Do all problems and show your work. Credit is not given for answers only. Don't spend too much time on any one problem. Use separate sheets of paper and don't cram your work into the spaces below. As in the notes, zF and z'F are the object and image distances measured from the front and rear focal points. The distances z and z' are the object and image distances measured from the front and rear principal plane.

Thin lens or general system $$\phi = \frac{1}{f}$$	Focal Lengths $$f'_R = n'f \qquad f_F = -nf$$
Single surface $$\phi = \frac{n' - n}{R}$$	Total power of two elements $$\phi = \phi_1 + \phi_2 - \left(\frac{t'_1}{n'_1}\right)\phi_1\phi_2$$
Front Principal Plane $$\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\left(\frac{t'_1}{n'_1}\right)$$	Rear Principal Plane $$\delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\left(\frac{t'_1}{n'_1}\right)$$
Front Focal Distance $$FFD = f_F + d$$	Back Focal Distance $$BFD = f'_R + d'$$
Gaussian Imaging Eq. $$\frac{n'}{z'} - \frac{n}{z} = \frac{1}{f}$$	Newtonian Imaging Eq. $$z_F z'_F = f_F f'_R$$
Transverse Magnification $$m = \frac{nz'}{n'z}$$	Longitudinal Magnification (small shift) $$\bar{m} = \frac{n'}{n} m^2$$
Nodal Points $$z_{PN} = z'_{PN} = f_F + f'_R$$	Mirrors Thickness and indices switch sign after reflection

1. Given the object distance zF = -120.000 mm, an object space index n = 1.50, and a lens of effective focal length f = -21.000 mm, what is the front focal length fF and the magnification m?

The front focal length is given by fF = -(1.50)(-21.000) = 31.500 mm.

The magnification is given by m = (1.50)(-21.000) / (-120.000) = 0.263.

2. Given the object distance zF = -120.000 mm, an object space index n = 1.00, and a lens of front focal length fF = -42.000 mm, what is the effective focal length f and the magnification m?

The effective focal length is given by f = -(-42.00) / 1.000 = 42.000 mm.

The magnification is given by m = (1.00)(42.000) / (-120.000) = -0.350.

3. Given the object distance zF = 160.000 mm, an object space index n = 1.33, and a magnification m = -1.500 mm, what is the effective focal length f and the front focal length fF?

The effective focal length is given by fE = -(-1.500)(160.000) / 1.33 = -180.451 mm.

The front focal length is given by fF = -1.33(-180.451) = 240.000.

4. Given the image distance z'F = 80.000 mm, an image space index n' = 1.33, and a lens of effective focal length f = 21.000 mm, what is the rear focal length f'R and the magnification m?

The rear focal length is given by f'R = 1.33(21.000) = 27.930 mm.

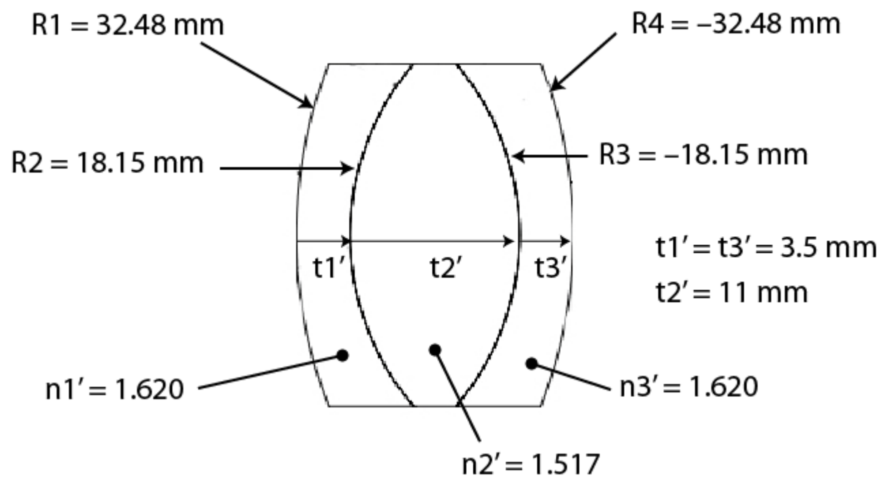
The magnification is given by m = -(80.000) / (1.33(21.000)) = -2.864.

