1.6 Optical Quality Metrics

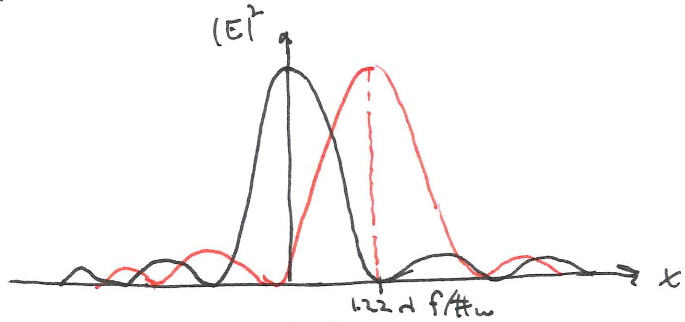
1.6.1 Resolution Targets

[Book 3.4]

Resolution Targets - The resolution of an optical system describes how close two points can be reliably distinguished. Resolution Targets have a series of patterns in which the resolution limit can be determined by analyzing the image of the target.

1.6.1.1 Rayleigh Criterion [Book 3.4.1]

Ultimately, the best we can hope to achieve in terms of resolution occurs for a diffraction limited system. In this case, the image of two points become two Airy disks.



The Rayleigh criterion says that two points can be resolved if the peak of one Airy disk falls on the first zero of the other Airy disk. Thus

$$\boxed{\text{Resolution} = 1.22\lambda f/\#_w}$$

Often we are interested in angular resolution (e.g. what is the angular separation between two stars that can just be resolved). In this case

$$\boxed{\text{Angular Resolution} = \frac{1.22\lambda}{D_E}}$$

radians

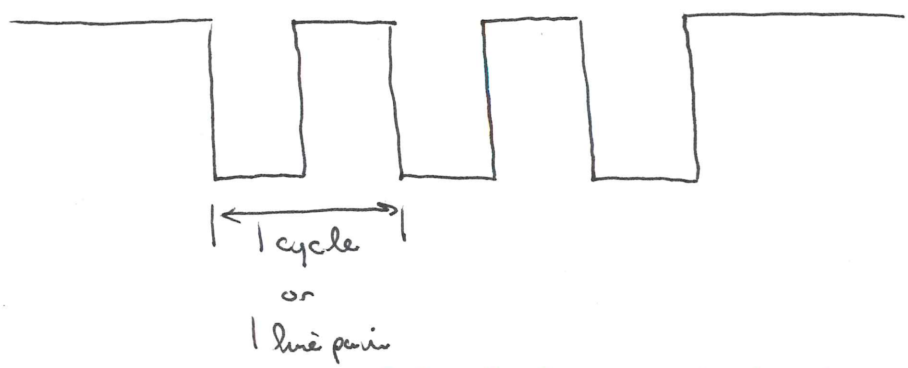
where D_E is entrance pupil diameter

multiply by $\frac{10800}{\pi}$ for arcmin

Examples of Resolution Targets include

1951 USAF TARGET [Book 3.4.2]

- 3-bar patterns of progressively diminishing size. Composed of Groups ~~of~~ of 6 Elements. Each element has standardized spatial frequency in cycles/mm or line pair/mm (lp/mm)



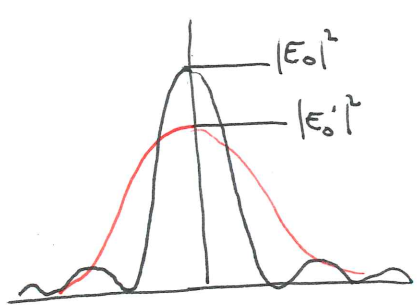
NEED TO ACCOUNT FOR SYSTEM MAGNIFICATION

IEEE RESOLUTION TARGET - CONFORMS TO STD 208-1995 "Measurement of Resolution of Camera Systems"

ISO Camera Resolution Target - conforms to Standard ISO-12233

[Book 3.5.1]

