Contents

Optical Specification, Fabrication and Testing is designed to unify concepts from geometrical optics, aberrations, Fourier optics, interference and diffraction. In the real world, a lens designer may provide an optical design, ideally with tolerances and the surface dimensions and materials. However, turning this design into a real system involves an array of further processes. This course draws on the elements described above to illustrate the process of turning a design into a working system. Namely:

- Creating specifications such as drawings that an optics shop can interpret correctly to fabricate the part(s).
- Creating specifications on the testing procedure which will be used to validate the performance of the part to ensure it was made correctly.
- Understanding fabrication technologies to determine what is possible, what the associated costs are with the fabrication technique and the limitations of materials and performance.

We will look at

- Performance metrics
- Simple and complex tests to verify performance
- Standards from creating drawings of optical components

1 Properties of Optical Systems

1.1 Optical Properties of a Single Spherical Surface

1.1.1 Planar Refractive Surfaces

Geometrical optics is a branch of optics which utilizes a ray picture to analyze optical systems. A ray is a specialized vector, having both an origin and a direction that illustrates the propagation of light. Rays are useful for assessing the optical properties of systems containing prisms, mirrors and lenses. In homogeneous media, rays travel in straight lines. When a ray reaches a boundary between two media, the ray is bent or refracted. Figure 1-1illustrates the refracting of a ray at the boundary between two regions of index n and n′. Snell's law, as shown in Equation 1.1, describes the degree of refraction

$$n \sin i = n' \sin i', \tag{1.1}$$

where i and i′ are the angles the ray forms with respect to the surface normal. The convention of using unprimed variables for values before the interface and primed variables for values after the interface will be used in the ensuing discussion. A ray in a medium of refractive index n that is incident on a surface at an angle i, will refract to leave the surface at angle i′. The newly refracted ray will then continue in a straight line until it intercepts a new boundary.

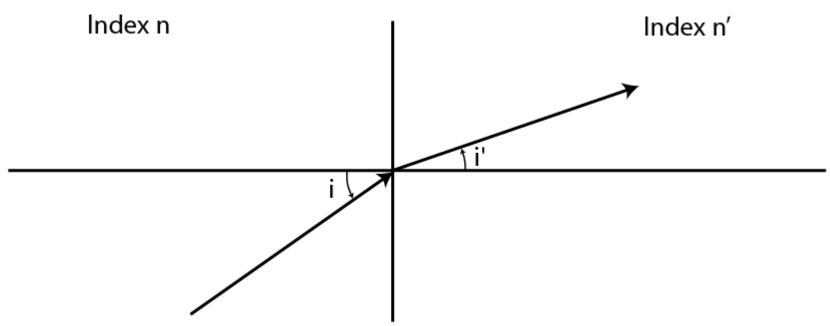


Figure 1-1 Refraction of a ray at the boundary between two materials

Figure 1-1 is also useful for the sign convention that will be used to describe angles in the ensuing discussion. Angles which are measured counter-clockwise from a reference line (the normal to the planar interface in this case) will be taken as positive, while clockwise rotations will represent negative angles Both i and i′ are positive in the figure above.

1.1.2 Spherical Refractive Surfaces

The spherical refracting surface shown in Figure 1-2 separates two optical spaces with refractive indices of n and n′, respectively. A line passing through the center of curvature of the spherical surface is deemed an optical axis. The intersection of the optical axis with the surface is called the vertex. This surface has a radius of curvature R, which is the distance from a point on the surface to the center of curvature. The value of R has units of length (e.g. millimeters (mm) for elements such as the optics found in camera lenses or meters (m) for mirrors found in astronomical telescopes). As R approaches infinity, the spherical surface approaches a plane. To avoid computational issues when describing infinite radius, spherical surfaces can also be specified in terms of surface curvature, C = 1 / R. Units for curvature for our examples above are mm^{-1} and m^{-1}. The unit m^{-1} is also called a diopter. When specifying an optical surface in terms of curvature, C = 0 for a planar surface.

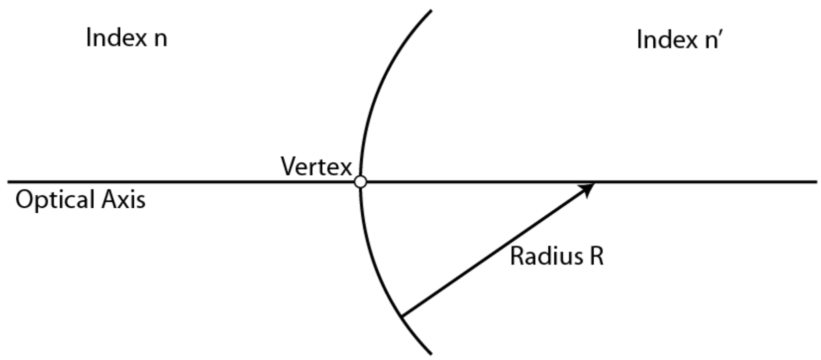


Figure 1-2 Spherical refractive surface of radius R, separating spaces with refractive index n and n′.