5. Given the image distance z'F = 80.000 mm, an object space index n' = 1.33, and a lens of rear focal length f'R = 42.000 mm, what is the effective focal length f and the magnification m?

The effective focal length is given by fE = (42.0) / 1.33 = 31.579 mm.

The magnification is given by m = -(80.000) / (1.33(31.579)) = -1.905.

6. The parameters of a Hastings triplet are shown in the figure below. Use Gaussian reduction on the lens above to find the location of all six cardinal points, as well as the FFD and BFD. Show all work.



First calculate all of the surface powers

$$\phi_1 = \frac{n'_1 - n_1}{R_1} = \frac{1.62 - 1.}{32.48\,\text{mm}} = 0.0190887\,\text{mm}^{-1}$$

$$\phi_2 = \frac{n'_2 - n_2}{R_2} = \frac{1.517 - 1.62}{18.15\,\text{mm}} = -0.00567493\,\text{mm}^{-1}$$

$$\phi_3 = \frac{n'_3 - n_3}{R_3} = \frac{1.62 - 1.517}{-18.15\,\text{mm}} = -0.00567493\,\text{mm}^{-1}$$

$$\phi_4 = \frac{n'_4 - n_4}{R_4} = \frac{1. - 1.62}{-32.48\,\text{mm}} = 0.0190887\,\text{mm}^{-1}$$

$$\text{Calculate } \phi_{12} = \phi_1 + \phi_2 - \frac{t'_1}{n'_1}\phi_1\phi_2 = 0.0136478\,\text{mm}^{-1}$$

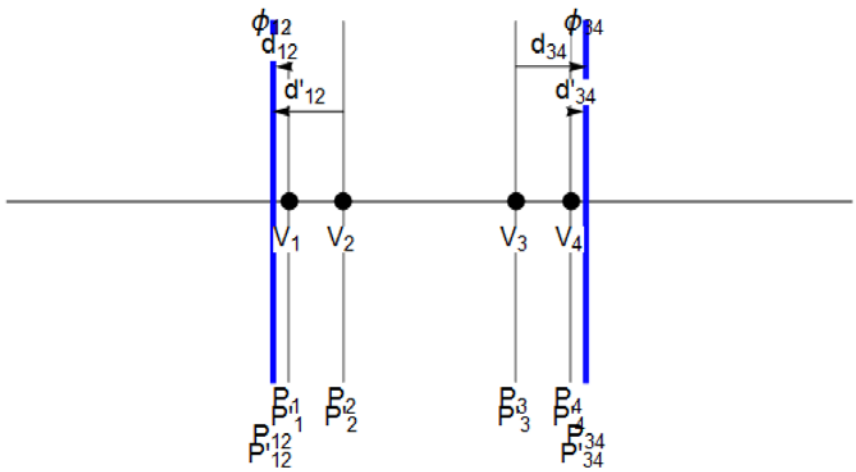
$$\text{Calculate } d_{12} = n_1\,\delta_{12} = n_1\frac{\phi_2}{\phi_{12}}\frac{t'_1}{n'_1} = -0.898363\,\text{mm}$$

$$\text{Calculate } d'_{12} = n'_2\,\delta'_{12} = -n'_2\frac{\phi_1}{\phi_{12}}\frac{t'_1}{n'_1} = -4.58408\,\text{mm}$$

$$\text{Calculate } \phi_{34} = \phi_3 + \phi_4 - \frac{t'_3}{n'_3}\phi_3\phi_4 = 0.0136478\,\text{mm}^{-1}$$

$$\text{Calculate } d_{34} = n_3\,\delta_{34} = n_3\frac{\phi_4}{\phi_{34}}\frac{t'_3}{n'_3} = 4.58408\,\text{mm}$$

$$\text{Calculate } d'_{34} = n'_4\,\delta'_{34} = -n'_4\frac{\phi_3}{\phi_{34}}\frac{t'_3}{n'_3} = 0.898363\,\text{mm}$$