$\boxed{1.6.2}$ STREHL RATIO - This is a single numeric metric for describing the quality of optical systems with small aberrations. It is the ratio of the peak of the PSF to the peak of the Airy disk. ~~It is a reasonable metric down to about 0.8~~

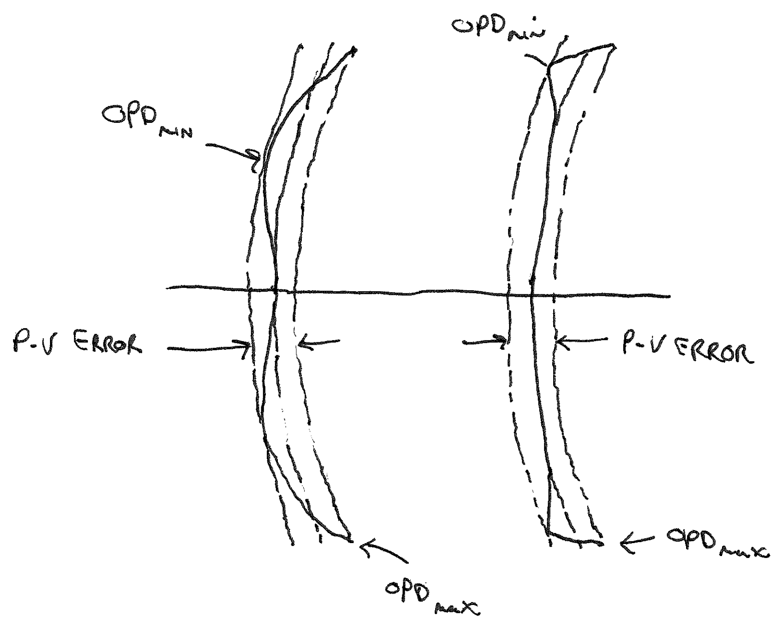


$$\text{STREHL RATIO} = \frac{|E_0'|^2}{|E_0|^2}$$

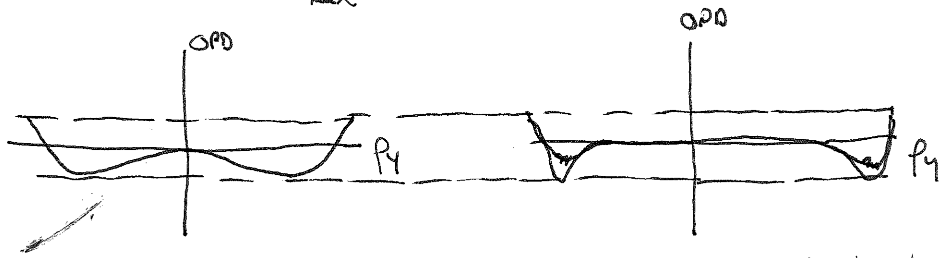
$$\text{STREHL RATIO} = \frac{1}{\pi^2}\left|\int_0^{2\pi}\int_0^1 e^{i\frac{2\pi}{\lambda}W(\rho,\theta)}\rho\,d\rho\,d\theta\right|^2$$

1.6.3 Peak-To-Valley and rms Wavefront Error [Book 3.5.2]

Peak-To-Valley wavefront error is a metric of system performance. It is defined as $OPD_{max} - OPD_{min}$ regardless of what occurs in the wavefront shape



Both of these wavefronts have the same P-V error, but the one on the right is smoother except at the very edges. P-V tends to underestimate performance for systems with localized errors.



