OPTI 600B Homework 1

1. Design two thin achromats with a focal length of 100 mm. For the first one, use N-BK7 and N-PSK53A for the two glasses. For the second one, use N-BK7 and F2 for the glasses.

a) What are the required powers of the two elements for each of the achromats?

From the notes, the following matrix equation needs to be solved

$$\begin{pmatrix} 1 & 1 \\ 1/\nu_1 & 1/\nu_2 \end{pmatrix}\begin{pmatrix} \phi_{1d} \\ \phi_{2d} \end{pmatrix} = \begin{pmatrix} \phi_d \\ 0 \end{pmatrix}$$

The determinant of the 2x2 matrix is given by

$$\frac{1}{\nu_2} - \frac{1}{\nu_1} = \frac{\nu_1 - \nu_2}{\nu_1 \nu_2}$$

The inverse of the 2x2 matrix is given by

$$\frac{\nu_1 \nu_2}{\nu_1 - \nu_2}\begin{pmatrix} 1/\nu_2 & -1 \\ -1/\nu_1 & 1 \end{pmatrix}$$

Solving for the individual lens powers gives

$$\begin{pmatrix} \phi_{1d} \\ \phi_{2d} \end{pmatrix} = \frac{\nu_1 \nu_2}{\nu_1 - \nu_2}\begin{pmatrix} 1/\nu_2 & -1 \\ -1/\nu_1 & 1 \end{pmatrix}\begin{pmatrix} \phi_d \\ 0 \end{pmatrix}$$

$$\phi_{1d} = \frac{\phi_d \nu_1}{\nu_1 - \nu_2} \quad \textit{and} \quad \phi_{2d} = -\frac{\phi_d \nu_2}{\nu_1 - \nu_2}$$

For the N-BK7 and N-PSK53A combination, $\nu_1 = 64.17$ *and* $\nu_2 = 63.39$*, so* $\phi_{1d} = 0.823\,\text{mm}^{-1}$ *and* $\phi_{2d} = -0.813\,\text{mm}^{-1}$.

For the N-BK7 and F2 combination, $\nu_1 = 64.17$ *and* $\nu_2 = 36.37$*, so* $\phi_{1d} = 0.023\,\text{mm}^{-1}$ *and* $\phi_{2d} = -0.013\,\text{mm}^{-1}$.

b) While a solution exists for both cases, why is one solution a bad design choice and the other is a good design choice? What is the driving factor affecting the bad design choice?

For the N-BK7 and N-PSK53A combination, the magnitudes of the individual powers are extremely large, leading to short focal lengths and small apertures. This stems from $\nu_1 \approx \nu_2$*, which in turn means that the two rows of*

$$\begin{pmatrix} 1 & 1 \\ 1/\nu_1 & 1/\nu_2 \end{pmatrix}$$

are nearly linearly dependent.

2. Through similar arguments made for the three element apochromat in the notes, we can try to make a two element apochromat. The matrix equation for this case looks like

$$\begin{pmatrix} 1 & 1 \\ 1/\nu_1 & 1/\nu_2 \\ P_{1\lambda F}/\nu_1 & P_{2\lambda F}/\nu_2 \end{pmatrix} \begin{pmatrix} \phi_{1d} \\ \phi_{2d} \end{pmatrix} = \begin{pmatrix} \phi_d \\ 0 \end{pmatrix}.$$

Now we have three equations and two unknowns. The only hope for a unique solution is that one of the rows is linearly dependent upon the other rows. We already know from the achromat case that the element powers go to infinity when $\nu_1 = \nu_2$. What condition is required for row 1 and row 3 to be linearly dependent assuming $\nu_1 \neq \nu_2$? What condition is required for row 2 and row 3 to be linearly dependent assuming $\nu_1 \neq \nu_2$?

For rows 1 and 3 to be linearly dependent, we need $P_{1\lambda F}/\nu_1 = P_{2\lambda F}/\nu_2$. *Unfortunately, known materials don't seem to achieve this without* $\nu_1 \approx \nu_2$. *For rows 2 and 3 to be linearly dependent, we need* $P_{1\lambda F} = P_{2\lambda F}$, *which is the typical strategy for creating apochromatic doublet.*

3. Calculate the Mueller matrix for a $\lambda/2$ plate by combining the Mueller matrices for two $\lambda/4$ plates with their fast axes aligned. Verify that the resultant matrix converts left circularly polarized light to right circularly polarized light, and vice versa.

A half wave plate with its fast axis vertical has a Mueller matrix given by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

A quarter wave plate with its fast axis vertical has a Mueller matrix given by

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Carrying out the matrix multiplication of two quarter wave plates gives

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The Stokes vector for left circularly polarized light is given by $\begin{pmatrix}1 & 0 & 0 & -1\end{pmatrix}^{\mathrm{T}}$ *and for right circularly polarized light* $\begin{pmatrix}1 & 0 & 0 & 1\end{pmatrix}^{\mathrm{T}}$. *Multiplying left circularly polarized light by the Mueller matrix for a half wave plate gives*

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\\ 0\\ -1\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\\ 1\end{pmatrix}$$

which is right circularly polarized light. Similarly, multiplying right circularly polarized light by the Mueller matrix for a half wave plate gives

$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\\ 0\\ 1\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 0\\ -1\end{pmatrix}$$

which is left circularly polarized light.