The power ϕ of an optical surface describes its ability to cause light to converge or diverge as it passes through the surface. The power is defined in eq. 1.2 as

$$\phi = (n' - n)C = \frac{n' - n}{R}, \tag{1.2}$$

which has units identical to the curvature. At this point, a sign convention needs to be invoked. In cases such as that shown in Figure 1-3, when $n' > n$, rays parallel to the optical axis will converge as they pass through the surface. This situation will be associated with a positive power. Consequently, the sign convention for radii of curvature requires that $R > 0$ when the center of curvature lies to the right of the surface and $R < 0$ when the center of curvature lies to the left of the surface. Figure 1-3 illustrates the surface sign convention. The convention of showing negative and positive distances with the arrowhead will be used. The left side of Figure 1-3 denotes a positive radius of curvature since the arrowhead points to the right of the surface, while the right side of the figure denotes a negative curvature since the arrowhead points to the left of the surface.

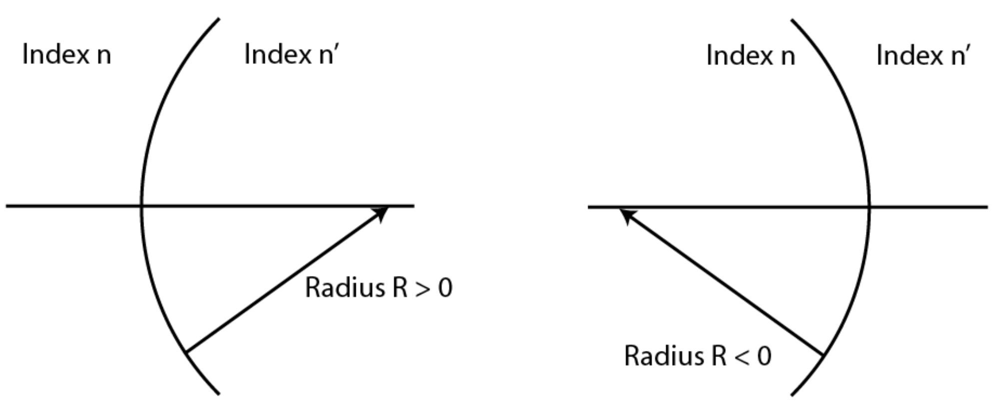


Figure 1-3 Sign convention for the radius of a spherical surface.

Example

n = 1.0 (Air), n′ = 1.5 and R = 80 mm

$$\phi = \frac{1.5 - 1.0}{80\,\text{mm}} = 0.00625\,\text{mm}^{-1}. \tag{1.3}$$

If the air at the interface is replaced with water such that n = 1.33, then the power is

$$\phi = \frac{1.5 - 1.33}{80\,\text{mm}} = 0.002125\,\text{mm}^{-1}. \tag{1.4}$$

Note that the power is reduced when the difference between n and n′ is reduced.

Rays traveling parallel to the optical axis are said to be collimated. When collimated light intercepts a surface with positive power, the rays will refract and converge to a point F′. Similarly, there exists a point F in which rays which diverge from F intercept the positive powered surface and are refracted so that they become collimated following the surface. The points F and F′ are called the front and rear focal points of the surface. The focal length is related to the distance from F or F' to the surface vertex. The Effective Focal Length (EFL, f_E) is defined as the reciprocal of the power

$$f_E = \frac{1}{\phi}. \tag{1.5}$$

The Front Focal Length (FFL, f_F) and the Rear (Back) Focal Length (BFL, f'_R) are defined as

$$f_F = -\frac{n}{\phi} = -nf_E \tag{1.6}$$

and

$$f'_R = \frac{n'}{\phi} = n'f_E. \tag{1.7}$$

The EFL describes the equivalent focal length for a media of index of one, whereas the FFL and the BFL incorporate the local refractive index of the optical space. For the example given by equation 1.3, the Effective Focal Length is f_E = 160 mm, the Front Focal Length f_F = -160 mm and the Rear Focal Length is f'_R = 240 mm. Figure 1-4 illustrates the meaning of the front and rear focal lengths. Figure 1-4 (a) shows a ray passing through the point F in front of the surface. This ray will ideally refract at the surface and emerge parallel to the optical axis. The Front Focal Length is the distance from the surface vertex to the point F. Figure 1-4 (b) shows a ray traveling through the region of index n, which is parallel to the optical axis. For the case where n′ > n, this ray will refract at the surface and ideally cross the optical axis at a point F′. The Rear Focal Length is the distance from the surface vertex to F′. The sign convention used for measuring these lengths suggests that distances to the left of the surface are negative and to the right of the surface are positive.