The Rayleigh Criterion is roughly equivalent to P-V Error $\leq \frac{\lambda}{4}$

RMS wavefront Error better captures the "smoothness" of the wavefront. To get RMS wavefront Error, we need to define several quantities first

Wavefront Variance - $\sigma_w^2 = \langle W^2 \rangle - \langle W \rangle^2$

$$\sigma_w^2 = \underbrace{\frac{1}{\pi}\int_0^{2\pi}\int_0^1 W^2(\rho,\theta)\,\rho\,d\rho\,d\theta}_{\text{Average value of square of wavefront}} - \Bigg[\underbrace{\frac{1}{\pi}\int_0^{2\pi}\int_0^1 W(\rho,\theta)\,\rho\,d\rho\,d\theta}_{\text{Average value of wavefront}}\Bigg]^2$$

RMS Wavefront Error $= \sigma_w = \sqrt{\sigma_w^2}$

Example 1 $\quad W(\rho,\theta) = \rho^4$

$$\langle W\rangle = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \rho^5 \, d\rho \, d\theta = 2\left[\frac{\rho^6}{6}\right]_0^1 = \frac{1}{3}$$

$$\langle W\rangle^2 = \frac{1}{9}$$

$$\langle W^2\rangle = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \rho^9 \, d\rho \, d\theta = 2\left(\frac{\rho^{10}}{10}\right)_0^1 = \frac{1}{5}$$

$$\sigma_W^2 = \frac{1}{5} - \frac{1}{9} = \frac{4}{45}$$

Example 2 $\quad W(\rho,\theta) = \rho^4 - \rho^2$

$$\langle W\rangle = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 (\rho^5 - \rho^3)\, d\rho \, d\theta = 2\left(\frac{\rho^6}{6} - \frac{\rho^4}{4}\right)_0^1 = -\frac{1}{6}$$

$$\langle W\rangle^2 = \frac{1}{36}$$

$$\langle W^2\rangle = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 (\rho^9 - 2\rho^7 + \rho^5)\, d\rho \, d\theta = 2\left[\frac{\rho^{10}}{10} - \frac{2\rho^8}{8} + \frac{\rho^6}{6}\right]$$

$$= \left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) = \left[\frac{6}{30} - \frac{15}{30} + \frac{10}{30}\right] = \frac{1}{30}$$

$$\sigma_W^2 = \frac{1}{30} - \frac{1}{36} = \frac{1}{180}$$

Example 1	Example 2
$\sigma_W^2 = \frac{4}{45} = \frac{16}{180}$	$\sigma_W^2 = \frac{1}{180}$

adding defocus reduced σ_W^2 by ~~more than~~ a factor of 16.

$\boxed{1.6.3.1}$ Relationship to Zernike Coefficients

Suppose we represent wavefront with Zernike polynomials

$$W = \sum a_{NM} Z_N^m(\rho, \theta)$$

$$\langle W \rangle = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \sum a_{NM} Z_N^m(\rho, \theta)\, \rho\, d\rho\, d\theta$$

$$\langle W \rangle = \frac{1}{\pi} \sum a_{NM} N_N^m \int_0^1 R_N^m(\rho)\, \rho\, d\rho \int_0^{2\pi} \left\{ \begin{array}{ll} 1 & \text{for } m = 0 \\ \cos m\theta & \text{for } m > 0 \\ \sin m\theta & \text{for } m < 0 \end{array} \right\} d\theta$$

$$\int_0^{2\pi} d\theta = 2\pi \quad \text{for } m = 0 \qquad \int_0^{2\pi} \cos m\theta\, d\theta = \left. \frac{\sin m\theta}{m} \right|_0^{2\pi} = 0 \quad \text{for } m > 0;\ m \text{ integer}$$

$$\int_0^{2\pi} \sin m\theta\, d\theta = \left. \frac{-\cos m\theta}{m} \right|_0^{2\pi} = 0 \quad \text{for } m < 0;\ m \text{ integer}$$

