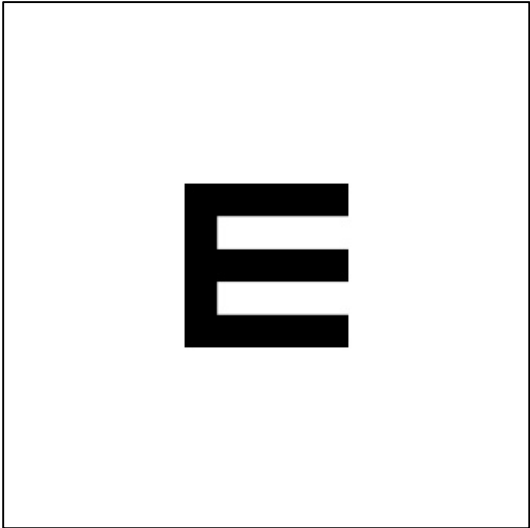
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# Harmonic Analysis 2D



Continue adding additional terms.