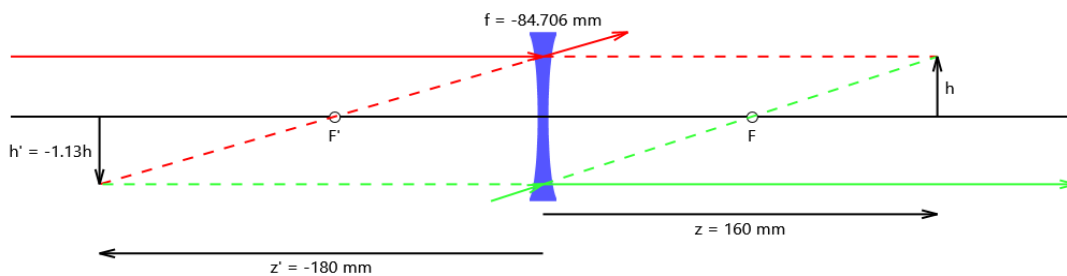


Answer all questions. Show your work. Partial credit will be given. Don't spend too much time on any one problem. Use separate sheets of paper and don't cram your work into the spaces below. Problems are worth 10 points each.

For problems 1-8, provide the requested values and draw a sketch of the system with the two rays passing through the front and rear focal points drawn. Label the relevant values in the sketch. For example, the system below has an object distance $z = 160\text{mm}$ and an image distance $z' = -180\text{mm}$. The lens focal length is $f = -84.706\text{mm}$ and the magnification is $m = -1.13$. The red ray shows a collimated ray entering the lens that passes through the rear focal point. The green ray shows a ray exiting the lens collimated which passes through the front focal point.



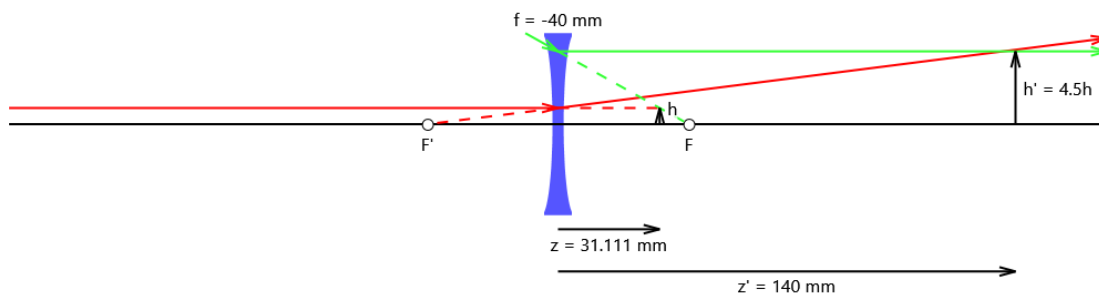
The basic thin lens in air equations are

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \quad \text{and} \quad m = \frac{z'}{z}$$

1. Given the image distance $z' = 140\text{mm}$ and a lens of focal length $f = -40\text{mm}$, what is the object distance z and the magnification m ?

The object distance is given by $z = \left[\frac{1}{140} - \frac{1}{-40} \right]^{-1} = 31.111\text{mm}$.

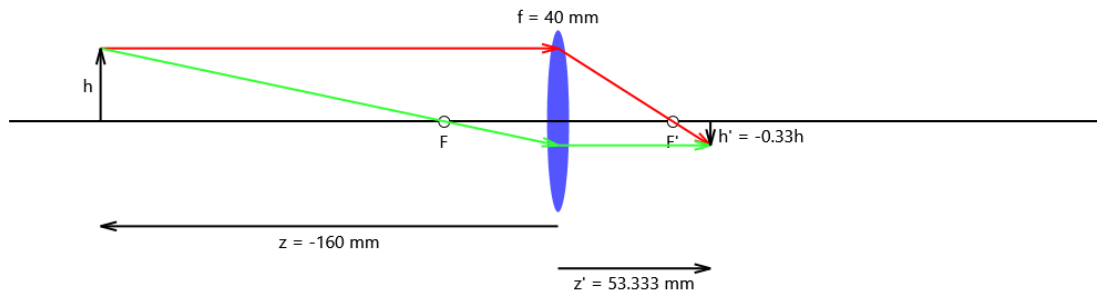
The magnification is given by $m = \frac{z'}{z} = \frac{140}{31.111} = 4.5$.



