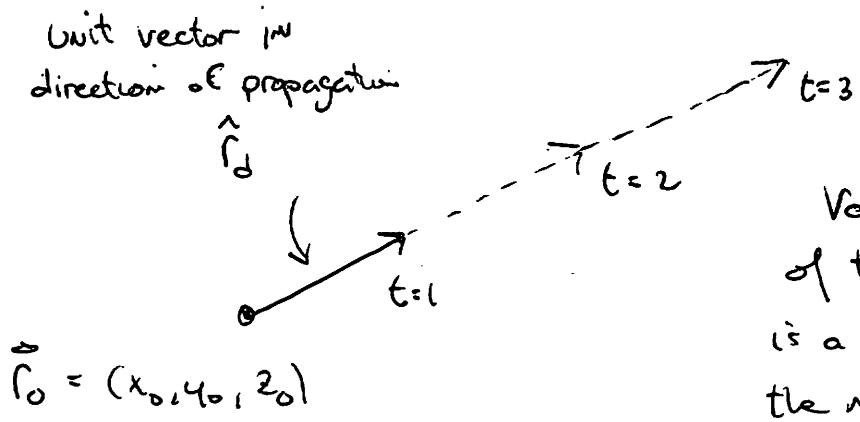
Real Raytracing

While paraxial raytracing gives the first order properties of an optical system, real raytracing is used to accurately refract, reflect and propagate rays through the system with the simplications of the paraxial approximations. In general, a ray will have a starting point $\vec{r}_0 = (x_0, y_0, z_0)$ in 3D space and a propagation direction $\vec{r}_d = (x_d, y_d, z_d)$. Note that $\vec{r}_d$ is typically normalized to a unit vector and the ray position at any time t is written as

$$\vec{r}(t) = \vec{r}_0 + t\hat{r}_d \qquad \text{where} \quad \hat{r}_d = \frac{\vec{r}_d}{|\vec{r}_d|} \quad \text{i.e. a unit vector}$$



Varying t changes the position of the end of the ray and is a measure of the length of the ray.

Real raytracing is a two-step process just like paraxial raytracing. First, t is varied until the ray intersects an object (transfer). Once the intersection point is determined, the ~~ob~~ object's normal is calculated. Snell's Law (Law of Reflection) is used to determine the ray's new direction after refraction (reflection). The intersection point becomes the new origin of the ray and the process is repeated

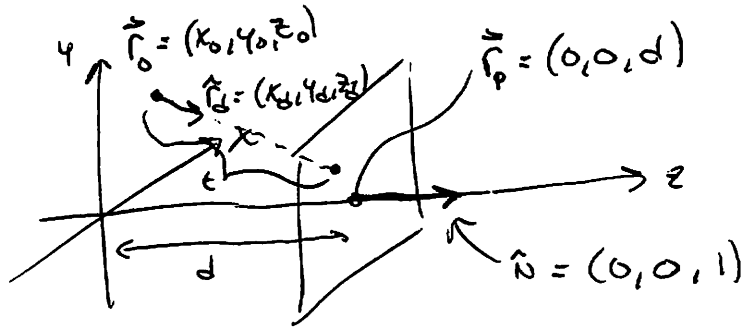
The two most common surfaces for real raytracing are the plane and the spherical surface. Let's start by figuring out where a ray intersects a plane

A plane is defined by two components: its normal vector $\vec{N}$ and a point in the plane $\vec{r}_p = (x_p, y_p, z_p)$. The formula for a plane is

$$Ax + By + Cz + D = 0$$

where $\vec{N} = (A, B, C)$ and $D = -\vec{N} \cdot \vec{r}_p = -Ax_p - By_p - Cz_p$

Any point (x, y, z) that satisfies the above expression lies in the plane.



To find where the ray intersects the plane, plug in values for x, y and z and solve for t.

$$A(x_0 + t x_d) + B(y_0 + t y_d) + C(z_0 + t z_d) + D = 0$$

Solving for t gives

$$\boxed{t = \frac{-(Ax_0 + By_0 + Cz_0 + D)}{Ax_d + By_d + Cz_d}}$$

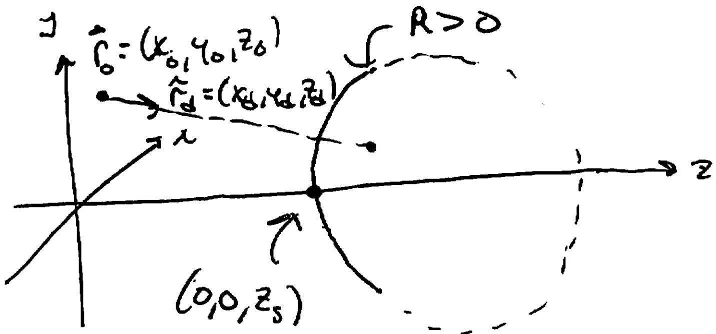
$t < 0$ means the plane is behind the ray origin

Note: if denominator is zero, then the ray is parallel to the plane and therefore doesn't intersect it.

