Do all problems and show your work. Credit is not given for answers only. Don't spend too much time on any one problem. Use separate sheets of paper and don't cram your work into the spaces below. As in the notes, z_F and z'_F are the object and image distances measured from the front and rear focal points. The distances z and z' are the object and image distances measured from the front and rear principal plane. Questions 1-5 are worth 8 points each. Questions 6-8 are worth 20 points each.

Thin lens or general system $\phi = \frac{1}{f}$	Focal Lengths $f'_R = n'f \qquad f_F = -nf$
Single surface $\phi = \frac{n' - n}{R}$	Total power of two elements $\phi = \phi_1 + \phi_2 - \left(\frac{t'_1}{n'_1}\right)\phi_1\phi_2$
Front Principal Plane $\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\left(\frac{t'_1}{n'_1}\right)$	Rear Principal Plane $\delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\left(\frac{t'_1}{n'_1}\right)$
Front Focal Distance $FFD = f_F + d$	Back Focal Distance $BFD = f'_R + d'$
Gaussian Imaging Eq. $\frac{n'}{z'} - \frac{n}{z} = \frac{1}{f}$	Newtonian Imaging Eq. $z_F z'_F = f_F f'_R$
Transverse Magnification $m = \frac{h'}{h} = \frac{nz'}{n'z} = -\frac{z'_F}{f'_R} = -\frac{f_F}{z_F} = \frac{nu}{n'u'}$	Longitudinal Magnification (small shift) $\overline{m} = \frac{n'}{n}m^2$
Nodal Points $z_{PN} = z'_{PN} = f_F + f'_R = (n' - n)f$	Mirrors Thickness and indices switch sign after reflection
Paraxial Refraction (Reflection) Equation $n'_j u'_j = n_j u_j - y_j \phi_j$	Paraxial Transfer Equation $y_{j+1} = y_j + n'_j u'_j \frac{t'_j}{n'_j}$

1. The object space index is $n = 1$ and the image space index is $n' = 1$. Given an object distance $z_F = -60mm$ and a lens of focal length $f = -40mm$, what is the image distance z'_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 40mm$.

The rear focal length is given by $f'_R = n'f = -40mm$.

The magnification is given by $m = -\frac{f_F}{z_F} = -\frac{40}{-60} = 0.667$.

The image distance is given by $z'_F = -mf'_R = -(0.667)(-40) = 26.667$.

2. The object space index is $n = 1.5$ and the image space index is $n' = 1$. Given an image distance $z'_F = 20mm$ and a lens of focal length $f = 40mm$, what is the object distance z_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = -60mm.$

The rear focal length is given by $f'_R = n'f = 40mm.$

The magnification is given by $m = -\frac{z'_F}{f'_R} = -\frac{20}{40} = -0.5.$

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{-60}{-0.5} = -120.$

3. The object space index is $n = 1.333$ and the image space index is $n' = 1$. Given a magnification $m = -3$ and a lens of focal length $f = -40mm$, what is the object distance z_F and the image distance z'_F? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 53.333mm.$

The rear focal length is given by $f'_R = n'f = -40mm.$

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{53.333}{-3} = 17.778.$

The image distance is given by $z'_F = -mf'_R = -(-3)(-40) = -120.$

4. The object space index is $n = 1$ and the image space index is $n' = 1$. Given an object distance $z_F = -40mm$ and a lens of focal length $f = -80mm$, what is the image distance z'_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = 80mm.$

The rear focal length is given by $f'_R = n'f = -80mm.$

The magnification is given by $m = -\frac{f_F}{z_F} = -\frac{80}{-40} = 2.$

The image distance is given by $z'_F = -mf'_R = -(2)(-80) = 160.$

5. The object space index is $n = 1.5$ and the image space index is $n' = 1$. Given an image distance $z'_F = 80mm$ and a lens of focal length $f = 60mm$, what is the object distance z_F and the magnification m? What are the front and rear focal lengths f_F and f'_R?

