Answer all three questions

1. A thin lens is used as an objective lens for an object at infinity. The system is $\mathrm{F} / 4$ with a focal length of 80 mm and a FFOV of $8^{\circ}$. The stop is at the lens. Answer the following:
(a) What is the entrance pupil diameter?

Based on the F-number and the focal length

$$
f / \#=\frac{f}{D_{E}} \Rightarrow D_{E}=\frac{80 \mathrm{~mm}}{4}=20 \mathrm{~mm}
$$

(b) Assuming the lens is planoconvex thin lens and has an index of $n=1.517$, what are the radii R1 and R2 of the lens?
Planoconvex means one surface is flat and the other in curved. Let's choose $R 2=\infty$ as the flat surface. The other radii R1 can be found as follows:

The power of the lens is $\phi=1 / f=0.0125 \mathrm{~mm}^{-1}$.
From the Lensmakers' formula

$$
\begin{gathered}
\phi=(n-1)\left(\frac{1}{R 1}-\frac{1}{R 2}\right) \\
0.0125=(1.517-1)\left(\frac{1}{R 1}\right) \\
R 1=41.36 \mathrm{~mm} \\
R 2=\infty
\end{gathered}
$$

(c) Based on the table of CCD sensors below, which sensor best matches (diagonal) the field of view of the lens?
The height of the chief ray at the image plane is

$$
\bar{y}=\operatorname{ftan}(F F O V / 2)=5.594 \mathrm{~mm}
$$

This suggests that the diagonal of the sensor should be about double this value. A $2 / 3$ " sensor has a diagonal of 11 mm , which best matches the FFOV.

Table 1.1 Common CMOS and CCD sensors and their dimensions.

| Sensor type | Width $(\mathrm{mm})$ | Height $(\mathrm{mm})$ | Diagonal $(\mathbf{m m})$ |
| :--- | :---: | :---: | :---: |
| $1 / 10 \mathrm{in}$. | 1.28 | 0.96 | 1.60 |
| $1 / 8 \mathrm{in}$. | 1.60 | 1.20 | 2.00 |
| $1 / 6 \mathrm{in}$. | 2.40 | 1.80 | 3.00 |
| $1 / 4 \mathrm{in}$. | 3.20 | 2.40 | 4.00 |
| $1 / 3.6$ in. | 4.00 | 3.00 | 5.00 |
| $1 / 3.2 \mathrm{in}$. | 4.54 | 3.42 | 5.68 |
| $1 / 3 \mathrm{in}$. | 4.80 | 3.60 | 6.00 |
| $1 / 2.7 \mathrm{in}$. | 5.37 | 4.29 | 6.72 |
| $1 / 2.5 \mathrm{in}$. | 5.76 | 4.80 | 7.18 |
| $1 / 2 \mathrm{in}$. | 6.40 | 5.32 | 8.00 |
| $1 / 1.8 \mathrm{in}$. | 7.18 | 5.70 | 8.93 |
| $1 / 1.7 \mathrm{in}$. | 7.60 | 6.01 | 9.50 |
| $1 / 1.6 \mathrm{in}$. | 8.08 | 6.60 | 10.07 |
| $2 / 3 \mathrm{in}$. | 8.80 | 9.60 | 11.00 |
| 1 in. | 12.80 |  | 16.00 |

(d) For a wavelength of $\lambda=0.5876 \mu m$, what is the Airy disk diameter?

The Airy disk diameter is given by

$$
\text { Diameter }=2.44 \lambda F / \#=5.735 \mu m
$$

(e) Which sensor resolution best matches the optics of the objective: $640 \times 480,1024 \times 768$, or 1600 x 1200 ?
Based on the Airy disk diameter and the dimensions of the $2 / 3^{\prime \prime}$ sensor, the pixels should about the same size as the Airy disk

$$
\frac{8.8 \mathrm{~mm}}{0.005735} \times \frac{6.6 \mathrm{~mm}}{0.005735}=1534 \times 1151
$$

So the $1600 \times 1200$ resolution best matches the optics.
2. The figure below shows three points in the exit pupil of an optical system. The exit pupil has a diameter of 4 mm .

(a) Based on the real coordinates for each point given in the table below, fill in the normalized coordinate values in the remainder of the table. Recall that $\theta$ is the polar angle measured counterclockwise from the horizontal axis and $\psi$ is the polar angle measured clockwise from the vertical axis.

|  | Real Cartesian |  | Normalized Cartesian |  |  | Normalized Polar |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Point | $x_{p}$ | $y_{p}$ | $\rho_{x}$ | $\rho_{y}$ | $\rho$ | $\theta$ | $\psi$ |  |
| A | 0 | 0 | 0 | 0 | 0 | $N A$ | $N A$ |  |
| B | 1 | 1 | 0.5 | 0.5 | 0.707 | $45^{\circ}$ | $45^{\circ}$ |  |
| C | 0 | 2 | 0 | 1 | 1.0 | $90^{\circ}$ | $0^{\circ}$ |  |

(b) The wavefront error for this system is $W(\rho, \psi)=W_{131} h \rho^{3} \cos \psi$ where $W_{131}=0.001 \mathrm{~mm}$. If the reference sphere radius is $R=100 \mathrm{~mm}$, derive the transverse ray errors $\varepsilon_{x}$ and $\varepsilon_{y}$.
This wavefront error is just Seidel coma and from the notes, the transverse ray errors are given by

$$
\begin{gathered}
\varepsilon_{x}(\rho, \psi)=-\frac{R W_{131}}{r_{\max }} h \rho^{2} \sin 2 \psi=-0.05 h \rho^{2} \sin 2 \psi \\
\varepsilon_{y}(\rho, \psi)=-\frac{R W_{131}}{r_{\max }} h \rho^{2}(2+\cos 2 \psi)=-0.05 h \rho^{2}(2+\cos 2 \psi)
\end{gathered}
$$

(c) For an object point at full field $(h=1)$, where do the rays passing the points A, B, and C in the pupil, intersect the paraxial image plane relative to the ideal image point?

For an object point at full field $(h=1)$, the values of $\varepsilon_{x}$ and $\varepsilon_{y}$ are the deviations from ideal image point. Based on the values in the table from part (a) and the results of part (b), the values of $\varepsilon_{x}$ and $\varepsilon_{y}$ are:

Point A: $(0 \mathrm{~mm}, 0 \mathrm{~mm})$ since $\rho=0$, which means this ray intersects the ideal image point.
Point B: $(-0.025 \mathrm{~mm},-0.05 \mathrm{~mm})$
Point C: $(0 \mathrm{~mm},-0.15 \mathrm{~mm})$
3. Sketch the lens specified in the ISO10110 drawing below. Note, the lens is 3 mm thick,


| Left Surface | Material | Right Surface |
| :---: | :---: | :---: |
| R $80 \quad$ CX Øe 40 8 $3 /-$ $4 /-$ $5 /-$ $6 /-$ | $\begin{aligned} & \text { GLASS: } \text { BK7 } \\ & \text { Nd }=1.516800 \\ & \text { Vd }=64.17 \\ & 0 /- \\ & 1 /- \\ & 2 /- \end{aligned}$ | R $40 \quad$ CC <br> Øe 40 <br> Q <br> $3 /-$ <br> $4 /-$ <br> $5 /-$ <br> $6 /-$ |
| SO Element Drawing Indications According to ISO 10110 |  |  |