2. Given the object distance $z = -160\text{mm}$ and a lens of focal length $f = 40\text{mm}$, what is the image distance z' and the magnification m ?

The image distance is given by $z' = \left[\frac{1}{40} + \frac{1}{-160} \right]^{-1} = 53.333 \text{ mm}$.

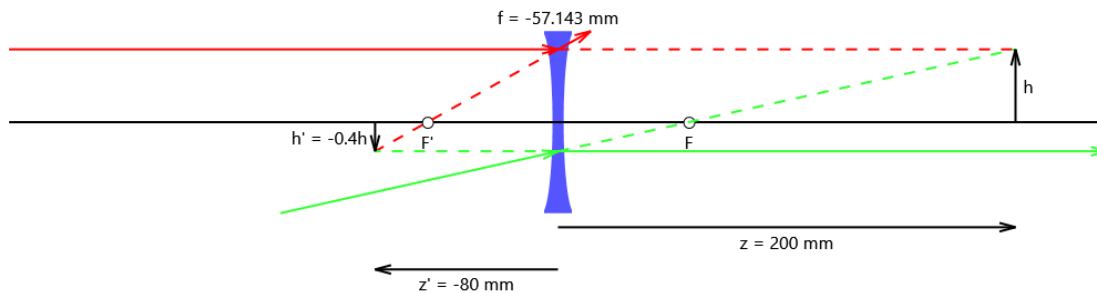
The magnification is given by $m = \frac{z'}{z} = \frac{53.333}{-160} = -0.333$.



3. Given the object distance $z = 200 \text{ mm}$ and the image distance $z' = -80 \text{ mm}$, what is the lens focal length f and the magnification m ?

The lens focal length is given by $f = \left[\frac{1}{-80} - \frac{1}{200} \right]^{-1} = -57.143 \text{ mm}$.

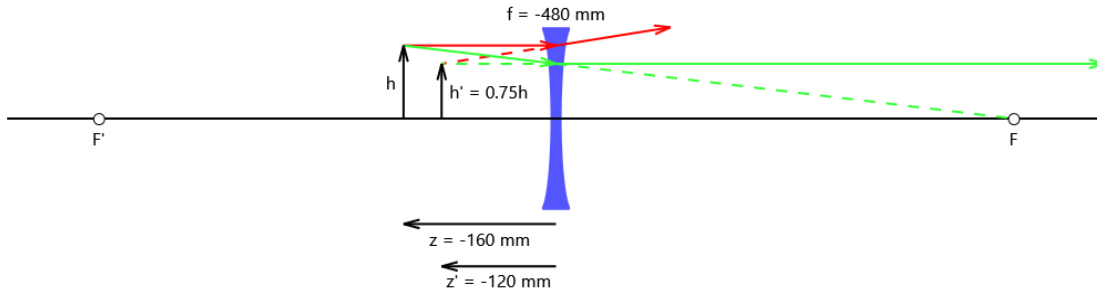
The magnification is given by $m = \frac{z'}{z} = \frac{-80}{200} = -0.4$.



4. Given the object distance $z = -160 \text{ mm}$ and the magnification $m = 0.75$, what is the lens focal length f and the image distance z' ?

The lens focal length is given by $f = \frac{mz}{1-m} = \frac{(0.75)(-160)}{1-0.75} = -480 \text{ mm}$.

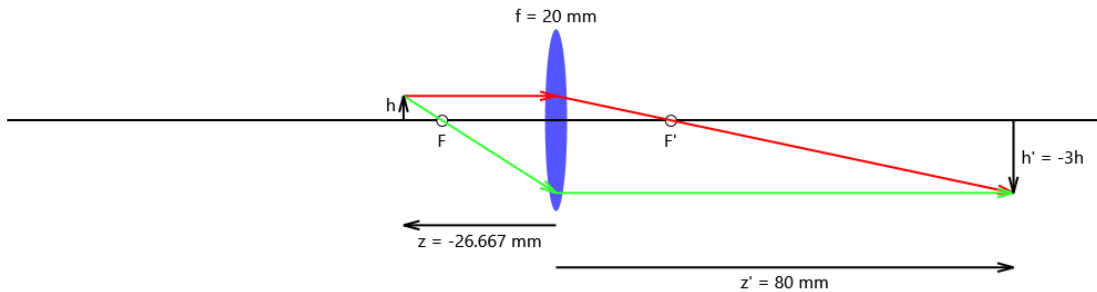
The image distance is given by $z' = m \cdot z = (0.75)(-160) = -120 \text{ mm}$.