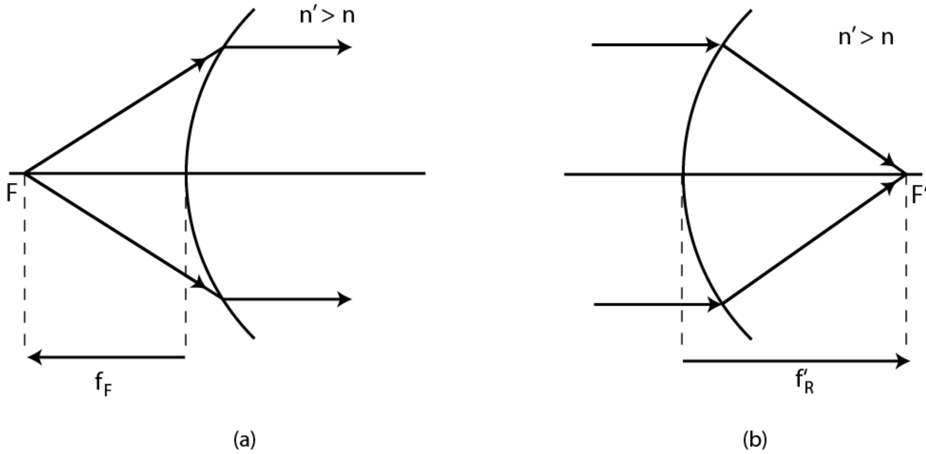


Figure 1-4 Illustration of the front and rear focal points. (a) Rays passing through the front focal point F are refracted by the surface and emerge parallel to the optical axis. (b) Rays traveling parallel to the optical axis are refracted by the surface and pass through the rear focal point, F′.

1.1.3 Reflective Surfaces

In the case of reflective surfaces, the definitions and sign conventions outlined above still hold as long as we assume that n′ = -n. Under this assumption, the power is given by

$$\phi = -2nC = -\frac{2n}{R}, \tag{1.8}$$

and the assorted focal lengths are

$$f_F = f'_R = -\frac{n}{\phi} = -nf_E = \frac{R}{2} = \frac{1}{2C}. \tag{1.9}$$

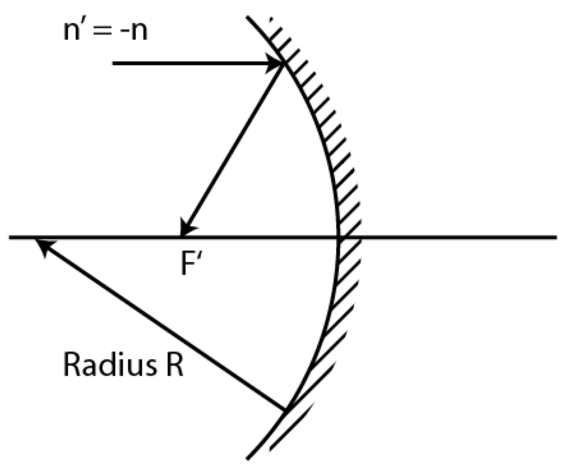


Figure 1-5 Collimated light focuses to a point halfway to the center of curvature of the mirror

Equation 1.9 shows that collimated light striking a mirror will focus to a point half way between the mirror's vertex and its center of curvature as shown in Figure 1-5. The subsequent discussion considers the imaging properties of a single refractive surface. However, all of the definitions and derivations hold for a reflective surface too as long as the preceding sign conventions and the assumption regarding n′ is used.

1.1.4 Gaussian Imaging Equation

The Gaussian imaging equations allows the determination of the object and image locations for a surface of power ϕ. The distance from the surface vertex to the object will be denoted by z and the distance from the surface to the image plane is denoted by z′. In keeping with the sign convention, these distances are negative if the distance is measured to the left of the surface and positive if it is to the right of the surface. The Gaussian imaging equation is

$$\frac{n'}{z'} - \frac{n}{z} = \phi. \tag{1.10}$$

Figure 1-6 shows the object and image distances for a single refracting surface.