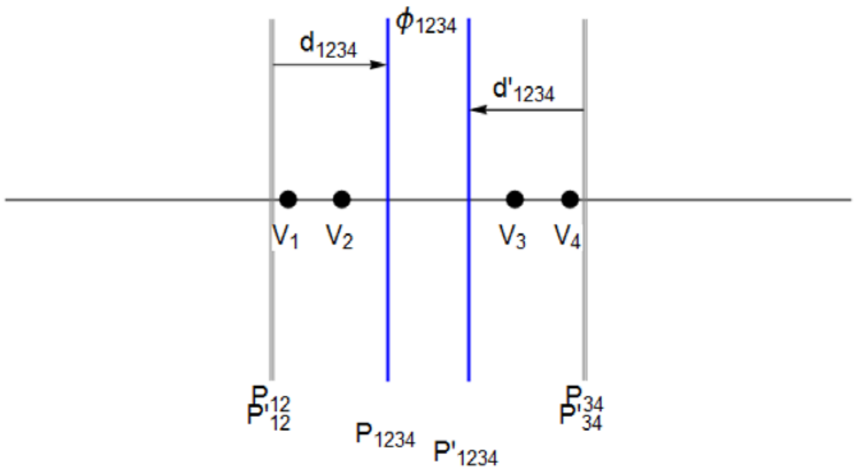
Next, we need to calculate the distance between these two reduced systems. The new distance t'_{12} is the separation between the rear principal plane of the first system P'_{12} and the front principal plane of the second system P_{34}. This distance is given by

$$t'_{12} = t'_2 - d'_{12} + d_{34} = 20.16816\,mm$$

Calculate $\phi_{1234} = \phi_{12} + \phi_{34} - \frac{t'_{12}}{n'_{12}} \phi_{12} \phi_{34} = 0.0248192 \, \text{mm}^{-1}$

Calculate $d_{1234} = n_{12} \delta_{1234} = n_{12} \frac{\phi_{34}}{\phi_{1234}} \frac{t'_{12}}{n'_{12}} = 7.31062 \, \text{mm}$

Calculate $d'_{1234} = n'_{34} \delta'_{1234} = -n'_{34} \frac{\phi_{12}}{\phi_{1234}} \frac{t'_{12}}{n'_{12}} = -7.31062 \, \text{mm}$



The effective focal length is $f_E = \frac{1}{\phi_{1234}} = 40.29 \text{ mm}$. The front and rear focal lengths are $f'_R = 40.29 \text{ mm}$ and $f_F = -40.29 \text{ mm}$.

The front principal plane is located $d_{12} + d_{1234} = 6.412 \text{ mm}$ to the right of V1.

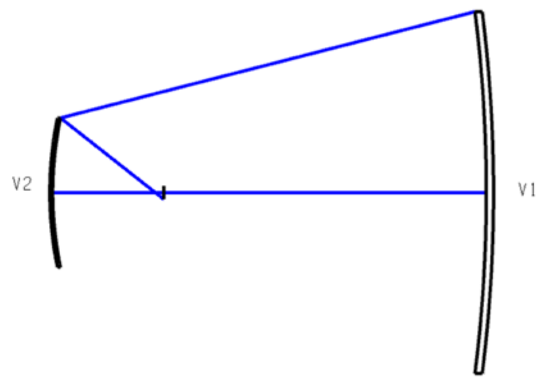
By symmetry, the rear principal plane is located 6.412 mm to the left of V4.

The $BFD = f'_R + d'_{1234} = 33.88 \text{ mm}$. By symmetry, the $FFD = -33.88 \text{ mm}$.

Since the system is in air, the nodal points are located at the principal planes.

7. A Gregorian telescope has two reflective mirrors. The first mirror has a radius of curvature of $R1 = -500$ mm. The second mirror lies to the left of the first mirror such that $t_1' = -150$ mm. The radius of curvature of the second mirror is $R2 = 125$ mm.

(a) Sketch the telescope and label the vertices, V1 and V2.



