1. The file `asphere515.txt` (available online) contains a set of data points from a stylus profilometer measurement of an aspheric surface. The data \( \{r_i, z_i\} \) with \( i = 1 \cdots N \) is in two columns, where \( r_i \) is the lateral distance from the origin and \( z_i \) is the measured surface sag at that point. Compute the apical radius \( R \) and conic constant \( K \) for a fit to this data using the following technique.

For an conoid, the sag equation is

\[
z = R - \sqrt{R^2 - (K + 1)r^2}.
\]

Rearranging gives

\[
(K + 1)z^2 - 2Rz = -r^2.
\]

For the data, we can write a system of equations as

\[
\begin{pmatrix}
z_1^2 & -2z_1 \\
\vdots & \vdots \\
z_N^2 & -2z_N
\end{pmatrix}
\begin{pmatrix}
K + 1 \\
R
\end{pmatrix}
= 
\begin{pmatrix}
-r_1^2 \\
\vdots \\
-r_N^2
\end{pmatrix}
\]

This matrix equation can be solved using a least squares technique as

\[
(K + 1)z^2 - 2Rz = -r^2.
\]

Plot the fit error between the original data and the conoid.

2. Alvarez lenses are a pair of cubic phase plates that form a variable power element when they are shifted by equal amounts in opposite directions. The sag of these surfaces are given by

\[
z = A \left( xy^2 + \frac{1}{3}x^3 \right)
\]

Rewrite the sag in terms of the Chebyshev polynomials.

3. The following page has a drawing of an element with aspheric surfaces. Fill out the sag values \( z \) in the sag table. What type of surfaces are used for this lens?
**Left aspheric surface**

\[
z = \frac{y^2}{R} + Ay^4 + By^6 + Cy^8 + Dy^{10} = Ey^{12} + Fy^{14} + Gy^{16} + Hy^{18} + Jy^{20}
\]

<table>
<thead>
<tr>
<th>(y)</th>
<th>(z)</th>
<th>(\Delta z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.4</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>0.8</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1.2</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Right aspheric surface**

<table>
<thead>
<tr>
<th>(y)</th>
<th>(z)</th>
<th>(\Delta z)</th>
</tr>
</thead>
<tbody>
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<tr>
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<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Left surface:**

- Asphere
- \(\phi_a = 2.21\)
- \(N_a = 1.6229 \pm 0.000\)
- \(\sigma_a = 2.37\)
- Coating:
- Rev level: Chk
- Date: yonjgian zhu
- Next assy:чреж
- Revisions: Eng
- Ltr descr dt: QA
- Scale 19:1
- Units=mm
- Ind. acc. ISO 10110