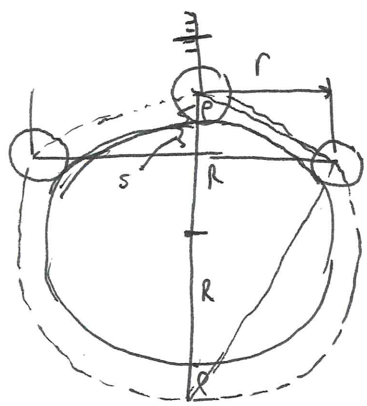
3. NON-INTERFEROMETRIC TESTING [Book Ch. 5]

3.1 SURFACE RADIUS OF CURVATURE [Book 5.2]

3.1.1 ~~SURFACE RADIUS~~ GENEVA GAUGE (SHOW SLIDE) [Book 5.2.1]

A ~~g~~ Geneva gauge is a simple device for measuring the radius of curvature of a surface. Suppose we have three legs that are tipped with spheres of radius ρ. The outer two legs are fixed and separated by $2r$. The center leg moves so that when the device is placed on a spherical surface, we can measure the displacement s of the center leg relative to the outer legs.



From the drawing, using similar triangles

$$\frac{r}{2(R+\rho)-s} = \frac{s}{r} \Rightarrow r^2 = 2s(R+\rho) - s^2$$

$$\boxed{R = \frac{r^2+s^2}{2s} - \rho}$$ FOR CONVEX SURFACE

$$\boxed{R = \frac{r^2+s^2}{2s} + \rho}$$ FOR CONCAVE SURFACE

A ~~g~~ Geneva gauge uses this principal but typically gives "power" under the assumption of a "crown" glass with

$n = 1.523$

$$\text{Power } \phi_g = \frac{0.523}{R}$$

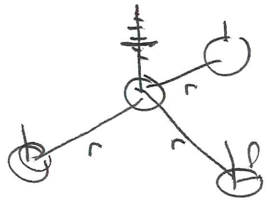
If we have a different material of known index n, then the true power is

$$\phi = \left(\frac{n-1}{0.523}\right)\phi_g$$

If gauge is not $\perp$ to surface, the wrong answer is optained

[Book 5.2.2]

3.1.2 Spherometer is a similar device to the Geneva gauge, but typically has 3 outer legs in triangular pattern to provide stability. It avoids errors related to tilting, but may miss astigmatism.



(SHOW SLIDE)

3.2 Wavefronts [Book 5.4]

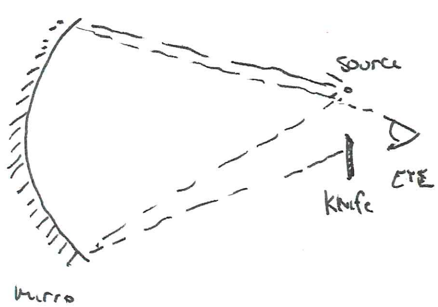
Non-interferometric testing of wavefronts are usually fast, inexpensive and qualitative means for assessing the aberrations of a system. There are two key concepts to understanding how these tests work.

1. A mask is placed near focus (but not necessarily at focus). The mask has a known pattern which blocks certain rays. We need to know the position of rays in the mask plane to determine if they are blocked or passed → transverse ray error $\varepsilon_x(\rho_x, \rho_y)$ $\varepsilon_y(\rho_x, \rho_y)$

2. For the Foucault, wire and Ronchi test, we view the exit pupil of the system so we see dark regions at points ρ_x, ρ_y for rays that are blocked and light regions at points (ρ_x, ρ_y) for rays that pass. For the Hartmann test, the pattern is recorded in the mask plane.

3.2.1 Foucault Knife Edge Test [Book 5.4.1]

A knife edge (e.g. razor blade) is used to block a portion of the beam near focus. The eye looks back into the system at the exit pupil and observes the pattern.



We are concerned with fabrication errors which destroy the rotational symmetry of the system, so we'll look at

W_{40}, W_{31} and W_{22}

We need to know the transverse ray error in the plane of the knife

$$\varepsilon_x = \frac{-R}{r_{max}}\left[2W_{020}\rho_x + 4W_{40}(\rho_x^3 + \rho_x\rho_y^2) + 2W_{31}\rho_x\rho_y\right]$$

$$\varepsilon_y = \frac{-R}{r_{max}}\left[2W_{020}\rho_y + 4W_{040}(\rho_x^2\rho_y + \rho_y^3) + W_{31}(\rho_x^2 + 3\rho_y^2) + 2W_{22}\rho_y\right]$$

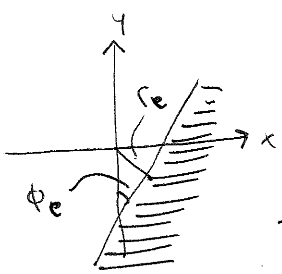
Need W_{020} term to handle defocus of knife from paraxial image plane

From page 53 of the notes we also showed

$$\delta z = -8(f/\#)^2 W_{020}$$

where δz is the distance between the mask and the paraxial image plane.

Next, we need a convenient way to describe the knife edge



r_e = distance of knife edge from optical axis

ϕ_e = orientation of knife edge

The boundary of the knife is given by

$$x\cos\phi_e - y\sin\phi_e = r_e$$

where x and y are coordinates in the mask plane.