The front focal length is given by $f_F = -nf = -90mm.$

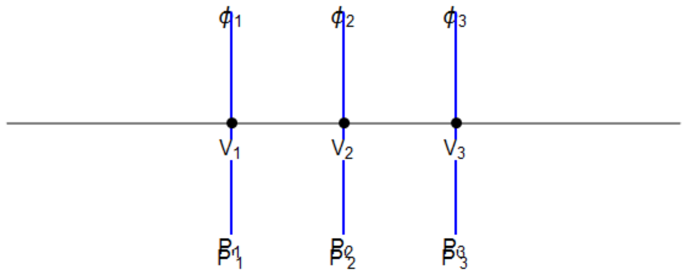
The rear focal length is given by $f'_R = n'f = 60mm.$

The magnification is given by $m = -\frac{z'_F}{f'_R} = -\frac{80}{60} = -1.333.$

The object distance is given by $z_F = -\frac{f_F}{m} = -\frac{-90}{-1.333} = -67.5.$

6. A Cooke triplet consists of three thin lenses in air. The effective focal lengths of the lenses are $f_1 = 44.4\ mm$, $f_2 = -22\ mm$ and $f_3 = 44.4\ mm$. The lenses are separated by $t_1' = t_2' = 15.3$ mm. Do the following:

(a) Sketch the lens and label the lens vertices, V_1, V_2 and V_3. Show the location of the principal planes for each of the three thin lenses.



(b) Find the powers of each individual thin lens.

The thin lens powers are given by

$$\phi_1 = \frac{1}{f_1} = 0.022523\ \text{mm}^{-1}$$

$$\phi_2 = \frac{1}{f_2} = -0.04545\ \text{mm}^{-1}$$

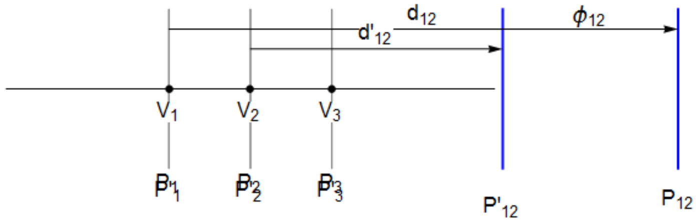
$$\phi_3 = \frac{1}{f_3} = 0.022523\ \text{mm}^{-1}$$

(c) Using Gaussian reduction, calculate the overall power of the system and the locations of the principal planes?

Calculate $\phi_{12} = \phi_1 + \phi_2 - \frac{t'_1}{n'_1}\phi_1\phi_2 = -0.00726863\,\text{mm}^{-1}$

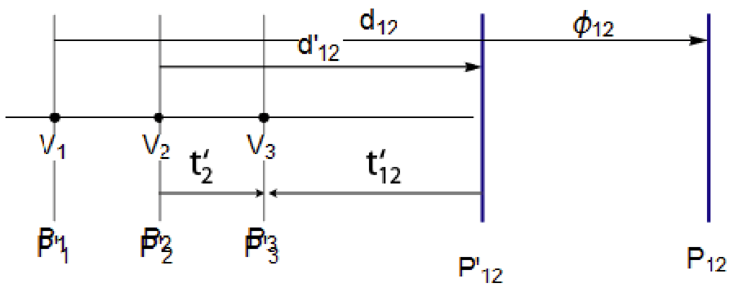
Calculate $d_{12} = n_1\delta_{12} = n_1\frac{\phi_2}{\phi_{12}}\frac{t'_1}{n'_1} = 95.6789\,\text{mm}$

Calculate $d'_{12} = n'_2\delta'_{12} = -n'_2\frac{\phi_1}{\phi_{12}}\frac{t'_1}{n'_1} = 47.4085\,\text{mm}$



Calculate the new distance t'_{12} *between* P'_{12} *and* P_3.

$t'_{12} = t'_2 - d'_{12} = 15.3 - 47.4085 = -32.1085\,\text{mm}$.