5. Given a lens of focal length $f = 20\text{mm}$ and a magnification $m = -3$, what is the object distance z and the image distance z' ?

The object distance is given by $z = f \left[\frac{1}{m} - 1 \right] = 20 \left[\frac{1}{-3} - 1 \right] = -26.667\text{mm}$.

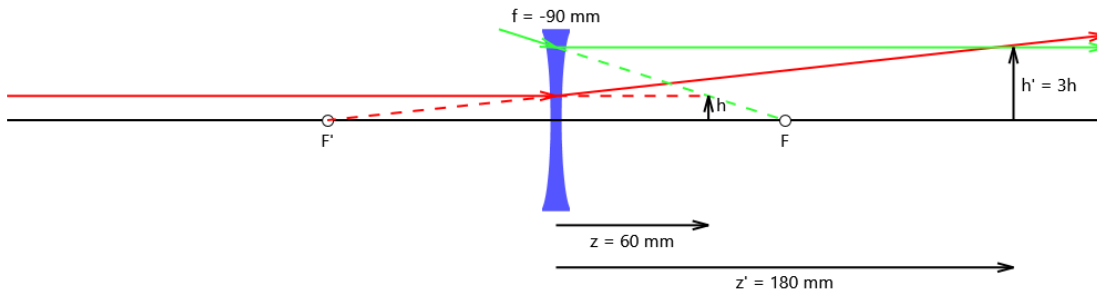
The image distance is given by $z' = m \cdot z = (-3)(-26.667) = 80\text{mm}$.



6. Given an image distance $z' = 180\text{mm}$ and a magnification $m = 3$, what is the focal length f and the object distance z ?

The lens focal length is given by $f = \frac{z'}{1-m} = \frac{180}{1-3} = -90\text{mm}$.

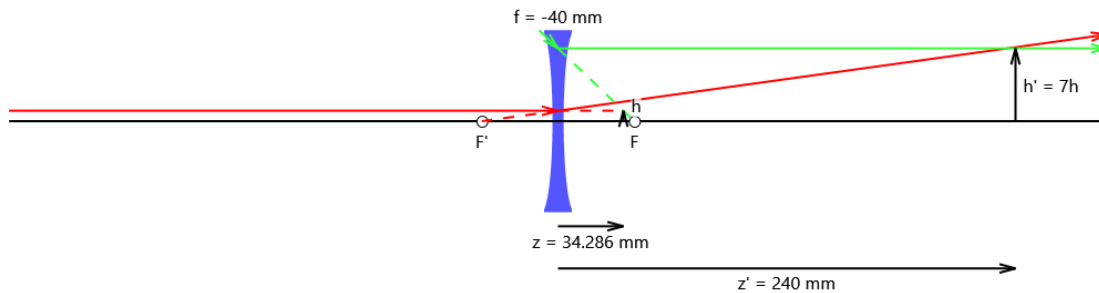
The object distance is given by $z = \frac{z'}{m} = \frac{180}{3} = 60\text{mm}$.



7. Given the image distance $z' = 240\text{mm}$ and a lens of focal length $f = -40\text{mm}$, what is the object distance z and the magnification m ?

The object distance is given by $z = \left[\frac{1}{240} - \frac{1}{-40} \right]^{-1} = 34.286 \text{ mm}$.