The transmission of the mask when W_{40}, W_{31} and W_{22} are present is given by

$$T = \begin{cases} 1 & \text{for } \varepsilon_x\cos\phi_e - \varepsilon_y\sin\phi_e < r_e \\ 0 & \text{for } \varepsilon_x\cos\phi_e - \varepsilon_y\sin\phi_e \geq r_e \end{cases}$$

$$T(\rho_x,\rho_y) = \begin{cases} 1 & \text{for } \frac{-R}{r_{max}}\cos\phi_e\left[\frac{-\delta z}{4(f/\#)^2}\rho_x + 4W_{40}(\rho_x^3 + \rho_x\rho_y^2) + 2W_{31}\rho_x\rho_y\right] \\ & \quad - \frac{R}{r_{max}}\sin\phi_e\left[\frac{-\delta z}{4(f/\#)^2}\rho_y + 4W_{40}(\rho_x^2\rho_y + \rho_y^3) + W_{31}(\rho_x^2 + 3\rho_y^2) + 2W_{22}\rho_y\right] < r_e \\ 0 & \text{OTHERWISE} \end{cases}$$

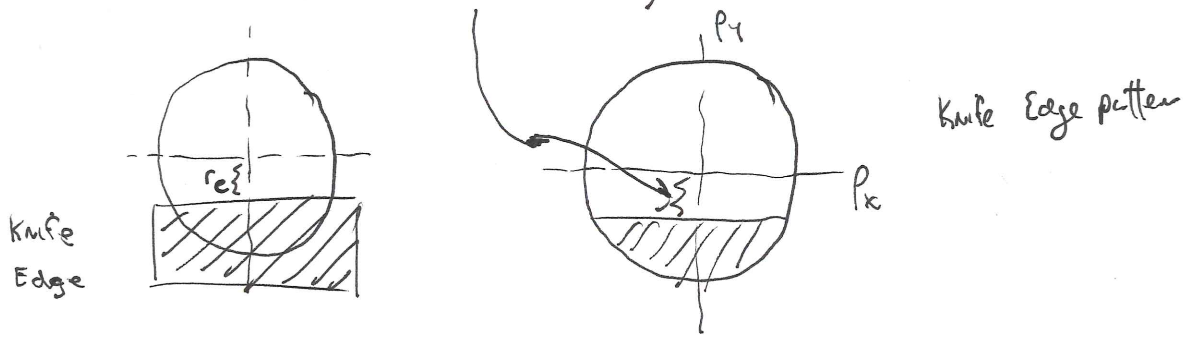
Can plot this for $\rho_x^2 + \rho_y^2 \leq 1$ i.e. over the exit pupil

Example ① $\phi_e = \frac{\pi}{2}$, Astigmatism Only

Boundary of shadow pattern occurs when

$$\frac{-R}{r_{max}}\left[\frac{-\delta z}{4(f/\#)^2}\rho_y + 2w_{22}\rho_y\right] = r_e$$

$$\rho_y = \frac{r_e}{\frac{-R}{r_{max}}\left[\frac{-\delta z}{4(f/\#)^2} + 2w_{22}\right]} \Rightarrow \text{Horizontal line}$$



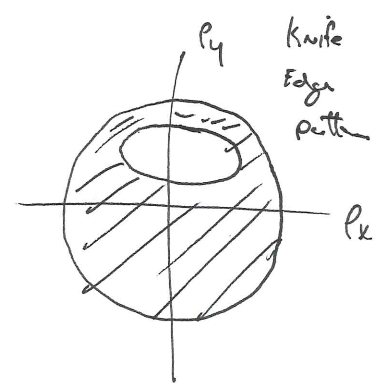
Example ② $\phi_e = \frac{\pi}{2}$, Coma only

Boundary occurs when

$$\frac{-R}{r_{max}}\left(\frac{-\delta z}{4(f/\#)^2}\rho_y + w_{31}(\rho_x^2 + 3\rho_y^2)\right) = r_e$$

$$\rho_y^2 + \frac{-\delta z}{12(f/\#)^2 w_{31}}\rho_y + \frac{\rho_x^2}{3} = -\frac{r_e r_{max}}{3Rw_{31}}$$

Define $A = \frac{+\delta z}{24(f/\#)^2 w_{31}}$; $B = \frac{-r_e r_{max}}{3Rw_{31}}$



$$\rho_y^2 - 2A\rho_y \boxed{+A^2} + \frac{\rho_x^2}{3} = B \boxed{+A^2}$$

$$\frac{(\rho_y - A)^2}{B + A^2} + \frac{\rho_x^2}{3(B+A^2)} = 1$$

Equation of ellipse centered at $+A$

semi-major axis $= \sqrt{3(B+A^2)}$

Semi-minor axis $= \sqrt{B+A^2}$

Example 3 $\phi_e = 0$, coma only

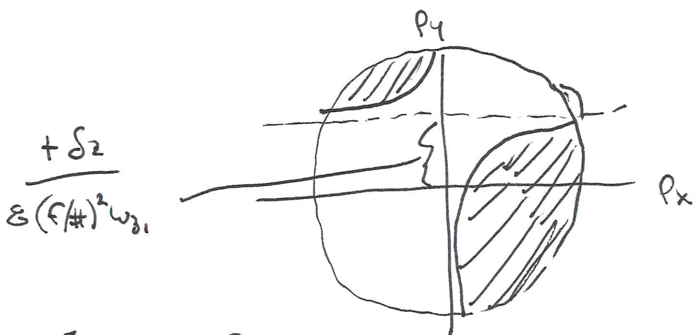
Boundary occurs where

$$\frac{-R}{r_{max}}\left[\frac{-\delta z}{4(f/\#)^2}\rho_x + 2w_{31}\rho_x\rho_y\right] = r_e$$