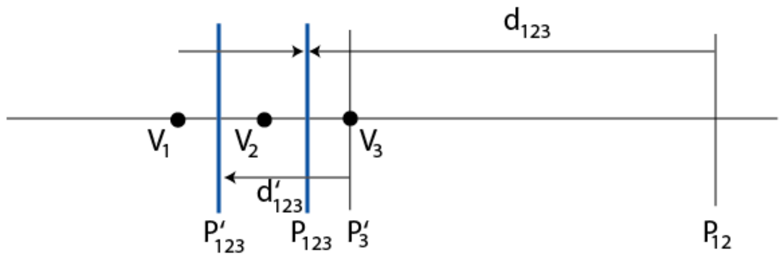
Next, reduce the two remaining systems

$\phi_{123} = \phi_{12} + \phi_3 - t'_{12}\phi_{12}\phi_3 = 0.01\,\text{mm}^{-1}$.

$$d_{123} = \frac{\phi_3}{\phi_{123}}t'_{12} = -72.3345\,\text{mm}$$

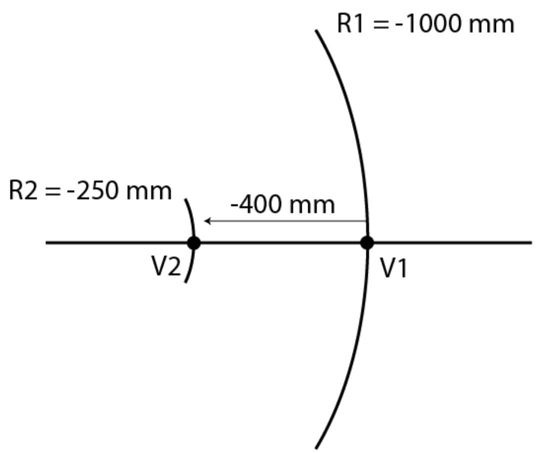
$$d'_{123} = -\frac{\phi_{12}}{\phi_{123}}t'_{12} = -23.3443\,\text{mm}$$

So, the rear principal plane is -23.3443 mm to left of V_3*. From the symmetry of the lens or from combining all of the results, the front principal plane is 23.3443 mm to the right of* V_1*. The total power of the system is just* ϕ_{123}.



7. A Cassegrain telescope has two reflective mirrors. The first mirror has a radius of curvature of $R1 = -1000$ mm. The second mirror lies to the left of the first mirror such that $t'_1 = -400\ mm$. The radius of curvature of the second mirror is $R2 = -250\ mm$.

(a) Sketch the telescope and label the vertices, V_1, V_2.



(b) Using Gaussian reduction, find the overall focal length of the telescope.

First calculate all of the surface powers

$$\phi_1 = \frac{n'_1 - n_1}{R_1} = \frac{-1. - 1.}{-1000\,mm} = 0.002\,mm^{-1}$$

$$\phi_2 = \frac{n'_2 - n_2}{R_2} = \frac{1. - -1.}{-250\,mm} = -0.008\,mm^{-1}$$

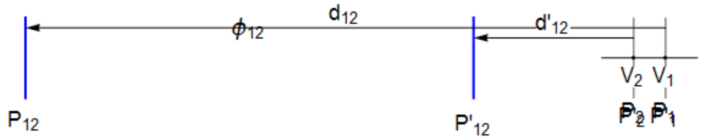
Calculate $\phi_{12} = \phi_1 + \phi_2 - \frac{t'_1}{n'_1}\phi_1\phi_2 = 0.0004\,mm^{-1}$

The effective focal length is $f_E = \frac{1}{\phi_{12}} = 2500\,\text{mm}$.

(c) Where are the principal planes located with respect to the vertices V1 and V2?

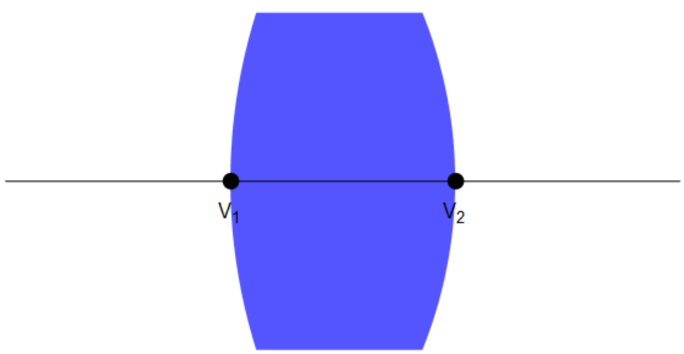
$$\text{Calculate } d_{12} = n_1\,\delta_{12} = n_1 \frac{\phi_2}{\phi_{12}} \frac{t'_1}{n'_1} = -8000.\ \text{mm}$$

$$\text{Calculate } d'_{12} = n'_2\,\delta'_{12} = -n'_2 \frac{\phi_1}{\phi_{12}} \frac{t'_1}{n'_1} = -2000.\ \text{mm}$$



