OPTI 201R Midterm 2 November 13, 2018

Do all problems and show your work. Credit is not given for answers only. Don't spend too much time on any one problem. Use separate sheets of paper and don't cram your work into the spaces below. As in the notes, z_F and z'_F are the object and image distances measured from the front and rear focal points. The distances z and z' are the object and image distances measured from the front and rear principal plane. Questions 1-5 are worth 8 points each. Questions 6-7 are worth 30 points each.

Thin lens or general system $\phi = \frac{1}{f}$	Focal Lengths $f'_R = n'f \qquad f_F = -nf$
Single surface $\phi = \frac{n'-n}{R}$	Total power of two elements $\phi = \phi_1 + \phi_2 - \left(\frac{t'_1}{n'_1}\right)\phi_1\phi_2$
Front Principal Plane $\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\left(\frac{t'_1}{n'_1}\right)$	Rear Principal Plane $\delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\left(\frac{t'_1}{n'_1}\right)$
Front Focal Distance $FFD = f_F + d$	Back Focal Distance $BFD = f'_R + d'$
Gaussian Imaging Eq. $\frac{n'}{z'} - \frac{n}{z} = \frac{1}{f}$	Newtonian Imaging Eq. $z_F z'_F = f_F f'_R$
Transverse Magnification $m = \frac{h'}{h} = \frac{nz'}{n'z} = -\frac{z'_F}{f'_R} = -\frac{f_F}{z_F} = \frac{nu}{n'u'}$	Longitudinal Magnification (small shift) $\bar{m} = \frac{n'}{n}m^2$
Nodal Points $z_{PN} = z'_{PN} = f_F + f'_R = (n'-n)f$	Mirrors Thickness and indices switch sign after reflection
Paraxial Refraction (Reflection) Equation $n'_j u'_j = n_j u_j - y_j \phi_j$	Paraxial Transfer Equation $y_{j+1} = y_j + n'_j u'_j \frac{t'_j}{n'_j}$

1. The object space index is $n = 1$ and the image space index is $n' = 1.333$. Given an object distance $z_F = -20mm$ and a lens of focal length $f = -80mm$, what is the image distance z'_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 80mm.$

The rear focal length is given by $f'_R = n'f = -106.667mm.$

The magnification is given by $m = -\frac{f_F}{z_F} = -\frac{80}{-20} = 4.$

The image distance is given by $z'_F = -mf'_R = -(4)(-106.667) = 426.667.$

2. The object space index is $n = 1$ and the image space index is $n' = 1$. Given an image distance $z'_F = 60mm$ and a lens of focal length $f = -20mm$, what is the object distance z_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 20mm.$

The rear focal length is given by $f'_R = n'f = -20mm.$

The magnification is given by $m = -\frac{z'_F}{f'_R} = -\frac{60}{-20} = 3.$

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{20}{3} = -6.667.$

3. The object space index is $n = 1$ and the image space index is $n' = 1$. Given a magnification $m = 0.5$ and a lens of focal length $f = -80mm$, what is the object distance z_F and the image distance z'_F? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 80mm.$

The rear focal length is given by $f'_R = n'f = -80mm.$

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{80}{0.5} = -160.$

The image distance is given by $z'_F = -mf'_R = -(0.5)(-80) = 40.$

4. The object space index is $n = 1$ and the image space index is $n' = 1$. Given an object distance $z_F = -80mm$ and a lens of focal length $f = 80mm$, what is the image distance z'_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = -80mm.$

The rear focal length is given by $f'_R = n'f = 80mm.$

The magnification is given by $m = -\frac{f_F}{z_F} = -\frac{-80}{-80} = -1.$

The image distance is given by $z'_F = -mf'_R = -(-1)(80) = 80.$

5. The object space index is $n = 1$ and the image space index is $n' = 1$. Given an image distance $z'_F = -40mm$ and a lens of focal length $f = 60mm$, what is the object distance z_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

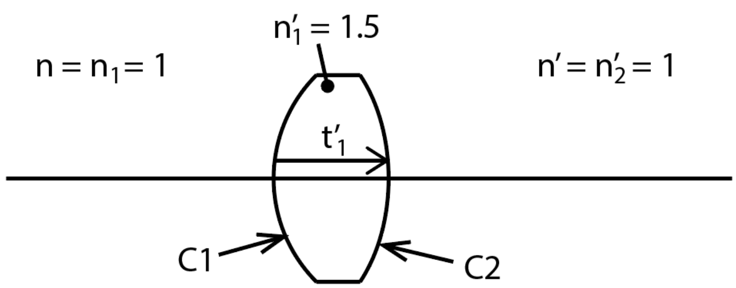
The front focal length is given by $f_F = -nf = -60mm$.