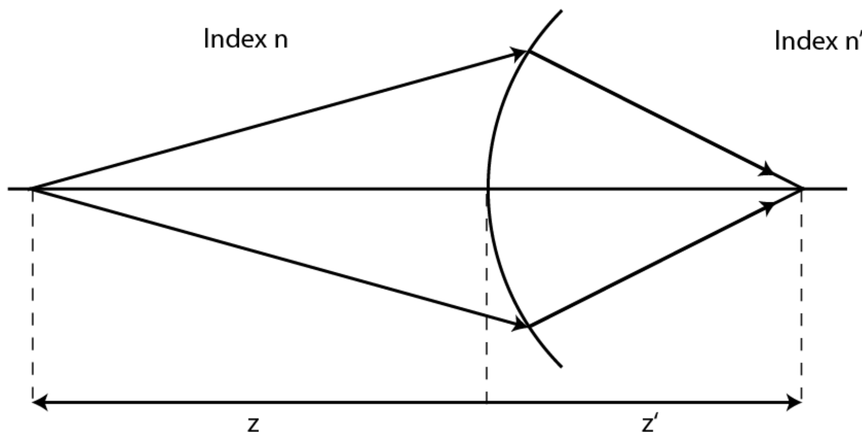


Figure 1-6 An object located a distance z from a single refracting surface of power ϕ will be imaged to a point located a z′. By convention z < 0 and z′ > 0 in this figure.

Continuing the previous example, where n = 1.0, n′ = 1.5 and $\phi = 0.00625\ \text{mm}^{-1}$, an object located 500 mm to the left of the surface would be imaged to

$$\frac{1.5}{z'} - \frac{1.0}{(-500\ \text{mm})} = 0.00625\ \text{mm}^{-1} \Rightarrow z' = +352.94\ \text{mm}. \tag{1.11}$$

Since z′ is positive, the image is formed to the right of the surface.

We can also examine the transverse (or lateral) magnification of the system for an extended object. Figure 1-7 shows an extended object of height y, located a distance z to the left of the surface. The image of this object has a height y′. The sign convention says that positive heights are above the optical axis and negative heights are below the optical axis. The transverse magnification is the ratio of the image height to the object height and is given by

$$\text{Transvere Magnification} = m = \frac{y'}{y}. \tag{1.12}$$

Figure 1-7 shows a case where m < 0, which means that the image is inverted relative to the object.

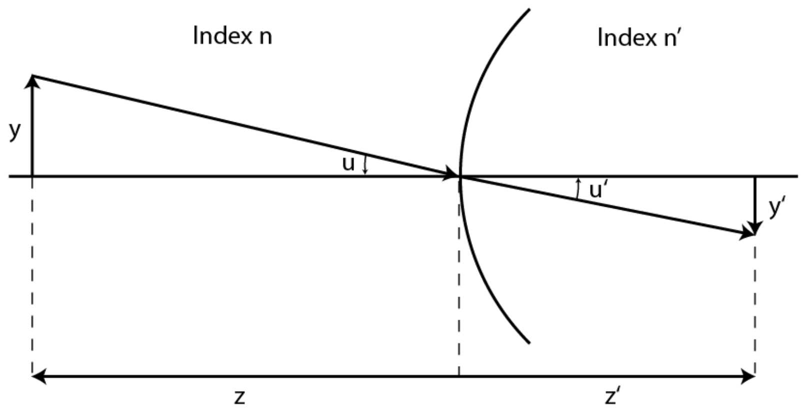


Figure 1-7 An extended object of height y results in an image of height y′. The ratio of these heights is the transverse magnification of the system. In this illustration, the magnification is negative, suggesting the image is inverted.

Figure 1-7 is also useful for developing an alternative expression for transverse magnification. First, the paraxial approximation needs to be developed. The ray leaving the tip of the object and intercepting the surface at its vertex undergoes refraction as described by Snell's law in eq. 1.1. If the object height h is small, then the small angle approximation can be used to define the paraxial angles $u \cong \sin i$ and $u' \cong \sin i'$. Furthermore, since the angles are small, $u \cong \tan i$ and $u' \cong \tan i'$, as well. Utilizing these approximations, Snell's law can be rewritten as

$$nu = n'u' \Rightarrow \frac{ny}{z} = \frac{n'y'}{z'}. \tag{1.13}$$

Rearranging eq. 1.13 gives

$$m = \frac{y'}{y} = \frac{nz'}{n'z}. \tag{1.14}$$

Continuing the example from eq. 1.11,

$$m = \frac{352.94}{1.5(-500)} = -0.47 \; . \tag{1.15}$$

The image height is therefore about 47% of the object height and inverted since m is negative. These relationships also enable alternative expressions for the Gaussian imaging equation to be derived:

$$\frac{z'}{n'}\left[\frac{n'}{z'} - \frac{n}{z}\right] = \phi\frac{z'}{n'} \Rightarrow 1 - m = \frac{z'}{f'_R} \tag{1.16}$$

so that

$$z' = (1 - m)f'_R . \qquad (1.17)$$

Similarly, multiplying the Gaussian imaging equation by z / n and rearranging gives

$$z = \left[\frac{m-1}{m}\right] f_F . \qquad (1.18)$$

1.1.5 Newtonian Imaging Equation

The Newtonian imaging equation provides yet another means of relating the object and image positions for a given refracting surface. For this relationship, however, the object and image distances $\tilde{z}$ and $\tilde{z}'$ are measured from the front and rear focal points, respectively. Figure 1-8 shows a typical imaging arrangement.