So $\langle W \rangle = 0$ for $m \neq 0$

What happens when $m = 0$

$$\langle W \rangle = 2 \sum a_{N0} N_N^0 \int_0^1 R_N^0(\rho)\, \rho\, d\rho$$

$$\int_0^1 R_0^0(\rho)\, \rho\, d\rho = \int_0^1 \rho\, d\rho = \frac{1}{2} \qquad N = 0$$

$$\int_0^1 R_2^0(\rho)\, \rho\, d\rho = \int_0^1 (2\rho^3 - \rho)\, d\rho = \frac{2}{4} - \frac{1}{2} = 0 \qquad N = 2$$

$$\int_0^1 R_4^0(\rho)\, \rho\, d\rho = \int_0^1 (6\rho^5 - 6\rho^3 + \rho)\, d\rho = \frac{6}{6} - \frac{6}{4} + \frac{1}{2} = 0 \qquad N = 4$$

In general,

$$\int_0^1 R_N^0(\rho)\, \rho\, d\rho = 0 \quad \text{for } N \neq 0$$

So $\langle w \rangle = 0$ unless $N=0$ and $m=0$

$$\langle w \rangle = 2\, a_{00}\, N_0^0\, \frac{1}{2} = a_{00}$$

piston coefficient describes the average value of the wavefront

wavefront variance

$$\sigma_w^2 = \langle w^2 \rangle - a_{00}^2$$

Now look at $\langle w^2 \rangle$

$$\langle w^2 \rangle = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 \left[\sum a_{NM}\, Z_N^m(\rho,\theta) \right] \left[\sum a_{N'M'}\, Z_{N'}^{m'}(\rho,\theta) \right] \rho\, d\rho\, d\theta$$

$$\langle w^2 \rangle = \cancel{\frac{1}{\pi}} \; \frac{1}{\pi} \sum a_{NM} \sum a_{N'M'} \underbrace{\int_0^{2\pi} \int_0^1 Z_N^m(\rho,\theta)\, Z_{N'}^{m'}(\rho,\theta)\, \rho\, d\rho\, d\theta}_{\text{This is orthogonality condition!}}$$

$$\langle w^2 \rangle = \frac{1}{\pi} \sum a_{NM} \sum a_{N'M'}\, \pi\, \delta_{N'N}\, \delta_{M'M}$$

$$\langle w^2 \rangle = \sum a_{NM}^2$$

So

$$\cancel{\sigma_w^2 = \langle w^2 \rangle - a_{00}^2}$$

$$\sigma_w^2 = \sum a_{NM}^2 - a_{00}^2$$

$$\boxed{\sigma_w^2 = \sum_{N \geq 1} a_{NM}^2}$$

Strictly positive terms since coefficients of squared. This means eliminating or reducing a given $|a_{ij}|$ reduces σ_w^2

11.6.3.2 Relationship to ~~Zernike Coefficients~~ Strehl Ratio [Book 3.5.4]

$$\text{Strehl Ratio} = \frac{1}{\pi^2}\left|\int_0^{2\pi}\!\!\int_0^1 e^{i\frac{2\pi}{\lambda}w(\rho,\theta)}\,\rho\,d\rho\,d\theta\right|^2$$

$$e^x = \sum_{N=0}^{\infty}\frac{x^N}{N!} = 1 + x + \frac{x^2}{2!} + \cdots$$

$$\text{Strehl Ratio} \simeq \frac{1}{\pi^2}\left|\int_0^{2\pi}\!\!\int_0^1\left(1 + \frac{i2\pi}{\lambda}w(\rho,\theta) - \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)^2 w^2(\rho,\theta)\right)\rho\,d\rho\,d\theta\right|^2$$

$$\simeq \frac{1}{\pi^2}\left|\int_0^{2\pi}\!\!\int_0^1 \rho\,d\rho\,d\theta + \frac{i2\pi}{\lambda}\int_0^{2\pi}\!\!\int_0^1 w(\rho\theta)\,\rho\,d\rho\,d\theta - \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)^2\int_0^{2\pi}\!\!\int_0^1 w^2(\rho,\theta)\,\rho\,d\rho\,d\theta\right|^2$$

$$\simeq \frac{1}{\pi^2}\left|\pi + \frac{i2\pi^2}{\lambda}\langle w\rangle - \frac{\pi}{2}\left(\frac{2\pi}{\lambda}\right)^2\langle w^2\rangle\right|^2$$