$$\rho_x\rho_y - \frac{\delta z}{8(f/\#)^2 w_{31}}\rho_x = -\frac{r_e r_{max}}{2Rw_{31}}$$

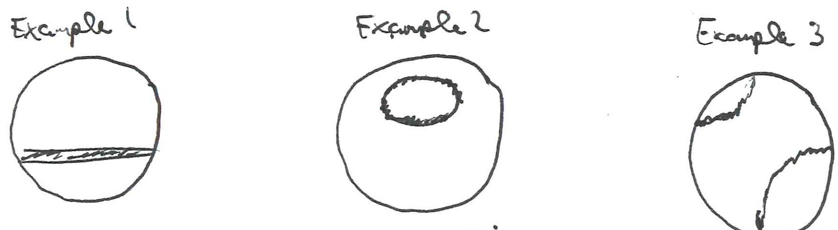
$$\rho_x\left(\rho_y - \frac{\delta z}{8(f/\#)^2 w_{31}}\right) = -\frac{r_e r_{max}}{2Rw_{31}}$$

Eq. of hyperbola with horizontal and vertical asymptotes

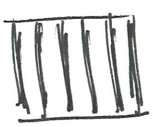


[Book 5.4.2]

$\boxed{3.2.2}$ Wire Test - Same concept as knife edge test, but thin wire used instead of knife. The patterns are similar, but only the boundaries between the light and dark regions are seen. From the previous examples, the wire test pattern would look like



$\boxed{3.2.3}$ Ronchi Test - Uses Ronchi Ruling as mask. This is like wire test with multiple wires simultaneously, which in turn is like knife edge test with with multiple values of r_1 and only the boundaries seen.



MASK

SHOW SLIDES

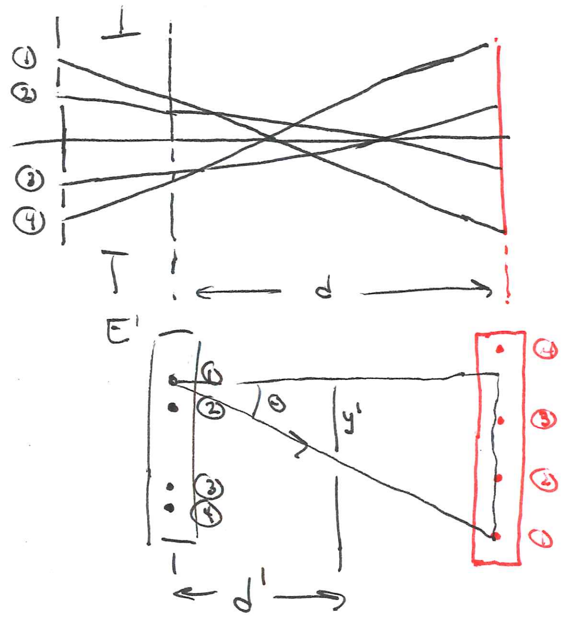
SHOW MOVIE

3.2.4 HARTMANN SCREEN TEST

- This test somewhat inverts the masking procedure seen in the previous tests. Here the mask is placed in the pupil to ~~at~~ form discrete values of ρ_x and ρ_y and then $\varepsilon_x(\rho_x, \rho_y)$, $\varepsilon_y(\rho_x, \rho_y)$ measured at planes inside and outside of focus.

Effectively, this is a spot diagram. If we capture patterns outside of the caustic, the spots can be related back to their pupil coordinates ρ_x, ρ_y. Two or more planes outside of the caustic define the ray ~~geometry~~ trajectory

SHOW SLIDES



The trajectory of ray ① is defined by d and the change in ray height Δy between the two planes.

$$\tan\theta = \frac{\Delta y}{d}$$

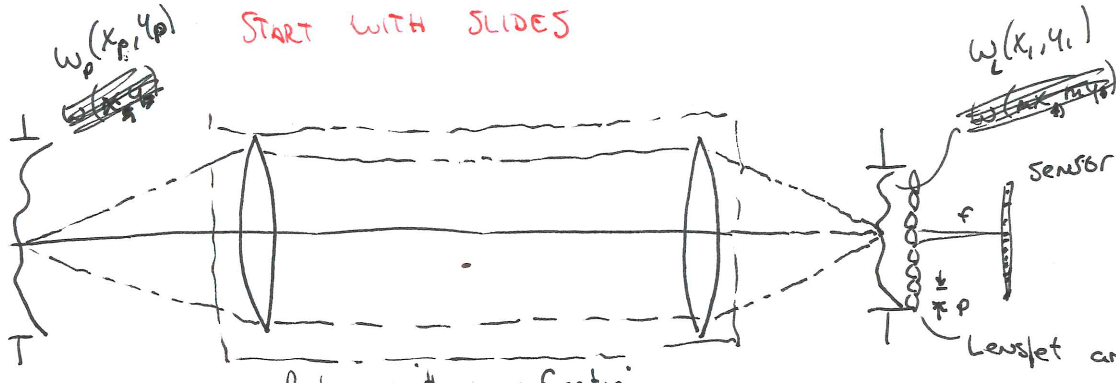
θ let's you determine the ray height for any position in space