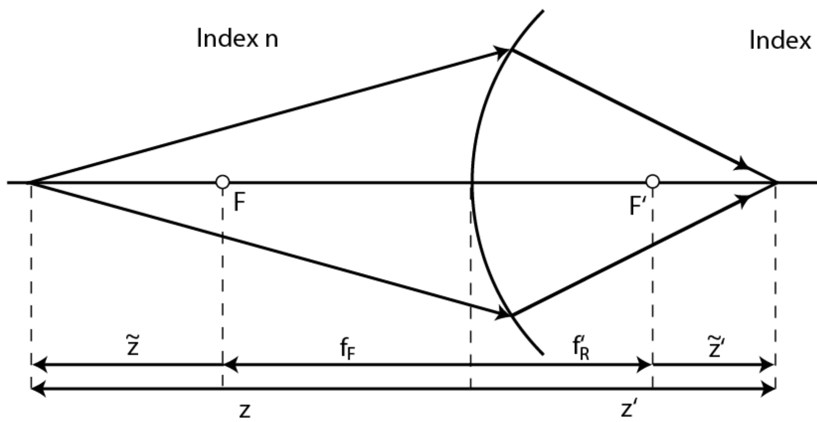


Figure 1-8 Imaging with the Newtonian equations measures the object and image distances from the front and rear focal points.

In referring to Figure 1-8, the *Gaussian* imaging equation can be written as either

$$\frac{n'}{f'_R + \tilde{z}'} - \frac{n}{f_F + \tilde{z}} = \frac{n'}{f'_R} \quad \text{or} \quad \frac{n'}{f'_R + \tilde{z}'} - \frac{n}{f_F + \tilde{z}} = -\frac{n}{f'_F} . \qquad (1.19)$$

The relations can be solved for $\tilde{z}$ and $\tilde{z}'$ and combined to give

$$\tilde{z}\tilde{z}' = f_F f'_R , \qquad (1.20)$$

which is known as the Newtonian imaging equation.

1.1.6 The Thin Lens

The thin lens is a useful concept in laying out optical systems. A thin lens is a lens whose thickness is much smaller than its radii of curvature. In making this approximation, the thin lens can be considered a planar surface with optical properties similar to a single refracting surface. For a thin lens with front radius R_1 and back radius R_2, the power of the lens is simply the sum of the powers of the two surfaces. If the index of refraction of the lens material is n_{lens} and the refractive index of the surrounding environment is n_o, then the power of the thin lens is given by

$$\phi = \frac{n_{lens} - n_o}{R_1} + \frac{n_o - n_{lens}}{R_2} = \left(n_{lens} - n_o\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right], \qquad (1.21)$$

which is known as the *Lensmaker's Equation*. The preceding derivations and definitions for focal lengths and imaging properties all hold for the thin lens when the power ϕ from eq. 1.21 is used.

1.2 Aperture and Field Stops

1.2.1 Aperture Stop Definition

The aperture stop is a mask within the system that limits the size of the bundle of rays that passes through an optical system. The mask is typically circular, but this is not a requirement. Figure 1-9(a) shows a single surface with the aperture stop located at the surface vertex. The object can be considered to consist of a series of discrete point sources. Each point source radiates light into all directions. The aperture stop in effect blocks most of these rays and only allows a limited cone of light from the point source to pass through the refractive surface. Figure 1-9(b) shows a case where the aperture stop is shifted towards the object. The object and image locations, as well as the magnification of the system remain unchanged, but the cone of light coming from the object is modified by the aperture stop location. As will be shown in later sections and chapters, the aperture stop dimensions and location affects the amount of light that reaches the image, as well as how well the rays come to a focus. In this latter case, note that the rays in Figure 1-9(a) pass through nearly the entire surface as drawn, while the rays in Figure 1-9(b) only pass through a peripheral portion of the surface. These different locations create different angles of incidence for the rays at the surface and consequently different angles of refraction following the surfaces. As will be shown below, a judicious choice of the location of the aperture stop can optimize the quality of an optical system.