$$\simeq \frac{1}{\pi^2}\left[\left(\pi - \frac{\pi}{2}\left(\frac{2\pi}{\lambda}\right)^2\langle w^2\rangle\right)^2 - \pi^2\left(\frac{2\pi}{\lambda}\right)^2\langle w\rangle^2\right]$$

$$\simeq \frac{1}{\pi^2}\left[\pi^2 - \pi^2\left(\frac{2\pi}{\lambda}\right)^2\langle w^2\rangle + \overset{\text{small}}{\cancel{\frac{\pi^2}{4}\left(\frac{2\pi}{\lambda}\right)^4\langle w^2\rangle^2}} - \pi^2\left(\frac{2\pi}{\lambda}\right)^2\langle w\rangle^2\right]$$

$$\simeq 1 - \left(\frac{2\pi}{\lambda}\right)^2\left(\langle w^2\rangle - \langle w\rangle^2\right)$$

$$\text{Strehl Ratio} \simeq 1 - \left(\frac{2\pi}{\lambda}\right)^2\sigma_w^2$$

This approximation only good for small variances. A better approximation occurs by recognizing these are the 1st two terms of a Taylor series expansion of exp( )

$$\text{Strehl Ratio} \simeq \exp\left[-\left(\frac{2\pi}{\lambda}\right)^2\sigma_w^2\right]$$

From Wyant's Notes

Fraction of Encircled Energy

$$\text{FractionOfEncircledEnergy} = \frac{1}{\text{totalEnergy}} \int_0^{2\pi} \int_0^r i[w]\, w\, dw\, d\phi$$

$$= \frac{1}{E_A \frac{\pi d^2}{4}} \frac{E_A \pi^2 d^2}{16 \lambda^2 (f\#)^2} \int_0^{2\pi} \int_0^r \left(\frac{2 J_1[\pi w / (\lambda f\#)]}{\pi w / (\lambda f\#)} \right)^2 w\, dw\, d\phi$$

Let

$$x = \frac{\pi}{\lambda f\#} w$$

$$\boxed{\text{FractionOfEncircledEnergy} = 2 \int_0^{\frac{\pi r}{\lambda f\#}} \frac{J_1[x]^2}{x} dx}$$

Well known recurrence relation

$$\frac{d}{dx}\left(x^{n+1} J_{n+1}[x]\right) = x^{n+1} J_n[x] \qquad (1)$$

or

$$\frac{d}{dx}\left(x^{-n} J_n[x]\right) = -x^{-n} J_{n+1}[x] \qquad (2)$$

Note

$$J_{-n}[x] = (-1)^n J_n[x]$$

From Eq. 1

$$(n+1)\, x^n J_{n+1}[x] + x^{n+1} \frac{d}{dx} J_{n+1}[x] = x^{n+1} J_n[x]$$

Let n=0

$$J_1[x] + x \frac{d}{dx} J_1[x] = x J_0[x]$$

$$\frac{J_1^2[x]}{x} = J_0[x] J_1[x] - J_1[x] \frac{d}{dx} J_1[x]$$

But from Eq. 2

$$\frac{d}{dx} J_0[x] = -J_1[x]$$

Therefore

98

$$\frac{J_1^2[x]}{x} = -\frac{1}{2}\frac{d}{dx}\left(J_0^2[x] + J_1^2[x]\right)$$

Remembering that $J_0[0] = 1$ and $J_1[0] = 0$

$$\boxed{\text{FractionOfEncircledEnergy} = 1 - J_0\left[\frac{\pi r}{\lambda f\#}\right]^2 - J_1\left[\frac{\pi r}{\lambda f\#}\right]^2}$$

SHOW ENCIRCLED ENERGY PLOTS

Ensquared Energy is a similar concept except the energy is integrated over a square region. Sometimes used in systems in which sensor pixels are the relevant unit. Also easier to calculate for numerical data.