8. A thick lens has air ($n_1 = 1$) on the object side and water ($n_2' = 1.33$) on the image side. The lens is 20 mm thick and has a refractive index of $n_1' = 1.5$. The radii of curvature of the lens are R1 = 50 mm and R2 = −40 mm, respective.

(a) Sketch the telescope and label the vertices, V1 and V2.



(b) Using Gaussian reduction, find the overall focal length of the lens.

First calculate all of the surface powers

$$\phi_1 = \frac{n'_1 - n_1}{R_1} = \frac{1.5 - 1.}{50\ \text{mm}} = 0.01\ \text{mm}^{-1}$$

$$\phi_2 = \frac{n'_2 - n_2}{R_2} = \frac{1.33 - 1.5}{-40\ \text{mm}} = 0.00425\ \text{mm}^{-1}$$

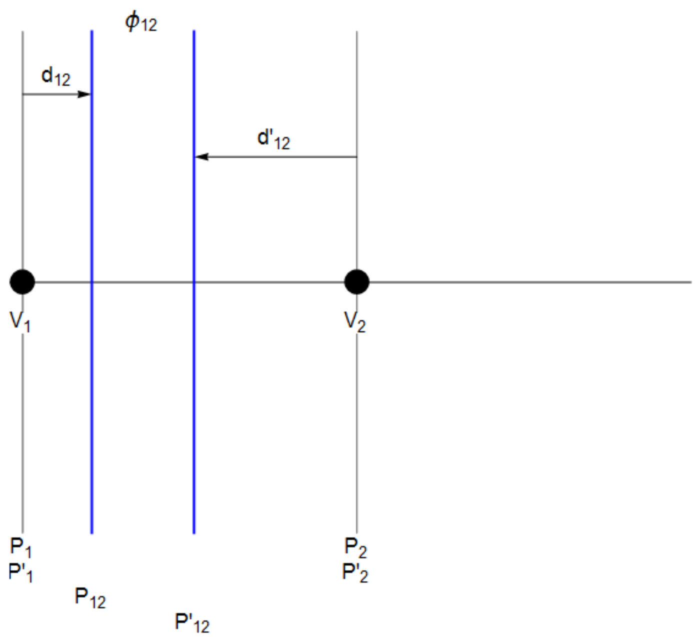
$$\text{Calculate } \phi_{12} = \phi_1 + \phi_2 - \frac{t'_1}{n'_1}\,\phi_1\,\phi_2 = 0.0136833\ \text{mm}^{-1}$$

The effective focal length is $f_E = \dfrac{1}{\phi_{12}} = 73.08\ \text{mm}$.

(c) Where are the principal planes located with respect to the vertices V1 and V2?

$$\text{Calculate } d_{12} = n_1\,\delta_{12} = n_1 \frac{\phi_2}{\phi_{12}} \frac{t'_1}{n'_1} = 4.14129\,\text{mm}$$

$$\text{Calculate } d'_{12} = n'_2\,\delta'_{12} = -n'_2 \frac{\phi_1}{\phi_{12}} \frac{t'_1}{n'_1} = -12.9598\,\text{mm}$$



(d) Where are the nodal points located?

The rear focal length is given by $f'_R = n'_2 f_E = 97.20\,\text{mm}$.

The front focal length is given by $f_F = -f_E = -73.08\,\text{mm}$.

The distance to the nodal points is $\overline{P_{12}N} = \overline{P'_{12}N'} = f'_R + f_F = 24.12\,\text{mm}$.