Do all problems and show your work. Credit is not given for answers only. Don’t spend too much time on any one problem. Use separate sheets of paper and don’t cram your work into the spaces below. As in the notes, z_F and z'_F are the object and image distances measured from the front and rear focal points. The distances z and z' are the object and image distances measured from the front and rear principal plane.

<table>
<thead>
<tr>
<th>Thin lens or general system</th>
<th>Focal Lengths</th>
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<tr>
<td>( \phi = \frac{1}{f} )</td>
<td>( f'_R = n'f )</td>
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<table>
<thead>
<tr>
<th>Single surface</th>
<th>Total power of two elements</th>
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<tbody>
<tr>
<td>( \phi = \frac{n' - n}{R} )</td>
<td>( \phi = \phi_1 + \phi_2 - \left( \frac{t'_1}{n'_1} \right) \phi_1 \phi_2 )</td>
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<table>
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<tr>
<th>Front Principal Plane</th>
<th>Rear Principal Plane</th>
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<td>( \delta = \frac{d}{n} = \frac{\phi_2}{\phi} \left( \frac{t'_1}{n'_1} \right) )</td>
<td>( \delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi} \left( \frac{t'_1}{n'_1} \right) )</td>
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<tr>
<th>Front Focal Distance</th>
<th>Back Focal Distance</th>
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<tbody>
<tr>
<td>( FFD = f_F + d )</td>
<td>( BFD = f'_R + d' )</td>
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<tr>
<th>Gaussian Imaging Eq.</th>
<th>Newtonian Imaging Eq.</th>
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<tr>
<td>( \frac{n'}{z'} - \frac{n}{z} = \frac{1}{f} )</td>
<td>( z_F z'_F = f_F f'_R )</td>
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<table>
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<tr>
<th>Transverse Magnification</th>
<th>Longitudinal Magnification (small shift)</th>
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<tr>
<td>( m = \frac{n z'}{n' z} )</td>
<td>( \bar{m} = \frac{n'}{n} m^2 )</td>
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<thead>
<tr>
<th>Nodal Points</th>
<th>Mirrors</th>
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<td>( z_{PN} = z'_{PN} = f_F + f'_R )</td>
<td>Thickness and indices switch sign after reflection</td>
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</table>

1. Given the object distance \( z_F = -120.000 \) mm, an object space index \( n = 1.50 \), and a lens of effective focal length \( f = -21.000 \) mm, what is the front focal length \( f_F \) and the magnification \( m \)?

The front focal length is given by \( f_F = -(1.50)(-21.000) = 31.500 \) mm.

The magnification is given by \( m = (1.50)(-21.000) / (-120.000) = 0.263 \).

2. Given the object distance \( z_F = -120.000 \) mm, an object space index \( n = 1.00 \), and a lens of front focal length \( f_F = -42.000 \) mm, what is the effective focal length \( f \) and the magnification \( m \)?
The effective focal length is given by \( f = \frac{-(-42.00)}{1.000} = 42.000 \text{ mm} \).

The magnification is given by \( m = \frac{(1.00)(42.000)}{(-120.000)} = -0.350 \).

3. Given the object distance \( z_F = 160.000 \text{ mm} \), an object space index \( n = 1.33 \), and a magnification \( m = -1.500 \text{ mm} \), what is the effective focal length \( f \) and the front focal length \( f_F \)?

The effective focal length is given by \( f_E = \frac{(-1.500)(160.000)}{1.33} = -180.451 \text{ mm} \).

The front focal length is given by \( f_F = -1.33(-180.451) = 240.000 \).

4. Given the image distance \( z'_F = 80.000 \text{ mm} \), an image space index \( n' = 1.33 \), and a lens of effective focal length \( f = 21.000 \text{ mm} \), what is the rear focal length \( f'R \) and the magnification \( m \)?

The rear focal length is given by \( f'R = 1.33(21.000) = 27.930 \text{ mm} \).

The magnification is given by \( m = \frac{-80.000}{(1.33(21.000))} = -2.864 \).

5. Given the image distance \( z'_F = 80.000 \text{ mm} \), an object space index \( n' = 1.33 \), and a lens of rear focal length \( f'R = 42.000 \text{ mm} \), what is the effective focal length \( f \) and the magnification \( m \)?

The effective focal length is given by \( f_E = \frac{42.0}{1.33} = 31.579 \text{ mm} \).

The magnification is given by \( m = \frac{-80.000}{(1.33(31.579))} = -1.905 \).