1.2.2 Marginal and Chief Rays

There are several special rays that provide useful information regarding the properties of an optical system. One set of rays are called marginal rays. These rays start at the object plane on the optical axis and pass through the edge of the aperture stop. In an ideal system, the marginal rays exiting the system will appear to intersect the optical axis at the image plane. In Figure 1-9, one particular marginal ray is labeled. This ray starts on axis at the object and passes through the upper edge of the aperture stop. This ray is sometimes referred to as the upper rim ray. The ray starting at the same location, but passing through the lower edge of the aperture stop (lower rim ray) is considered a marginal ray as well. Marginal rays will propagate through the optical system and ultimately converge to the optical axis at the image plane. Furthermore, if the marginal rays intersect the optical axis at some location between the object and image, then an intermediate image is formed. These intermediate image planes can useful in that a mask or reticle can be placed at the location, having the effect of superimposing the reticle pattern onto the final image. This technique is often used in microscope eyepieces for example to allow scaled rulings to be placed over the image to measure features. Care should also be taken when designing optical systems with intermediate image planes so that these planes are not located at or near a lens surface. Any imperfections or debris located on this lens surface would be superimposed onto the final image. Figure

1-10(a) shows an example of a reticle inside a World War I era artillery sight. The grid aids the gunner in targeting. Figure 1-10(b) shows a close up of the image where dirt particles on the reticle appear superimposed on the image.

A second special ray defined with regard to the aperture stop is the chief ray (principal ray). The chief ray is defined as the ray that starts at the edge of the object and passes through the center of the aperture stop. Where the chief ray crosses the optical axis, is called a pupil plane. The size of this pupil is defined by the height of the marginal ray in the pupil plane.

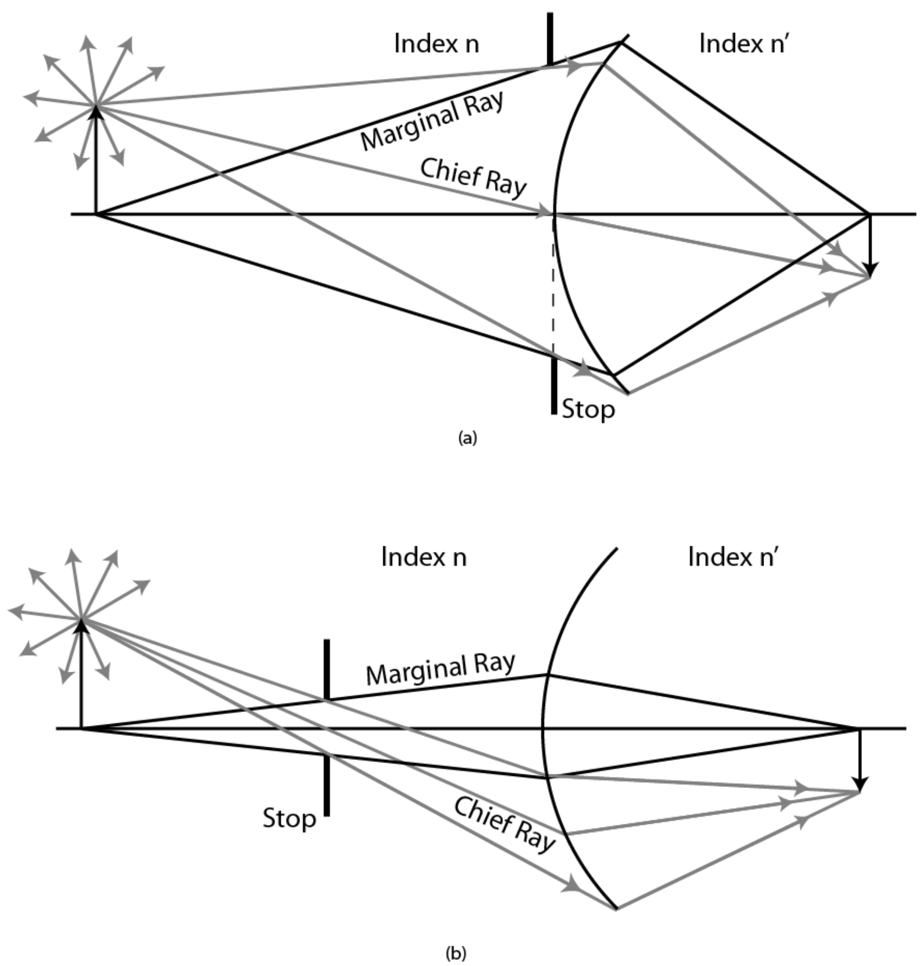
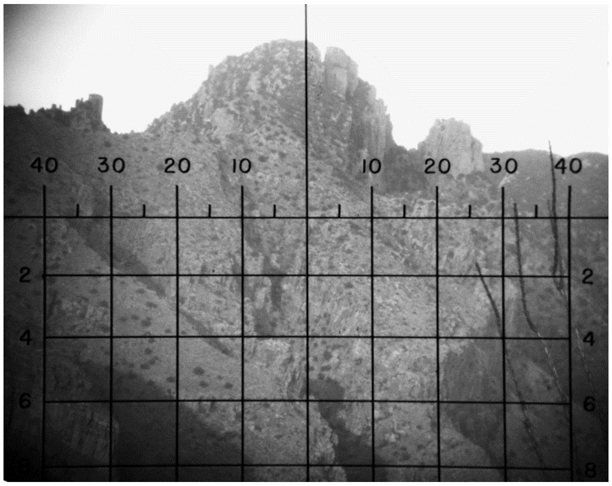


Figure 1-9 Limiting the bundle of rays passing through a refracting surface. (a) Aperture stop located at the surface. (b) Aperture stop located in front of surface.

