OPTI 512 Homework 4 Due: October 7, 2020

1. Prove that $rect_1(x) * rect_2(x) = tri(x)$ using the following procedure. Note the subscripts are just so we can keep track of the individual *rect()* functions.
   (a) Write the convolution integral with these two functions.
   (b) Plot $rect_1(\alpha)$.
   (c) Plot $rect_2(-\alpha)$. Note that this is just the second function flipped about the y-axis for the case when $x = 0$.
   (d) Plot $rect_2(x - \alpha)$ for the cases where $x = -1, x = -1/2, x = 0, x = 1/2,\ x = 1.$ This shows how the second functions slides horizontally depending on the value of x.
   (e) What are the values of the areas of $rect_1(\alpha)rect_2(x - \alpha)$ for $x = -1, x = -1/2, x = 0, x = 1/2,\ x = 1.$ The areas correspond to the integration of the product of the two functions.
   (f) What is the general formula for the area of $rect_1(\alpha)rect_2(x - \alpha)$ for $-1 < x \leq 0$? This will be a function of x.
   (g) What is the general formula for the area of $rect_1(\alpha)rect_2(x - \alpha)$ for $0 < x < 1$? This will also be a function of x.
   (h) Show that preceding results are consistent with the definition of $tri(x)$.
2. Use the Fourier integral to show that $Gaus(\xi) = \mathcal{F}\{Gaus(x)\}$. Hint: Use completing the square and that

$$\int_{-\infty}^{\infty} exp[-\pi u^2]du = 1$$

3. Based on the results of question 2, using the scaling and shifting properties of Fourier transforms to calculate

   (a) $\mathcal{F}\{Gaus(4x)\}$. Plot $Gaus(4x)$ and its Fourier transform for a horizontal range of -6 to 6 and a vertical range from 0 to 1.

   (b) $\mathcal{F}\left\{Gaus\left(\frac{x}{4}\right)\right\}$. Plot $Gaus\left(\frac{x}{4}\right)$ and its Fourier transform for a horizontal range of -6 to 6 and a vertical range from 0 to 4.

   (c) Based on (a) and (b) how are the widths of the Gaussian function and its Fourier transform related?

   (d) $\mathcal{F}\{Gaus(4x-2)\}$. Plot $Gaus(4x-2)$ and the magnitude and phase of its Fourier transform. Furthermore, plot the real and imaginary parts of the transform. How does the magnitude and phase of this function compare to the transform in part (a)? How does the envelope of the real and imaginary parts relate to the magnitude? Use a horizontal range of -6 to 6 for all plots. Use a vertical range from 0 to 1 for $Gaus(4x-2)$ and the modulus if its transform. Use a vertical range from -1 to 1 for the real and imaginary parts of the Fourier transform.