$$y' = \frac{d'}{d}\Delta y$$

In regions where the rays cross (caustic), relating which spot belongs to which hole in the mask becomes challenging. The general technique for this test is to measure two planes outside of the caustic and the predict how the rays cross the optical axis. Good systems have the rays cross at the same location.

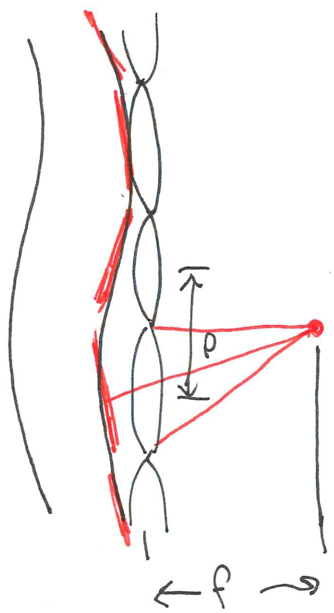
$\boxed{3.2.5}$ SHACK-HARTMANN SENSOR [Book 5.4.4]

START WITH SLIDES

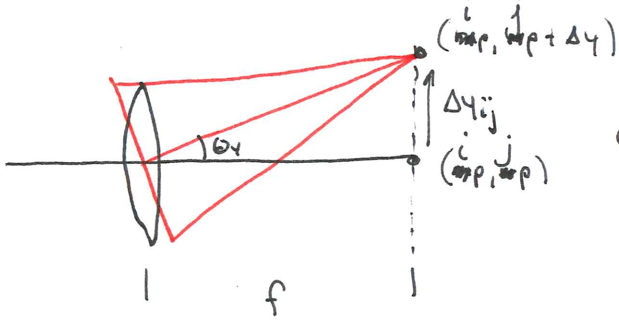


$$\boxed{x_1 = m x_p \qquad y_1 = m y_p}$$

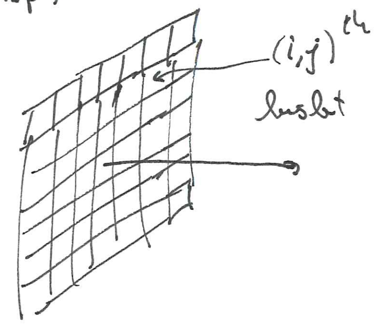
wavefront from exit pupil is relayed to lenslet array. If relay has magnification m, then a wavefront error $w_p(x_p, y_p)$ gets projected onto the lenslet array as $w_L(x_1, y_1)$. This means the wavefront is compressed in the transverse direction, but the phase remains the same.



Assume that the wavefront is slowly varying over the aperture of a given lenslet.
We can assume that the wavefront over a given lenslet is given by a tilted plane wave. The tilt is given by the local wavefront slope. A plane wave is focused to the rear focal plane of the lenslet. The focal spot, however, is displaced from the optical axis of the lenslet due to the tilt. By measuring the spot displacements, the gradient (i.e. x and y derivatives) of the wavefront error can be obtained. In turn, the gradient can be numerically integrated to recover the wavefront error.



Consider the $(i,j)^{th}$ lenslet in the array. It is centered at a point (ip, jp)



$$\tan \theta_y = \frac{\Delta y}{f}$$

but $$\tan \theta_y = -\frac{dw(ip,jp)}{dy}$$

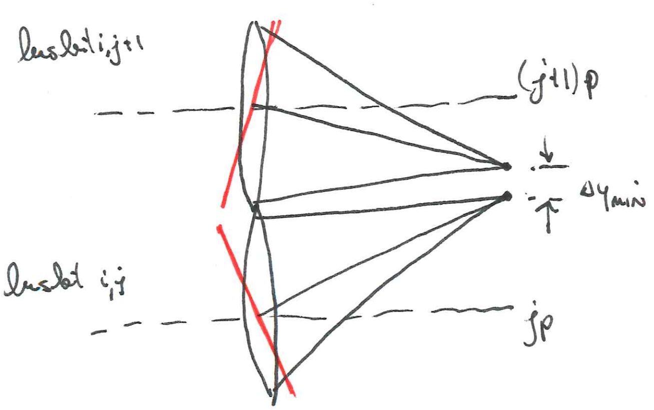
so
$$\boxed{\begin{aligned} \Delta y_{ij} &= -f \frac{dw_L}{dy_1}(ip,jp) \\ \text{Similarly} \quad \Delta x_{ij} &= -f \frac{dw_L}{dx_1}(ip,jp) \end{aligned}}$$

Practically, we don't get perfect point but instead some finite size to the focal spot. In this case, the spot centroid is used. There is a maximum change in slope that can occur between two adjacent lenslets.

