Answer all three questions

1. Microprinting is a technique for thwarting counterfeiters. The image below shows the words "THE UNITED STATES OF AMERICA" microprinted in Benjamin Franklin's collar on the $\$ 100$ bill. The height of these letters is $282 \mu \mathrm{~m}$. Let's design an imaging system capable of capturing the image below. The system will consist of an infinity-corrected microscope objective, a tube lens and a digital sensor.

(a) The specs for the digital sensor along with a chart of common sensor sizes are below. Based on this information, what are the horizontal and vertical pixel dimensions of the sensor?


Table 1.1 Common CMOS and CCD sensors and their dimensions.

| Sensor type | Width $(\mathrm{mm})$ | Height $(\mathrm{mm})$ | Diagonal $(\mathrm{mm})$ |
| :--- | :---: | :---: | :---: |
| $1 / 10 \mathrm{in}$. | 1.28 | 0.96 | 1.60 |
| $1 / 8 \mathrm{in}$. | 1.60 | 1.20 | 2.00 |
| $1 / 6 \mathrm{in}$. | 2.40 | 1.80 | 3.00 |
| $1 / 4 \mathrm{in}$. | 3.20 | 2.40 | 4.00 |
| $1 / 3.6 \mathrm{in}$. | 4.00 | 3.00 | 5.00 |
| $1 / 3.2 \mathrm{in}$. | 4.54 | 3.42 | 5.68 |
| $1 / 3 \mathrm{in}$. | 4.80 | 3.60 | 6.00 |
| $1 / 2.7 \mathrm{in}$. | 5.37 | 4.04 | 6.72 |
| $1 / 2.5 \mathrm{in}$. | 5.76 | 4.80 | 7.18 |
| $1 / 2 \mathrm{in}$. | 6.40 | 5.32 | 8.00 |
| $1 / 1.8 \mathrm{in}$. | 7.18 | 5.70 | 8.93 |
| $1 / 1.7 \mathrm{in}$. | 7.60 | 6.01 | 9.50 |
| $1 / 1.6 \mathrm{in}$. | 8.08 | 6.60 | 10.07 |
| $2 / 3 \mathrm{in}$. | 8.80 | 9.60 | 11.00 |
| 1 in. | 12.80 |  | 16.00 |

Based on the specs, the sensor has a $1 / 3$ " sensor with $2.2 \mu \mathrm{~m}$ square pixels. Based on the chart, a $1 / 3$ " sensor has dimensions of $4.8 \mathrm{~mm} \times 3.6 \mathrm{~mm}$. To determine the sensor dimensions in pixels simply divide the dimensions of the sensor by the size of each pixel to get $2182 \times 1636$ pixels. Based on the F-number and the focal length.
(b) The specification for the system says that a height of 32 pixels are needed to adequately resolve the microprinting. What is the required magnification of the system (don't forget the minus sign)?

The microprinting is $282 \mu \mathrm{~m}$ high and this size times the system magnification $m$ needs to correspond to 32 pixels on the sensor. Also, we expect the image to be inverted so the magnification is negative. So

$$
m=-\frac{32 p i x \times 2.2 \mu m}{282 \mu m}=-0.25
$$

(c) What is the focal length of the tube lens if the microscope objective has a focal length of 40 mm ?

The magnification is given by

$$
m=-\frac{f_{\text {tube }}}{f_{\text {objective }}} \Rightarrow f_{\text {tube }}=10 \mathrm{~mm}
$$

(d) The Numerical Aperture (NA) of the objective is 0.15 . What is the diameter of the aperture stop in the objective? (Hint: recall that the $F / \#=1 / 2 N A$ )

From the hint

$$
F / \#=\frac{f_{\text {objective }}}{D}=\frac{1}{2 N A} \Rightarrow D=2 \times 0.15 \times 40 \mathrm{~mm}=12 \mathrm{~mm}
$$

2. The figure below shows a double Gauss lens with three rays traced through it. Ray 1 is parallel to the optical axis as it enters the lens and passes through the edge of the aperture stop. Ray 2 is parallel to the optical axis as it exits the lens and passes through the edge of the aperture stop. Ray 3 passes through the center of the aperture stop. You will be drawing the cardinal points and pupils on the figures below. Be sure to be neat and illustrate how you determine the locations and diameters.

(a) On the figure above, draw the positions of the front and rear focal points and label them F and F , respectively.

(b) On the figure above, draw the front and rear principal planes and label them P and $\mathrm{P}^{\prime}$, respectively. Label the front and rear nodal points N and $\mathrm{N}^{\prime}$, as well.

(c) On the figure above, draw the location and diameter of the entrance pupil. Denote them EP and $\mathrm{D}_{\mathrm{EP}}$, respectively.

(d) On the figure above, draw the location and diameter of the exit pupil. Denote them $X P$ and $\mathrm{D}_{\mathrm{XP}}$, respectively.
(e) Why do we calculate the cardinal points? Once the cardinal points are determined, the system can be treated as a black box. The image location and magnification can be determined for any arbitrary object without the need to trace additional rays.
(f) Why do we calculate the entrance and exit pupil locations and size? The pupils determine how much energy passes through the system. The OPD in the exit pupil describes the aberrations of the optical system.
3. Sketch the lens specified in the ISO10110 drawing below. Note, the lens is 3 mm thick and has a diameter of $\phi=25.4 \mathrm{~mm}$. Why is $\phi_{e}<\phi$ ?