1.6.5 | Optical Transfer Function (OTF) [Book 3.6]

Up to now, the image quality metrics have been mainly geared towards characteristics of the PSF either directly or indirectly

Direct

Resolution - How close can two PSFs get and still be distinguished?

Strehl Ratio - What's the height of the PSF relative to the Airy disk? To conserve energy height of PSF must go down as PSF spreads out.

Encircled / Ensquared Energy - Again how spread out is the PSF.

Indirect

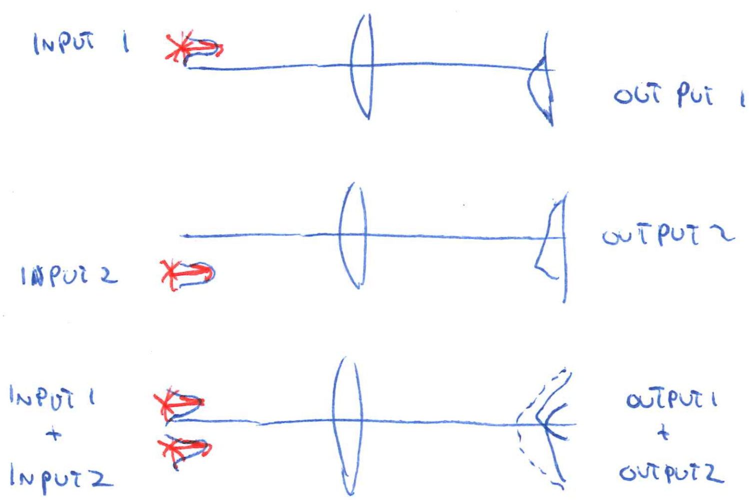
Wavefront Variance - How wiggly is the wavefront relative to reference sphere? We were able to relate this back to Strehl ratio

Instead of measuring spread, the OTF is a measure of contrast and how it is degraded by the optical system. Often in our imaging systems, we are interested in targets well below the resolution limit but with limited contrast relative to the background.

SHOW AERIAL PHOTOS

OTF calculations assume linear shift-invariant (LSI) systems

Linear means the output of a system with multiple inputs is just the sum of the outputs for each individual input.

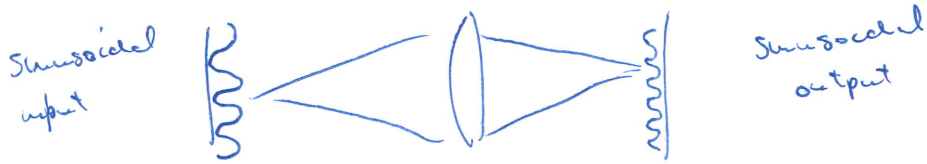


What is example where linearity breaks down? Saturation

Shift-invariant means PSF stays the same regardless of the transverse position of the input.

In general, this is not true for optical systems, but is approximately true over localized regions called isoplanatic regions.

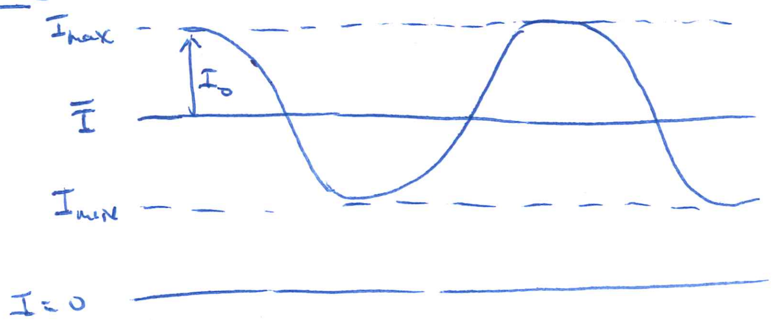
For an LSI system, if we put in a sinusoidal pattern, the output is sinusoidal too