Dynamic Range

MAXIMUM SLOPE CHANGE BETWEEN LENSLETS

Spots can merge or cross over if slope change is too big between lenslets



Want

$$(j+1)p + \Delta y_{i,j+1} - \left[jp + \Delta y_{i,j} \right] \geq \Delta y_{min}$$

where Δy_{min} is some minimum separation between adjacent spots

So,

$$p + \Delta y_{i,j+1} - \Delta y_{i,j} \geq \Delta y_{min}$$

$$p - f\frac{\partial w_L}{\partial y}(ip,(j+1)p) + f\frac{\partial w_L}{\partial y}(ip,jp) \geq \Delta y_{min}$$

$$\boxed{\frac{\partial w_L}{\partial y_1}(ip,(j+1)p) - \frac{\partial w_L}{\partial y_1}(ip,jp) \leq \frac{p - \Delta y_{min}}{f}}$$

MAXIMUM SLOPE CHANGE BETWEEN LENSLETS

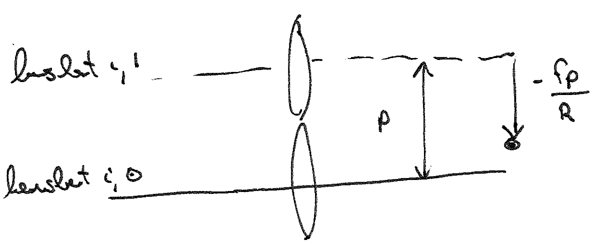
$$\frac{\frac{\partial w_L}{\partial y_1}(ip,(j+1)p) - \frac{\partial w_L}{\partial y_1}(ip,jp)}{p} \simeq \boxed{\frac{\partial^2 w_L}{\partial y_1^2}(ip,jp) \leq \frac{p - \Delta y_{min}}{pf}}$$

MAXIMUM WAVEFRONT CURVATURE

Simple Example w/ 1-D

$$W_L = \frac{y_1^2}{2R} \Rightarrow \frac{\partial w_L}{\partial y_1} = \frac{y_1}{R} \Rightarrow \frac{\partial^2 w_L}{\partial y_1^2} = \frac{1}{R}$$

Suppose $\Delta y_{min} = 0$



$$\Delta y_{i,0} = 0$$

$$\Delta y_{i,1} = -\frac{fp}{R}$$

Maximum Slope Change Says

$$\frac{p}{R} - 0 \leq \frac{p}{f} \Rightarrow \frac{1}{R} \leq \frac{1}{f} \Rightarrow R \geq f$$

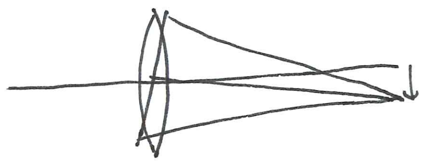
means $\Delta y_{i,1} \leq p$

MAXIMUM WAVEFRONT CURVATURE Says

$$\frac{1}{R} \leq \frac{1}{f}$$ same thing

Practically, Δy_{min} will be limited by the pixel size on the detector or the size of the focal spot (Like Rayleigh criterion how close can these spots get and still be resolved?)

Sensitivity - what's the smallest absolute slope that can be measured?



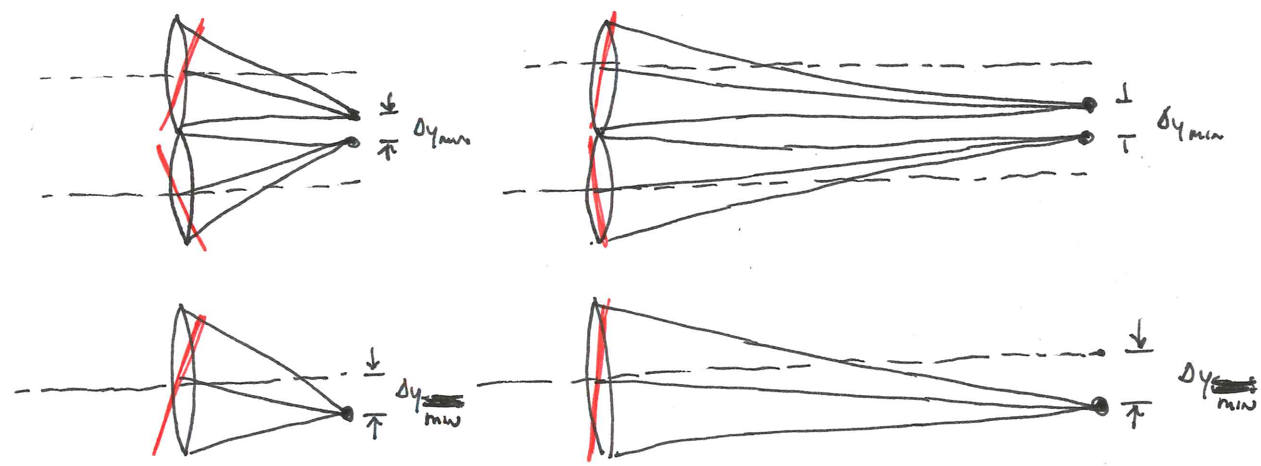
$$\Delta y_{sens} = -f \frac{d\omega_{min}}{dy_1}$$

$$\boxed{\left|\frac{\partial\omega_L(ip,jp)}{\partial y}\right|_{min} = \frac{\Delta y_{sens}}{f}}$$