Next, let's look at a spherical surface. Here, we'll simplify things a little and assume the sphere is on axis and has its normal pointing along the z-axis. This surface can be written as

$$z - z_s = R\left[1 - \sqrt{1 - \frac{x^2+y^2}{R^2}}\right]$$

where R is the radius of the sphere and z_s is the location of its vertex



R controls ~~whether the~~ the direction the surface curves using our standard sign convention.

Follow same steps as before

$$(z_o + t z_d) - z_s = R\left[1 - \sqrt{1 - \frac{(x_o+tx_d)^2 + (y_o+ty_d)^2}{R^2}}\right]$$

Solve for t

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Two answers in general

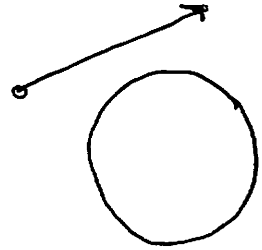
~~smallest positive t is correct answer~~

sign of R tells which solution to pick.

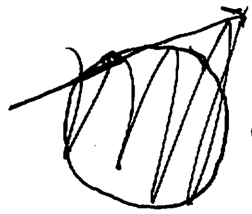
where $A = x_d^2 + y_d^2 + z_d^2 = 1$ since unit vector

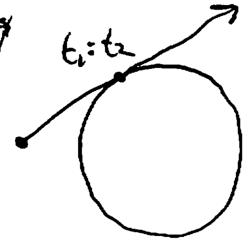
$$B = 2\left[x_d x_o + y_d y_o + z_d\left(z_o - (z_s + R)\right)\right]$$

$$C = x_o^2 + y_o^2 + \left(z_o - (z_s+R)\right)^2 - R^2$$



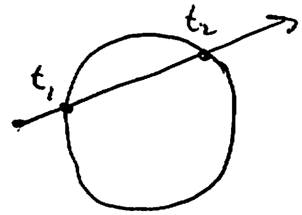
$B^2 - 4AC < 0$





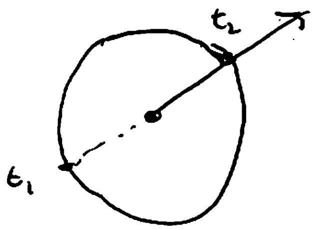
$B^2 - 4AC = 0$

$t_1 > 0$



$B^2 - 4AC > 0$

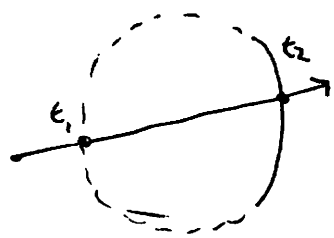
$t_1 > 0 \, , \; t_2 > 0$



$B^2 - 4AC > 0$

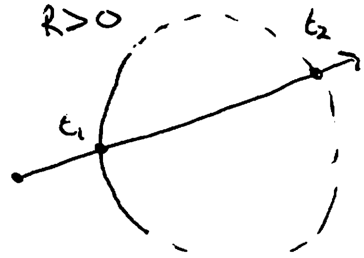
$t_1 < 0 \, , \; t_2 > 0$

$R < 0$



if $R < 0$ and both t_1 and t_2 positive, choose largest t.

$R > 0$



if $R > 0$ and both t_1 and t_2 positive, choose smallest t.

