OPTI 415 Midterm 2   March 23, 2017

1. Suppose you have a rotationally symmetric wavefront error given by $W(\rho) = W_{020}\rho^2 + W_{040}\rho^4$.
   Do the following:

   a) If the wavefront error at $\rho = 0.707$ is $W(0.707) = 0$ and at $\rho = 1.0$ is $W(1.0) = 1$,
      calculate the values of the coefficients W_{020} and W_{040}.
      The values of the wavefront error give two equations
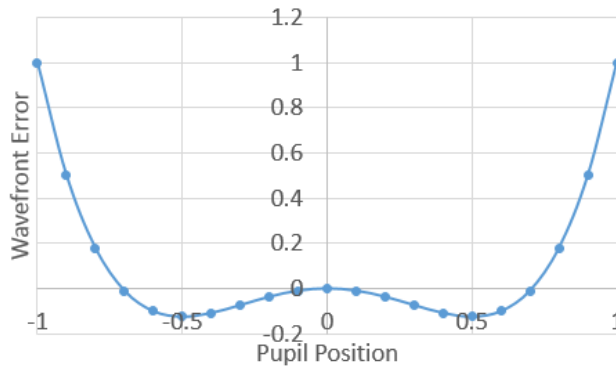
      $$W(0.707) = 0.5W_{020} + 0.25W_{040}\rho^4 = 0$$

      and

      $$W(1.0) = W_{020} + W_{040} = 1$$

      Solving these two equations gives $W_{020} = -1.0$ and $W_{040} = 2.0$.

   b) Sketch a plot of the wavefront error.

   c) Find the equivalent wavefront error in terms of the Zernike polynomials.
      Equate the wavefront error to the expected Zernike terms

      $$-\rho^2 + 2\rho^4 = a_{40}\sqrt{5}(6\rho^4 - 6\rho^2 + 1) + a_{20}\sqrt{3}(2\rho^2 - 1) + a_{00}$$

      Compare like powers

      $$2\rho^4 = 6\sqrt{5}a_{40}\rho^4$$

      $$-\rho^2 = -6\sqrt{5}a_{40}\rho^2 + 2\sqrt{3}a_{20}\rho^2$$

      $$0 = \sqrt{5}a_{40} - \sqrt{3}a_{20} + a_{00}$$

      Solve for the expansion coefficients

      $$a_{40} = \frac{1}{3\sqrt{5}}$$

$$a_{20} = \frac{1}{2\sqrt{3}}$$

$$a_{00} = \frac{1}{6}$$

d) What is the variance of this wavefront error?

The variance is the sum of the squares of the Zernike coefficients, excluding the piston term

$$\sigma^2 = a_{20}^2 + a_{40}^2 = 0.106$$

e) What is the rms wavefront error?

The rms wavefront error is the square root of the variance

$$rms = \sqrt{\sigma^2} = 0.325$$

f) What is the Peak-to-Valley value for this wavefront error.

Based on the plot in part (b), the maximum wavefront error is 1 and occurs at the edge of the pupil. The location of the minimum value of the wavefront error is found by taking the radial derivative and setting it equal to zero.

$$\frac{\partial W(\rho)}{\partial \rho} = -2\rho + 8\rho^3 = 0$$

$$\rho = \frac{1}{2}$$

The peak to valley error is then

$$PV\ Error = 1 - W\left(\frac{1}{2}\right) = \frac{9}{8}$$