SHORT FOCAL LENGTH   LONG FOCAL LENGTH

DYNAMIC RANGE

SENSITIVITY



sensitivity ↓   sensitivity ↑

dynamic range ↑   dynamic range ↓

Effect of Relay System

$\omega_p(x_p, y_p) = \omega_L(x_1, y_1)$ where $x_1 = m x_p$; $y_1 = m y_p$ m = relay magnification

$$\frac{dx_1}{dx_p} = m \; ; \; \frac{dy_1}{dy_p} = m$$

chain rule

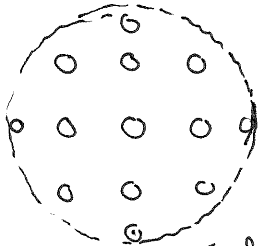
$$\frac{\partial\omega_p}{\partial x_p} = \frac{\partial\omega}{\partial x_1}\frac{\partial x_1}{\partial x_p} \Rightarrow \boxed{\begin{aligned}\frac{\partial\omega_L}{\partial x_1} &= \frac{1}{m}\frac{\partial\omega_p}{\partial x_p}\\ \frac{\partial\omega_L}{\partial y_1} &= \frac{1}{m}\frac{\partial\omega_p}{\partial y_p}\end{aligned}}$$

Slope at lenslet   slope at E'

For $m<1$, slope is steeper at lenslet array than at E'

Dynamic Range ↓

SHACK HARTMANN SPOT PATTERN



$$\mathrm{cyl}\left(\frac{r_1}{mD_{E'}}\right)\sum_{i,j}\left[\delta\left(x_1 - ip + f\frac{dW_L}{dx_1}(ip,jp),\ y_1 - jp + f\frac{dW_L}{dy_1}(ip,jp)\right) * B(x_1,y_1)\right]$$

scaled exit pupil

spot shifts due to aberrations

blur due to source size and diffraction

$$r_1^2 = x_1^2 + y_1^2$$

$$\mathrm{cyl}\left(\frac{r}{mD_{E'}}\right)\sum_{i,j} B\left(x_1 - ip + f\frac{dW_L}{dx_1}(ip,jp),\ y_1 - jp + f\frac{dW_L}{dy_1}(ip,jp)\right)$$

Centroid of $B(x_1,y_1)$ is at (d_x, d_y) where

$$d_x = \frac{\int x_1 B(x_1,y_1)\,dx_1 dy_1}{\int B(x_1,y_1)\,dx_1 dy_1} \qquad d_y = \frac{\int y_1 B(x_1,y_1)\,dx_1 dy_1}{\int B(x_1,y_1)\,dx_1 dy_1}$$

Centroids of the spots are given by

$$\mathrm{cyl}\left(\frac{r_1}{mD_{E'}}\right)\sum_{i,j}\delta\left(x_1 - ip + f\frac{dW_L}{dx_1}(ip,jp) - d_x,\ y_1 - jp + f\frac{dW_L}{dy_1}(ip,jp) - d_y\right)$$

Centroid of $B(x_1,y_1)$ shifts entire pattern by constant amount.
The preceding is only true for ~~interior~~ lenslets interior to the cyl() function.
Spots that are clipped by the cyl() function will have displaced centroids and induce slight errors.

<u>Example</u> $\quad W_p(x_p,y_p) = W_{020}(x_p^2 + y_p^2) \Rightarrow$ Defocus in Exit Pupil

$$W_L(x_1,y_1) = \frac{W_{020}}{m^2}(x_1^2 + y_1^2)$$

$$\frac{dW_L}{dx_1} = \frac{2W_{020}}{m^2}x_1 \qquad \frac{dW_L}{dy_1} = \frac{2W_{020}}{m^2}y_1$$

SPOT PATTERN

$$\mathrm{cyl}\left(\frac{r_1}{mD_{E'}}\right)\sum_{ij}\delta\left(x_1-\left(1-\frac{2W_{020}}{m^2}f\right)ip-d_x,\ y_1-\left(1-\frac{2W_{020}}{m^2}f\right)jp-d_y\right)$$

$\left[1-\frac{2W_{020}}{m^2}f\right]p$ uniformly spaced grid

when $W_{020}=0$ spots ~~fall~~ separation $=p$

when $W_{020}>0$ spots uniform but compress

when $W_{020}<0$ spots uniform but expand

$\boxed{3.2.5.1}$ Fitting Shack-Hartmann Data to Zernike Polynomials [Book 5.4.5]

STEPS TO MEASURING A WAVEFRONT WITH A SHACK HARTMANN SYSTEM

1. Calibrate system with a perfect plane wave
   This gives a set of spots for the case where $W(x_1,y_1)=0$
   Only need to do this once
2. Measure the test system
   This gives a set of spots that are displaced by the aberrations of the system
3. Measure the distance between the ideal and aberrated spots to get
   $\{\Delta x_{ij}, \Delta y_{ij}\}_{ij}$
4. Convert ~~this~~ this set to $\left\{ \frac{\partial W}{\partial x_1}^{(ip,jp)}, \frac{\partial W}{\partial y}^{(ip,jp)} \right\}_{ij}$ with eqs. from pg. $\boxed{129}$