The rear focal length is given by $f'_R = n'f = 60mm$.

The magnification is given by $m = -\frac{z'_F}{f'_R} = -\frac{-40}{60} = 0.667$.

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{-60}{0.667} = 90$.

6. This question explores the effect of object and image space refractive index on the Cardinal points of a thick lens. The figure below shows a thick lens in air. The lens has a refractive index of $n'_1 = 1.5$ and a thickness $t'_1 = 10\ mm$. The curvatures of the surfaces are $C1 = 0.03\ mm^{-1}$ and $C2 = -0.01\ mm^{-1}$.



(a) Using Gaussian reduction, find the total power ϕ of the lens and the locations of the Cardinal points P, P', F, F', N, N'.

Here $n = n' = 1$ *and* $n'_1 = n_2 = 1.5$. *Given the curvatures of the two surfaces, the powers of the two surfaces are*

$$\phi_1 = (n'_1 - n_1)C_1 = 0.015\ mm^{-1}$$

$$\phi_2 = (n'_2 - n_2)C_2 = 0.005\ mm^{-1}.$$

The separation between the two lenses is $t'_1 = t_2 = 10\ mm$.

The total system power is $\phi = \phi_1 + \phi_2 - \frac{t_2}{n_2}\phi_1\phi_2 = 0.0195\ mm^{-1}$.

The focal length of the system is $f = 1/\phi = 51.282\ mm$.

The rear focal length of the lens is $f'_R = n'f = 51.282\ mm$.

The front focal length of the lens is $f_F = -nf = -51.282\ mm$.

The distance from the first surface to the front principle plane is

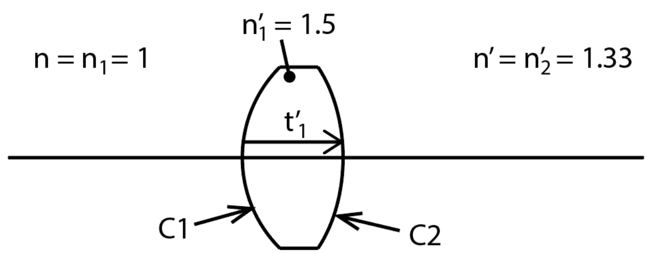
$$d = n\frac{\phi_2}{\phi}\frac{t_2}{n_2} = \frac{0.005}{0.0195}\frac{10}{1.5} = 1.709\,mm.$$

The distance from the second surface to the rear principle plane is

$$d' = -n'\frac{\phi_1}{\phi}\frac{t_2}{n_2} = -\frac{0.015}{0.0195}\frac{10}{1.5} = -5.128\,mm.$$

The nodal points are located at their respective principal planes since $n' = n$.

(b)The image space index is now switched to water with $n'_2 = 1.33$, as in the figure below. Find the new total power ϕ of the lens and the locations of the Cardinal points P, P', F, F', N, N'.



Here $n = 1$ and $n' = 1.33$ and $n_1' = n_2 = 1.5$. Given the curvatures of the two surfaces, the powers of the two surfaces are

$$\phi_1 = (n_1' - n_1)C_1 = 0.015\,mm^{-1}\ (unchanged\ from\ part(a))$$

$$\phi_2 = (n_2' - n_2)C_2 = 0.0017\,mm^{-1}.$$

The separation between the two lenses is still $t_1' = t_2 = 10\,mm$.

The total system power is $\phi = \phi_1 + \phi_2 - \frac{t_2}{n_2}\phi_1\phi_2 = 0.01653\,mm^{-1}$.

The focal length of the system is $f = 1/\phi = 60.496\,mm$.

The rear focal length of the lens is $f_R' = n'f = 80.460\,mm$.