Several things happen in this process

- spatial frequency of output is scaled by $\frac{1}{m}$; m = system magnification
- Contrast of sinusoid is reduced
- phase shift is possible

Contrast

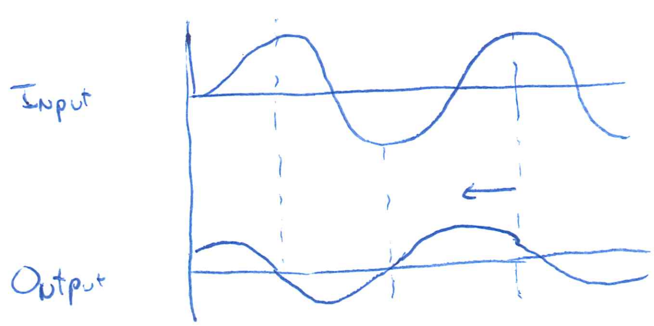


$$\text{Contrast} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2I_0}{2\bar{I}} = \frac{I_0}{\bar{I}}$$

Contrast = 1 when $I_{min} = 0 \Rightarrow \bar{I} = I_0$

Contrast = 0 when $I_{max} = I_{min} \Rightarrow I_0 = 0$

Phase Shift



Peaks and valleys of output don't line up with input.

OTF is complex function that describes how both the contrast and the phase is modified by the LSI system.

$\boxed{1.6.5.1}$ Modulation Transfer Function (MTF) [Book 3.6.1]

The MTF describes the contrast reduction

$$MTF = |OTF|$$

In general MTF is a function of spatial frequency

Cutoff frequency $= \frac{1}{\lambda f/\#}$ above this frequency ~~on~~ $MTF = 0$

$\boxed{1.6.5.2}$ Phase Transfer Function (PTF) [Book 3.6.2]

The PTF describes the phase shift that occurs

$$PTF = \mathrm{Arg}|OTF| \quad \text{where} \quad \mathrm{Arg}[A e^{i\theta}] = \theta$$

Again PTF is a function of spatial frequency

We can write $OTF = MTF\, e^{i(PTF)}$

Show why phase is important

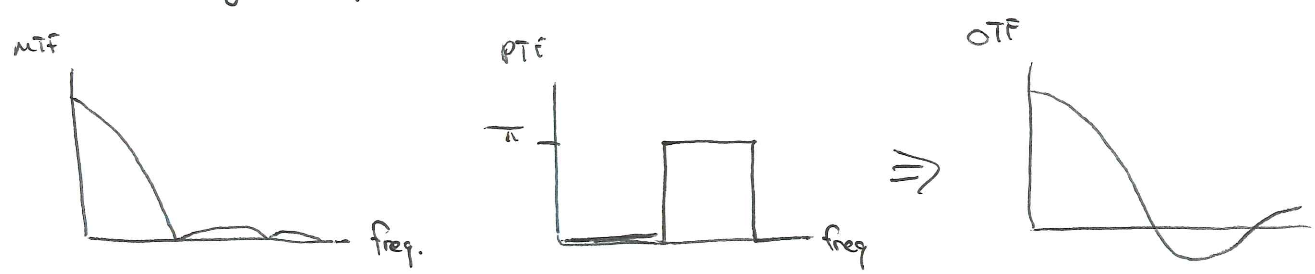
PTF of rotational symmetry (i.e. defocus, astigmatism) ~~symmetric system~~ can only 0 or π