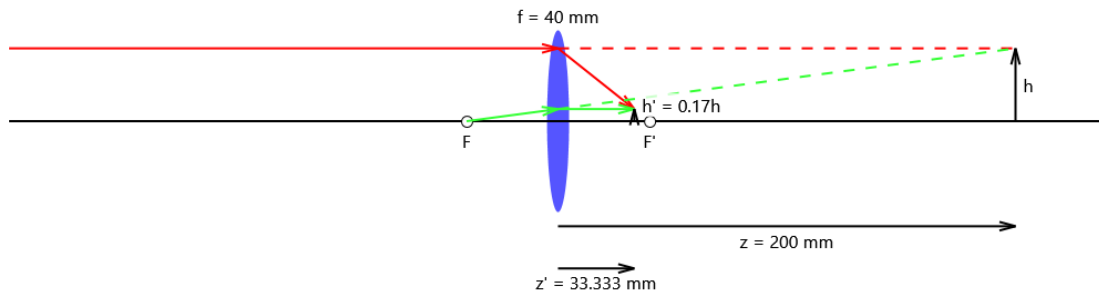
The magnification is given by $m = \frac{z'}{z} = \frac{240}{34.286} = 7$.



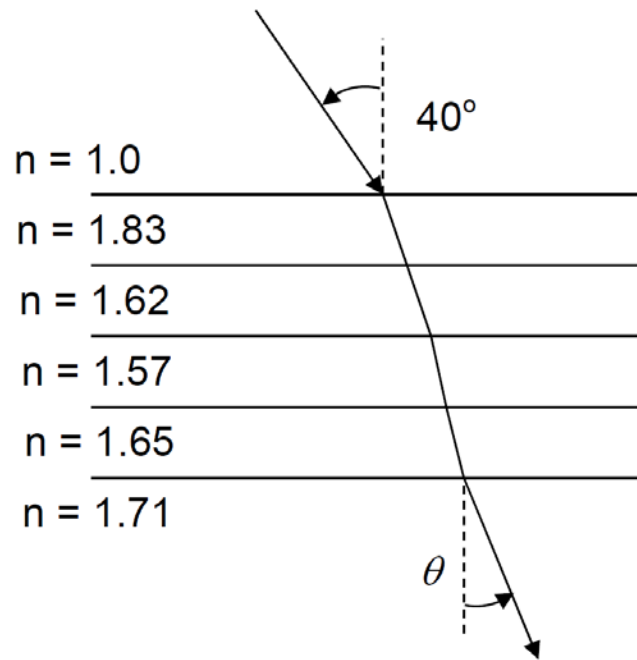
8. Given the object distance $z = 200 \text{ mm}$ and a lens of focal length $f = 40 \text{ mm}$, what is the image distance z' and the magnification m ?

The image distance is given by $z' = \left[\frac{1}{40} + \frac{1}{200} \right]^{-1} = 33.333 \text{ mm}$.

The magnification is given by $m = \frac{z'}{z} = \frac{33.333}{200} = 0.167$.



9. A series of glass plates with refractive indices shown in the figure below are stacked upon one another. A ray in air is incident on the stack at an angle of 40° . What is the angle θ for the ray in the last glass plate?

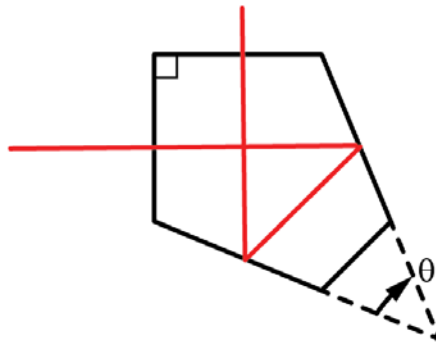


For each interface, $n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3 = \dots$, so only the first and last indices and angles are need. From the figure

$$\sin 40^\circ = 1.71 \sin \theta$$

$$\theta = \sin^{-1} \left[\frac{\sin 40^\circ}{1.71} \right] = 22.08^\circ$$

10. The Figure below shows a pentaprism prism with a ray traced through it (in red). The incoming beam is deviated by 90° .



a) Is parity conserved for this prism? Why?

There are an even number of reflections (two), so parity is conserved.

b) What is the angle θ (use the sign convention) between the two reflective surfaces?

Since the beam is deviation 90° , the dihedral angle $\theta = -45^\circ$ with the sign convention.

c) Does total internal reflection occur at the two surfaces if the prism has an index $n = 1.517$?

From the geometry, the magnitude of the angle that the ray strikes each reflective surface is 22.5° , which is well below the critical angle of 41.2° for this refractive index. Consequently, this prism needs to have a reflective coating applied to the two surfaces to work properly.

d) Circle the correct tunnel diagram for the pentaprism. Answer is C