The front focal length of the lens is $f_F = -nf = -60.496\,mm$.

The distance from the first surface to the front principle plane is

$$d = n\frac{\phi_2}{\phi}\frac{t_2}{n_2} = \frac{0.0017}{0.01653}\frac{10}{1.5} = 0.685\,mm.$$

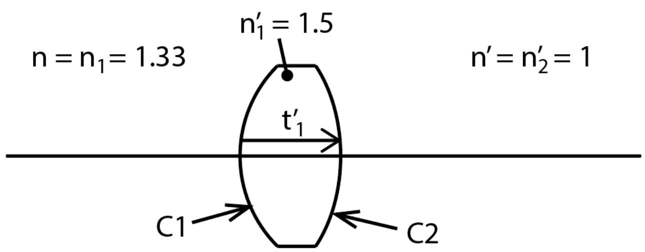
The distance from the second surface to the rear principle plane is

$$d' = -n' \frac{\phi_1}{\phi} \frac{t_2}{n_2} = -1.33 \frac{0.015}{0.01653} \frac{10}{1.5} = -8.046\, mm.$$

The nodal points are shifted from their respective principal planes such that

$$z_{PN} = z'_{PN} = (n' - n)f = 19.964\, mm.$$

(c) The object space index is now switched to water with $n_1 = 1.33$ and the image space index is again air with $n'_2 = 1$, as in the figure below. Find the new total power ϕ of the lens and the locations of the Cardinal points P, P', F, F', N, N'.



Here $n = 1.33$ *and* $n' = 1$ *and* $n'_1 = n_2 = 1.5$. *Given the curvatures of the two surfaces, the powers of the two surfaces are*

$$\phi_1 = (n'_1 - n_1)C_1 = 0.0051 mm^{-1}$$

$$\phi_2 = (n'_2 - n_2)C_2 = 0.005\, mm^{-1}\, (unchanged\ from\ part(a)).$$

The separation between the two lenses is still $t'_1 = t_2 = 10\, mm$.

The total system power is $\phi = \phi_1 + \phi_2 - \frac{t_2}{n_2}\phi_1\phi_2 = 0.00993\, mm^{-1}$.

The focal length of the system is $f = 1/\phi = 100.705\, mm$.

The rear focal length of the lens is $f'_R = n'f = 100.705\, mm$.

The front focal length of the lens is $f_F = -nf = -133.938\, mm$.

The distance from the first surface to the front principle plane is

$$d = n \frac{\phi_2}{\phi} \frac{t_2}{n_2} = 1.33 \frac{0.0051}{0.00993} \frac{10}{1.5} = 4.465\, mm.$$

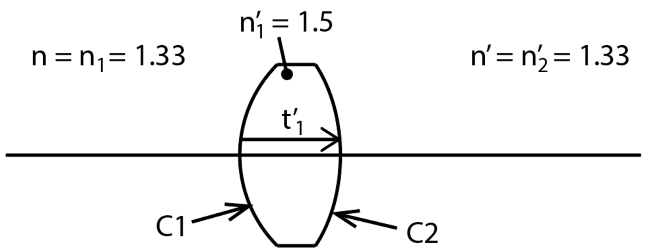
The distance from the second surface to the rear principle plane is

$$d' = -n'\frac{\phi_1}{\phi}\frac{t_2}{n_2} = -\frac{0.0051}{0.00993}\frac{10}{1.5} = -3.424\ mm.$$

The nodal points are shifted from their respective principal planes such that

$$z_{PN} = z'_{PN} = (n' - n)f = -33.233\ mm.$$

(d) Finally, both the object and image space indices are now switched to water with $n_1 = n'_2 = 1.33$, as in the figure below. Find the new total power ϕ of the lens and the locations of the Cardinal points P, P', F, F', N, N'.



Here $n = 1.33$ *and* $n' = 1.33$ *and* $n'_1 = n_2 = 1.5$. *Given the curvatures of the two surfaces, the powers of the two surfaces are*

$$\phi_1 = (n'_1 - n_1)C_1 = 0.0051\ mm^{-1}\ (like\ part(c))$$

$$\phi_2 = (n'_2 - n_2)C_2 = 0.0017\ mm^{-1}\ (like\ part(b)).$$

The separation between the two lenses is still $t'_1 = t_2 = 10\ mm.$

The total system power is $\phi = \phi_1 + \phi_2 - \frac{t_2}{n_2}\phi_1\phi_2 = 0.00674\ mm^{-1}$.