(a) 

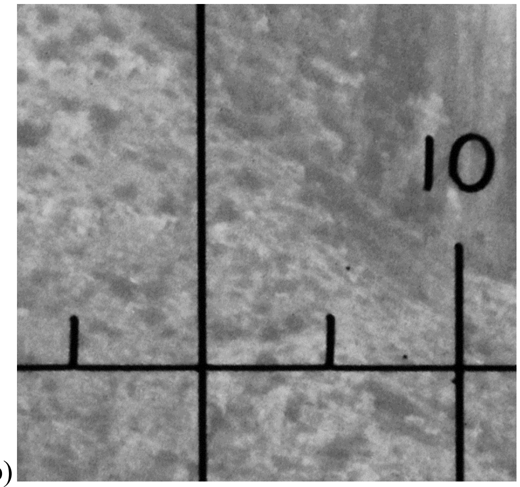
(b) 

Figure 1-10 (a) Reticle inside a panoramic sight. (b) Close up of reticle with small particles that get superimposed onto the image through the sight.

1.2.3 Vignetting

Vignetting occurs when a surface other than the aperture stop limits the cone of rays passing through an optical system. In Figure 1-9(b), the aperture stop is located in front of the refracting surface. The size of the aperture was chosen to so that the bundle of rays passing through it intersects the surface and refracts towards the image. In Figure 1-11, the aperture stop diameter is increased. In this case, the rays falling into the hashed region of the figure miss the refracting surface and consequently, do not continue to the image plane. In reality, this refracting surface is mounted, and the mount will clip this portion of the beam. The consequence of vignetting is to reduce the amount of light reaching the image plane. In Figure 1-11, the bundle of rays starting on the optical axis is unclipped by the refracting surface and all of the light reaches the on-axis image point. The bundle of rays starting from the edge of the object is vignetted and

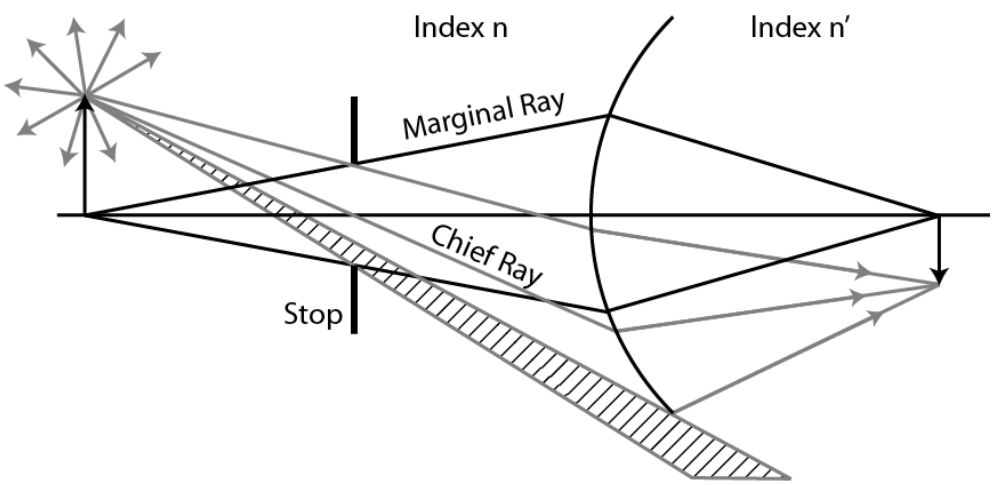


Figure 1-11 Vignetting occurs when a portion of the cone defined by the aperture stop misses or is clipped by another element in the system. Light in the hashed portion of the beam above misses the refracting surface.

consequently only a fraction of the rays reach the image plane. Intermediate starting points on the object will have varying degrees of clipping, so the effect of vignetting in general is to cause a decrease or "roll off" of the image brightness towards the edge on the image. This effect is illustrated in Figure 1-10(a) as well. The darkness of the image in the upper and lower left corners of the image is caused by vignetting. Vignetting can be caused by size constraints of various optical elements in a system, or it can be intentionally introduced by the lens designer to block certain rays from reaching the image plane. Often, the rays that are most difficult to bring into focus come from the edge of the object. Blocking these rays may improve image quality with only minimal or imperceptible reductions in image brightness.