If (x_1, y_1, z_1) is the intersect point.

$$x_1 = x_0 + t\, x_d$$

$$y_1 = y_0 + t\, x_d \qquad \text{where } t \text{ is the value predicted above}$$

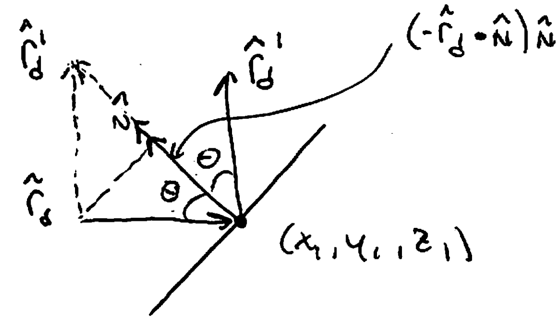
$$z_1 = z_0 + t\, z_d$$

The normal to the sphere at this point is

$$\vec{N} = \left(\frac{x_1}{R} \, , \; \frac{y_1}{R} \, , \; \frac{z_1 - (z_s + R)}{R} \right)$$

usually want to have unit normal $\hat{N} = \dfrac{\vec{N}}{|\vec{N}|}$.

The law of reflection in vector form.

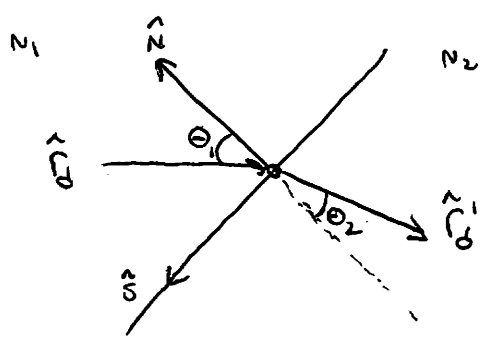


$\cos\theta = -\hat{r}_d \bullet \hat{N}$ since unit vectors

$$\hat{r}_d' = \hat{r}_d - 2(-\hat{r}_d \bullet \hat{N})\hat{N}$$

Snell's Law in vector form

Easy to calculate cosine of the incident ray and normal, so let's try to take advantage of that



Define a vector $\hat{s}$ perpendicular to $\hat{N}$

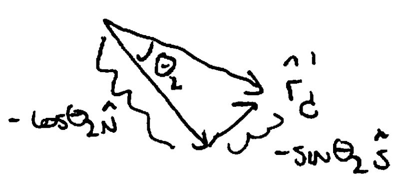
Snell's Law

$$N_1 \sin\theta_1 = N_2 \sin\theta_2$$

$$\sin\theta_2 = \left(\frac{N_1}{N_2}\right)\sin\theta_1 = \left(\frac{N_1}{N_2}\right)\sqrt{1-\cos^2\theta_1}$$

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\left(\frac{N_1}{N_2}\right)^2(1-\cos^2\theta_1)}$$

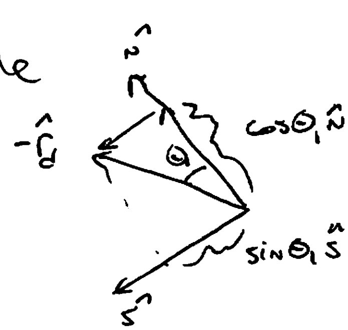
$$\hat{r}_d' = -\cos\theta_2\hat{N} - \sin\theta_2\hat{s}$$



minus signs since opposite direction as $\hat{N}$ and $\hat{s}$

$$\hat{r}_d' = -\sqrt{1-\left(\frac{N_1}{N_2}\right)^2(1-\cos^2\theta_1)}\;\hat{N} - \left(\frac{N_1}{N_2}\right)\sin\theta_1\hat{s} \qquad (1)$$

Now look on the incident side



$$\sin\theta_1\hat{s} + \cos\theta_1\hat{N} = -\hat{r}_d$$

$$\hat{s} = \frac{1}{\sin\Theta_1}\left(-\hat{r}_d - \cos\Theta_1 \hat{N}\right)$$

Plug into (1)

$$\hat{r}_d' = -\sqrt{1-\left(\frac{N_1}{N_2}\right)^2\left(1-\cos^2\Theta_1\right)}\ \hat{N} - \left(\frac{N_1}{N_2}\right)\left(-\hat{r}_d - \cos\Theta_1\hat{N}\right)$$

$$\boxed{\hat{r}_d' = \left(\frac{N_1}{N_2}\right)\hat{r}_d + \left[\left(\frac{N_1}{N_2}\right)\cos\Theta_1 - \sqrt{1-\left(\frac{N_1}{N_2}\right)^2\left(1-\cos^2\Theta_1\right)}\right]\hat{N}}$$

Completely in terms of $\cos\Theta_1 = -\hat{r}_d \cdot \hat{N}$

TIR occurs when argument under square root is negative