The focal length of the system is $f = 1/\phi = 148.320\ mm.$

The rear focal length of the lens is $f'_R = n'f = 197.265\ mm.$

The front focal length of the lens is $f_F = -nf = -197.265\ mm.$

The distance from the first surface to the front principle plane is

$$d = n\frac{\phi_2}{\phi}\frac{t_2}{n_2} = 1.33\frac{0.0051}{0.00674}\frac{10}{1.5} = 2.236\ mm.$$

The distance from the second surface to the rear principle plane is

$$d' = -n'\frac{\phi_1}{\phi}\frac{t_2}{n_2} = -1.33\frac{0.0017}{0.00674}\frac{10}{1.5} = -6.707\ mm.$$

The nodal points are located at their respective principal planes since $n' = n$.

(e) Which cases have the nodal points coinciding with the principal planes?

Parts (a) and (d) since $n' = n$.

(f) Which case has the highest optical power?

Part (a) since both surfaces are in air.

7. A telephoto lens is made from two thin lenses in air. The lenses have focal lengths of $f_1 = 75mm$ and $f_2 = -60mm$, respectively. The lenses are separated by a distance of $35mm$. Use Gaussian reduction to determine the focal length of the system, and the locations of the *rear* principal plane P' and the *rear* focal point F'. Draw a sketch of the system and label P' and F' relative to the position of the lenses.

Given the focal lengths, the powers of the two thin lenses are

$$\phi_1 = {}^1/_{f_1} = 0.01333\, mm^{-1}$$

$$\phi_2 = {}^1/_{f_2} = -0.01666\, mm^{-1}.$$

The separation between the two lenses is $t_2 = 35mm$.

The total system power is $\phi = \phi_1 + \phi_2 - t_2\phi_1\phi_2 = 0.00444\, mm^{-1}$.

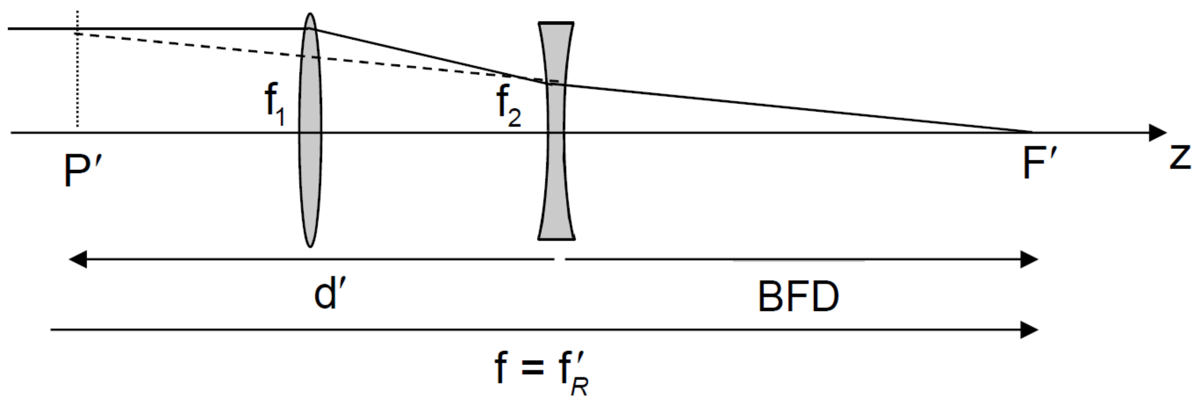
The focal length of the system is $f = 1/\phi = 225mm$.

The rear focal length of the lens is $f_R' = 225mm$.

The distance from the second thin lens to the rear principle plane is

$$d' = -\frac{\phi_1}{\phi} t_2 = -105mm.$$

The $BFD = f_R' + d' = 120mm$. A sketch of the system is below.



f_1
f_2
P′
F′
z
d′
BFD
$f = f'_R$