Want to integrate these slopes to get $W(x_1,y_1)$ back

One way: to do this is to fit to Zernike polynomials

Before we did $W(\bar{x},\bar{y}) = \sum a_{nm} Z_n^m(\bar{x},\bar{y})$

$$\frac{\partial W(\bar{x},\bar{y})}{\partial \bar{x}} = \sum a_{nm}\frac{\partial Z_n^m(\bar{x},\bar{y})}{\partial \bar{x}} \qquad \frac{\partial W(\bar{x},\bar{y})}{\partial \bar{y}} = \sum a_{nm}\frac{\partial Z_n^m(\bar{x},\bar{y})}{\partial \bar{y}}$$

Set up a matrix system

$$
\underbrace{\begin{pmatrix}
\cancel{\frac{dZ_0^0(\bar{x}_1,\bar{y}_1)}{d\bar{x}}} & \frac{dZ_1^{-1}(\bar{x}_1,\bar{y}_1)}{d\bar{x}} & \cdots & \frac{dZ_N^m(\bar{x}_1,\bar{y}_1)}{d\bar{x}} \\
\vdots & \vdots & & \vdots \\
\cancel{\frac{dZ_0^0(\bar{x}_N,\bar{y}_N)}{d\bar{x}}} & \frac{dZ_1^{-1}(\bar{x}_N,\bar{y}_N)}{d\bar{x}} & \cdots & \frac{dZ_N^m(\bar{x}_N,\bar{y}_N)}{d\bar{x}} \\
\cancel{\frac{dZ_0^0(\bar{x}_1,\bar{y}_1)}{d\bar{y}}} & \frac{dZ_1^{-1}(\bar{x}_1,\bar{y}_1)}{d\bar{y}} & \cdots & \frac{dZ_N^m(\bar{x}_{N+1},\bar{y}_{N+1})}{d\bar{y}} \\
\vdots & \vdots & & \vdots \\
\cancel{\frac{dZ_0^0(\bar{x}_N,\bar{y}_N)}{d\bar{y}}} & \frac{dZ_1^{-1}(\bar{x}_N,\bar{y}_N)}{d\bar{y}} & \cdots & \frac{dZ_N^m(\bar{x}_N,\bar{y}_N)}{d\bar{y}}
\end{pmatrix}}_{\vec{A}}
\underbrace{\begin{pmatrix}
\cancel{} \\ a_{1,-1} \\ a_{1,1} \\ \vdots \\ a_{NM}
\end{pmatrix}}_{\vec{x}}
=
\underbrace{\begin{pmatrix}
\frac{dw(\bar{x}_1,\bar{y}_1)}{d\bar{x}} \\
\frac{dw(\bar{x}_2,\bar{y}_2)}{d\bar{x}} \\
\vdots \\
\frac{dw(\bar{x}_N,\bar{y}_N)}{d\bar{x}} \\
\frac{dw(\bar{x}_1,\bar{y}_1)}{d\bar{y}} \\
\vdots \\
\frac{dw(\bar{x}_N,\bar{y}_N)}{d\bar{y}}
\end{pmatrix}}_{\vec{b}}
$$

2N rows

DON'T USE $\frac{dZ_0^0}{dx}$ or $\frac{dZ_0^0}{dy}$ because it gives a column of zeroes.

$\vec{D}\vec{A} = \vec{w}$ $\quad \vec{A}\vec{x} = \vec{b}$ $\quad \bar{x}, \bar{y}$ Normalized coordinates

$$\vec{x} = \left[\vec{A}^T \vec{A}\right]^{-1} \vec{A}^T \vec{b}$$

NOTE: The coordinates $\bar{x}, \bar{y}$ are normalized coordinates

$$\bar{x} = \frac{x_1}{\left[\frac{mD_E'}{2}\right]} \qquad \bar{y} = \frac{y_1}{\left[\frac{mD_E'}{2}\right]}$$

$$\frac{dZ_N^m}{d\bar{x}} = \frac{dZ_N^m}{dx}\frac{dx}{d\bar{x}} = \frac{mD_E'}{2}\frac{dZ_N^m}{dx}$$

Similar for $\frac{dZ_N^m}{d\bar{y}}$ and $\frac{dw}{d\bar{x}}$ $\frac{dw}{d\bar{y}}$

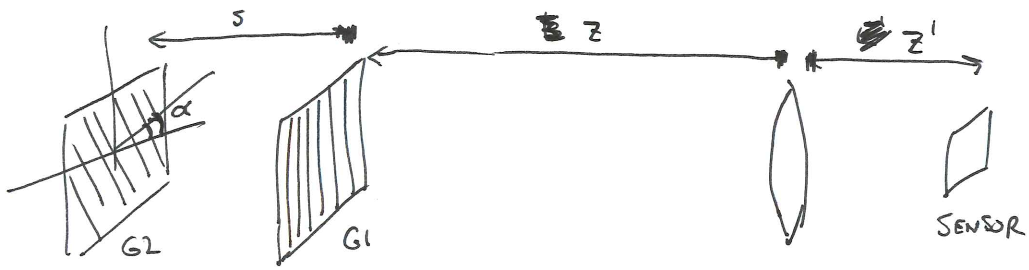
NOTE IN BOOK SECTION 5.4.5 unnormalized coordinates are used instead. Just be consistent between both sides of matrix equations