(b) Using Gaussian reduction, find the overall focal length of the telescope.

First calculate all of the surface powers

$$\phi_1 = \frac{n'_1 - n_1}{R_1} = \frac{-1. - 1.}{-500\,\text{mm}} = 0.004\,\text{mm}^{-1}$$

$$\phi_2 = \frac{n'_2 - n_2}{R_2} = \frac{1. - -1.}{125\,\text{mm}} = 0.016\,\text{mm}^{-1}$$

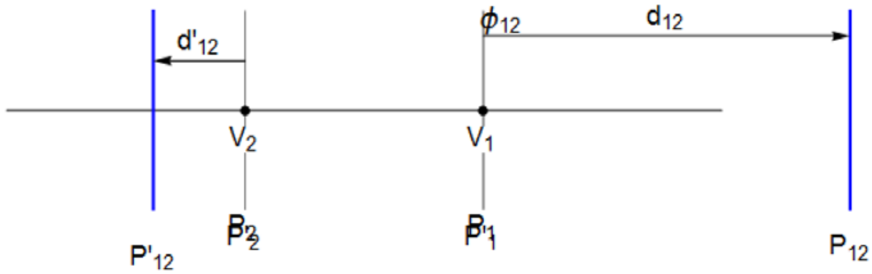
Calculate $\phi_{12} = \phi_1 + \phi_2 - \frac{t'_1}{n'_1}\phi_1\,\phi_2 = 0.0104\,\text{mm}^{-1}$

The effective focal length is $f_E = \frac{1}{\phi_{12}} = 96.15\,\text{mm}$.

(c) Where are the principal planes located with respect to the vertices V1 and V2?

Calculate $d_{12} = n_1\,\delta_{12} = n_1 \frac{\phi_2}{\phi_{12}} \frac{t'_1}{n'_1} = 230.769\,\text{mm}$

Calculate $d'_{12} = n'_2\,\delta'_{12} = -n'_2 \frac{\phi_1}{\phi_{12}} \frac{t'_1}{n'_1} = -57.6923\,\text{mm}$



8. A system consists of two thin lenses separated by a distance of 8 mm. The focal length of the first lens is 3.75 mm and the focal length of the second lens is 10 mm. An object is located 4 mm to the left of the first lens. The object has a height of 0.5 mm.

(a) What is the focal length of the system?

The powers of each of the lenses are given by

$$\phi_1 = \frac{1}{3.75\ mm} = 0.267\ mm^{-1}$$

$$\phi_2 = \frac{1}{10\ mm} = 0.100\ mm^{-1}$$

The total power of the system is given by

$$\phi = \phi_1 + \phi_2 - t\phi_1\phi_2 = 0.153\ mm^{-1}$$

The focal length of the system is

$$f = \frac{1}{\phi} = 6.522\ mm.$$

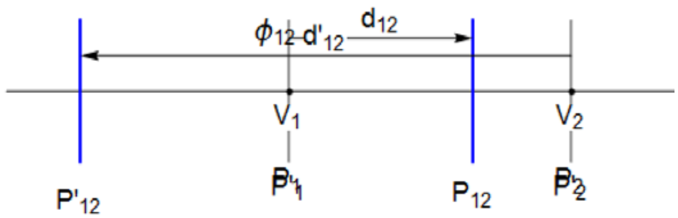
(b) Where is the image formed with respect to the last surface?

For this, we first need to find the location of the principal planes

$$d = \frac{\phi_2 t}{\phi} = 5.217\ mm$$

$$d' = -\frac{\phi_1 t}{\phi} = -13.913\ mm$$

The principal planes look like



So the object distance is $z = -d - 4 = 9.217\ mm$. Using the Gaussian imaging eq.

$$\frac{1}{z'} - \frac{1}{-9.217\ mm} = 0.153.$$

Solving gives $z' = 22.302\ mm$ from the rear principal plane. This puts the image 8.389 mm to right of last surface.

(c) How big and what is the orientation of the image?

The magnification is

$$m = \frac{z'}{z} = -2.42$$

This means the image is 1.21 mm high, but inverted.