6. The parameters of a Hastings triplet are shown in the figure below. Use Gaussian reduction on the lens above to find the location of all six cardinal points, as well as the FFD and BFD. Show all work.
First calculate all of the surface powers

\[
\phi_1 = \frac{n_1' - n_1}{R_1} = \frac{1.62 - 1.0}{32.48\ mm} = 0.0190887\ mm^{-1}
\]
\[
\phi_2 = \frac{n_2' - n_2}{R_2} = \frac{1.517 - 1.62}{18.15\ mm} = -0.00567493\ mm^{-1}
\]
\[
\phi_3 = \frac{n_3' - n_3}{R_3} = \frac{1.62 - 1.517}{-18.15\ mm} = -0.00567493\ mm^{-1}
\]
\[
\phi_4 = \frac{n_4' - n_4}{R_4} = \frac{1.0 - 1.62}{-32.48\ mm} = 0.0190887\ mm^{-1}
\]
Next, we need to calculate the distance between these two reduced systems. The new distance $t'_{12}$ is the separation between the rear principal plane of the first system $P'_{12}$ and the front principal plane of the second system $P_{34}$. This distance is given by

$$t'_{12} = t'_{2} - d'_{12} + d_{34} = 20.16816 \text{ mm}$$
The effective focal length is \( f'_E = \frac{1}{\phi_{1234}} = 40.29 \text{ mm} \). The front and rear focal lengths are

\[
f'_R = 40.29 \text{ mm} \quad \text{and} \quad f'_F = -40.29 \text{ mm}.
\]

The front principal plane is located \( d_{12} + d_{1234} = 6.412 \text{ mm} \) to the right of V1.

By symmetry, the rear principal plane is located 6.412 mm to the left of V4.

The \( BFD = f'_R + d'_{1234} = 33.88 \text{ mm} \). By symmetry, the \( FFD = -33.88 \text{ mm} \).

Since the system is in air, the nodal points are located at the principal planes.
7. A Gregorian telescope has two reflective mirrors. The first mirror has a radius of curvature of $R_1 = -500 \text{ mm}$. The second mirror lies to the left of the first mirror such that $t_1' = -150 \text{ mm}$. The radius of curvature of the second mirror is $R_2 = 125 \text{ mm}$.

(a) Sketch the telescope and label the vertices, V1 and V2.

(b) Using Gaussian reduction, find the overall focal length of the telescope.

First calculate all of the surface powers

\[
\phi_1 = \frac{n_1' - n_1}{R_1} = \frac{-1.1}{-500 \text{ mm}} = 0.004 \text{ mm}^{-1}
\]
\[
\phi_2 = \frac{n_2' - n_2}{R_2} = \frac{1.1}{125 \text{ mm}} = 0.016 \text{ mm}^{-1}
\]

Calculate $\phi_{12} = \phi_1 + \phi_2 - \frac{t_1'}{n_1'} \phi_1 \phi_2 = 0.0104 \text{ mm}^{-1}$

The effective focal length is $f_E = \frac{1}{\phi_{12}} = 96.15 \text{ mm}$.

(c) Where are the principal planes located with respect to the vertices V1 and V2?

Calculate $d_{12} = n_1 \delta_{12} = n_1 \frac{\phi_2}{\phi_{12}} \frac{t_1'}{n_1'} = 230.769 \text{ mm}$

Calculate $d_{12}' = n_2 \delta_{12}' = -n_2 \frac{\phi_1}{\phi_{12}} \frac{t_1'}{n_1'} = -57.6923 \text{ mm}$
8. A system consists of two thin lenses separated by a distance of 8 mm. The focal length of the first lens is 3.75 mm and the focal length of the second lens is 10 mm. An object is located 4 mm to the left of the first lens. The object has a height of 0.5 mm.

(a) What is the focal length of the system?

The powers of each of the lenses are given by
\[
\phi_1 = \frac{1}{3.75 \text{ mm}} = 0.267 \text{ mm}^{-1}
\]
\[
\phi_2 = \frac{1}{10 \text{ mm}} = 0.100 \text{ mm}^{-1}
\]

The total power of the system is given by
\[
\phi = \phi_1 + \phi_2 - t\phi_1\phi_2 = 0.153 \text{ mm}^{-1}
\]

The focal length of the system is
\[
f = \frac{1}{\phi} = 6.522 \text{ mm}.
\]

(b) Where is the image formed with respect to the last surface?

For this, we first need to find the location of the principal planes
\[
d = \frac{\phi_2 t}{\phi} = 5.217 \text{ mm}
\]
\[
d' = -\frac{\phi_1 t}{\phi} = -13.913 \text{ mm}
\]

The principal planes look like
So the object distance is \( z = -d - 4 = 9.217 \text{ mm} \). Using the Gaussian imaging eq.

\[
\frac{1}{z'} - \frac{1}{-9.217 \text{ mm}} = 0.153.
\]

Solving gives \( z' = 22.302 \text{ mm} \) from the rear principal plane. This puts the image 8.389 mm to right of last surface.

(c) How big and what is the orientation of the image?

The magnification is

\[
m = \frac{z'}{z} = -2.42
\]

This means the image is 1.21 mm high, but inverted.