1.2.4 Field Stop Definition

The field stop limits the size of the object that can be used with an optical system. The field stop is another mask with the system that blocks or clips light coming from outside a give region of the object. Often, the field stop is located at an intermediate image plane or at the final image plane. In the case of the intermediate image plane, the field stop is useful for controlling stray light that may be entering the system. This location is also useful for visual instruments where the eye serves as the final detector. Since the eye has an enormous field of view, the field stop serves to define the overall subtense of the image formed on the retina without the roll off effects of vignetting. When the field stop is located at the final image plane, the image sensor (typically CMOS or CCD arrays in today's digital imaging systems) defines the dimensions of the field stop. Figure 1-12 illustrates the effect of the field stop. The field stop is located at the final image plane in this case. In Figure 1-9(b) this imaging configuration had an object defined by the black arrow. Placing the field stop only allows a portion of the object to reach the image plane, so that only the gray arrow can be recorded in the final image.

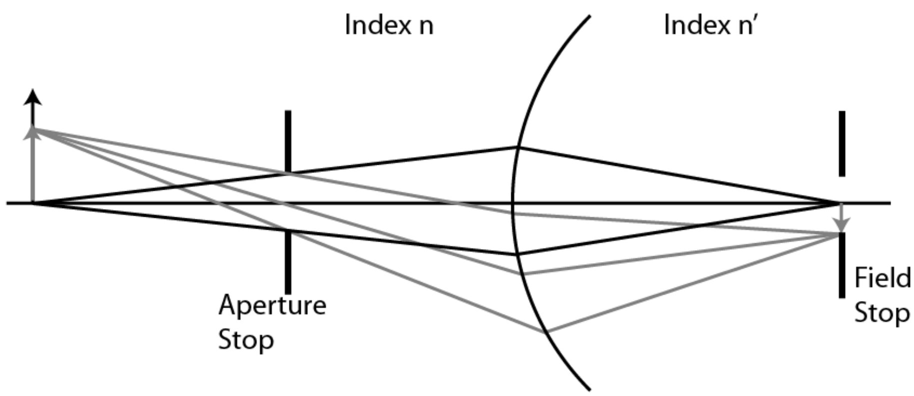


Figure 1-12 A field stop limits the size of the object that can be imaged with an optical system.

CMOS and CCD arrays often serve as the field stop and image recording device in modern imaging systems. The specification of the size of these arrays is somewhat convoluted and care should be used when determining the dimension of the arrays. The dimensions of these digital arrays are typically specified in units of inches. For example, 1/4" and 1/2" (read as one-quarter inch and one-half inch) sensors are common. The dimensions here are *not* the length of the diagonal of the sensor. The dimensions for these sensors are 3.2 x 2.4 mm and 6.4 x 4.8 mm, respectively. The naming convention for these sensors comes from their predecessor, vidicon tubes. Vidicon tubes were originally developed in the 1950s and were used to record images in video cameras. Figure 1-13 shows a photograph of a vidicon tube. An image was projected onto the circular end of the tube. The intensity variations of the image changed the local capacitance of the tube. An electron beam was raster scanned across a rectangular region of the tube end. Variations in read-out voltage encoded the image. The inch dimension for specifying CMOS and CCD arrays refer to the diameter of the equivalent vidicon tube. The true active area of the tube, and consequently the digital arrays is smaller. Table 1-1 provides a listing of common CMOS and CCD sensors and their corresponding array dimensions.

Figure 1-13 Vidicon tube. An image is projected onto the circular region at the left end of the tube leading to variations if capacitance on the surface. An electron beam within the tube scans across this surface and the resulting voltage variations encode the image.

Sensor Type	Width (mm)	Height (mm)	Diagonal (mm)
1/10”	1.28	0.96	1.60
1/8”	1.60	1.20	2.00
1/6”	2.40	1.80	3.00
1/4”	3.20	2.40	4.00
1/3.6”	4.00	3.00	5.00
1/3.2”	4.54	3.42	5.68
1/3”	4.80	3.60	6.00
1/2.7”	5.37	4.04	6.72
1/2.5”	5.76	4.29	7.18
1/2”	6.40	4.80	8.00
1/1.8”	7.18	5.32	8.93
1/1.7”	7.60	5.70	9.50
1/1.6”	8.08	6.01	10.07
2/3”	8.80	6.60	11.00
1”	12.80	9.60	16.00

Table 1-1 Common CMOS and CCD sensors and their dimension

The dimensions of a digital image sensor and its corresponding resolution provide insight into the size of the individual pixels on the sensor. For example, a common commercially available image sensor is specified as a 1/3” CCD array with a resolution of 1296 x 964 pixels. From Table 1-1, the dimensions of this sensor are 4.8 x 3.6 mm. If the horizontal and vertical dimensions are divided by the number of pixels in the corresponding directions, then each pixel is 3.75 μm square.