$\boxed{3.2.6}$

Moiré Deflectometry

[Book 5.4.6]

Show slides / demo



Moiré Deflectometry is a technique in which a Moiré pattern is created between two gratings G1 and G2 and imaged by a camera. A test lens can be placed in the path ~~to detem~~ and it power determined from the effect it has on the Moiré Pattern

Let $m_1 = \frac{z'}{z}$ magnification of G1 on sensor

$m_2 = \frac{z'}{z+s}$ magnification of G2 on sensor

For sinusoidal gratings, the ~~pattern~~ ~~transmissions~~ transmissions of the gratings are given by

$$\left.\begin{aligned} T_{G1} &= \tfrac{1}{2}\left(1 + \cos\left(\frac{2\pi x}{m_1 g_1}\right)\right) \\ T_{G2} &: \tfrac{1}{2}\left(1 + \cos\left(\frac{2\pi}{m_2 g_2}(x\cos\alpha + y\sin\alpha)\right)\right) \end{aligned}\right\} \text{ on sensor plane.}$$

where α is rotation of G2 relative to G1

Perceptually, we see a "dark" fringe when both T_{G1} and T_{G2} are zero, and a "bright" fringe when $T_{G1} = T_{G2} = 1$. There is no interference here. To capture this effect, we multiply T_{G1} and T_{G2}.

$$T_{G1} \cdot T_{G2} = \frac{1}{4}\left(1 + \cos\phi_1 + \cos\phi_2 + \cos\phi_1 \cos\phi_2\right)$$

where $\phi_1 = \frac{2\pi x}{m_1 g_1}$ and $\phi_2 = \frac{2\pi}{m_2 g_2}(x\cos\alpha + y\sin\alpha)$

Let $\phi_1 = \frac{\alpha_0 + \beta_0}{2}$ and $\phi_2 = \frac{\alpha_0 - \beta_0}{2}$ where $\alpha_0 = \phi_1 + \phi_2$; $\beta_0 = \phi_1 - \phi_2$

and use $\frac{1}{2}\left[\cos\alpha_0 + \cos\beta_0\right] = \cos\left(\frac{\alpha_0+\beta_0}{2}\right)\cos\left(\frac{\alpha_0-\beta_0}{2}\right)$

$$T_{G1} \cdot T_{G2} = \frac{1}{4}\left[1 + \cos\phi_1 + \cos\phi_2 + \frac{1}{2}\cos(\phi_1+\phi_2) + \frac{1}{2}\cos(\phi_1 - \phi_2)\right]$$

The last term represents the low frequency Moiré fringes we perceive.

$$\phi_1 - \phi_2 = \cancel{2\pi}\left(\frac{1}{m_1 g_1} - \frac{\cos\alpha}{m_2 g_2}\right)x - \cancel{2\pi}\frac{\sin\alpha}{m_2 g_2}y = \cancel{2}K\cancel{\pi}$$

K integer for peaks

Equation of a line

$$y = \left[\frac{\frac{1}{m_1 g_1} - \frac{\cos\alpha}{m_2 g_2}}{\frac{\sin\alpha}{m_2 g_2}}\right]x - \frac{K\, m_2 g_2}{\sin\alpha}$$

The slope of this line is

$$m_0 = \tan\theta_0 = \frac{\frac{m_2 g_2}{m_1 g_1} - \cos\alpha}{\sin\alpha}$$

if $g_1 = g_2$

$$\tan\theta_0 = \frac{\frac{Z}{Z+s} - \cos\alpha}{\sin\alpha}$$

What happens if we place a lens of power ϕ at G1?

m_1 and g_1 are unchanged

m_2 and g_2 change.

The new G2' is basically the image of G2 through the test lens

Use image location for s as above and scale g_2 by the magnification

$$\frac{1}{s'} - \frac{1}{s} = \phi$$

$$\frac{1}{s'} = \frac{s\phi}{s} + \frac{1}{s}$$

$$s' = \frac{s}{1+s\phi} \qquad m_s = \frac{s'}{s} = \frac{1}{1+s\phi}$$

So G2 appears to be at $s \to \frac{s}{1+s\phi}$ and $g_2 \to \frac{g_2}{1+s\phi}$

$$\frac{m_2 g_2}{m_1 g_1} \Rightarrow \frac{z}{z + \frac{s}{1+s\phi}} \cdot \frac{g_2}{1+s\phi} \frac{1}{g_1}$$

$$\frac{m_2 g_2}{m_1 g_1} \Rightarrow \frac{z}{z(1+s\phi)+s} \frac{g_2}{g_1}$$

$$\boxed{\tan\Theta_0 = \frac{\frac{z}{z(1+s\phi)+s}\frac{g_2}{g_1} - \cos\alpha}{\sin\alpha}}$$

SHOW MOIRÉ SLIDE WITH LENSES

FRINGES ROTATE AS FUNCTION OF ϕ

Can invert this function so that

$$\boxed{\phi = \frac{1}{s(\sin\alpha\tan\Theta_0 + \cos\alpha)} - \frac{1}{z} - \frac{1}{s}}$$

POWER OF LENS GIVEN L, s, α and Θ_0