~~π~~ 0 means we have $e^{i0} = 1$

π means we have $e^{i\pi} = -1$

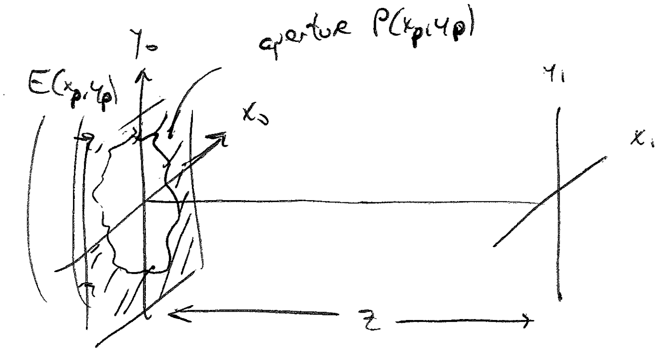
all other cases complex

with rotational symmetry



[1.6.5.3] Fourier Transform Relationship to PSF [BOOK 3.6.3]

Way back in section 1.5.3, we used Fresnel diffraction to show



Electric field

$$E(x_{img},y_{img}) = \frac{e^{ikz}}{i\lambda z}\, e^{\frac{i\pi}{\lambda z}(x_{img}^2+y_{img}^2)}\, \mathcal{F}\left\{ P(x_p,y_p)\,E(x_p,y_p)\, e^{\frac{i\pi}{\lambda z}(x_p^2+y_p^2)} \right\}_{\substack{\xi_{img}=\frac{x_{img}}{\lambda z}\\ \eta_{img}=\frac{y_{img}}{\lambda z}}}$$

where $\mathcal{F}\{f(x,y)\} = F(\xi_{img},\eta_{img})$ = Fourier Transform

Irradiance

$$PSF = I(x_{img},y_{img}) = |E(x_{img},y_{img})|^2 = \frac{1}{(\lambda z)^2}\left|\mathcal{F}\left\{P(x_p,y_p)\,E(x_p,y_p)\,e^{\frac{i\pi}{\lambda z}(x_p^2+y_p^2)}\right\}\right|^2_{\substack{\xi_{img}=\frac{x_{img}}{\lambda z}\\ \eta_{img}=\frac{y_{img}}{\lambda z}}}$$

This is the Point Spread Function of our optical system when

1. $P(x_p,y_p)$ describes the amplitude transmission of the exit pupil (often cyl())
2. $E(x_p,y_p) = e^{i\frac{2\pi}{\lambda}W(x_p,y_p)}\, e^{-\frac{i\pi}{\lambda R}(x_p^2+y_p^2)}$ where $W(x_p,y_p)$ is the wavefront error and the quadratic term describes the phase of the reference sphere
3. $z = R$ where R is the radius of the reference sphere.

$$\boxed{PSF(x_{img},y_{img}) = \frac{1}{(\lambda R)^2}\left|\mathcal{F}\left\{P(x_p,y_p)\,e^{-\frac{i2\pi}{\lambda}W(x_p,y_p)}\right\}\right|^2_{\substack{\xi_{img}=\frac{x_{img}}{\lambda R}\\ \eta_{img}=\frac{y_{img}}{\lambda R}}}}$$

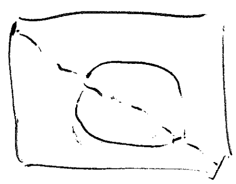
Complex Pupil Function : $P_{E1}(x_p,y_p)\, e^{-i\frac{2\pi}{\lambda} w(x_p,y_p)} = P(x_p,y_p)$

The OTF is given by

$$OTF(\xi_{img},\eta_{img}) = \mathcal{F}\{PSF(x_{img},y_{img})\}\Big|_{\text{normalized to 1 for } \xi_{img}=\eta_{img}=0}$$

Three properties (1) $OTF(0,0) = 1$ Basically a statement about conservation of energy $\xi_{img}=\eta_{img}=0$ means spatial frequency of zero ⇒ constant. If you put a constant object into a LSI system you get a constant image.

(2) $OTF(-\xi_{img},-\eta_{img}) = OTF^{*}(\xi_{img},\eta_{img})$ ⇒ ~~Hermitian~~ symmetry about diagonal



(3) $|OTF(\xi_{img},\eta_{img})| \leq |OTF(0,0)|$ ⇒ means always less than one for non-zero spatial frequencies

$\boxed{1.6.5.4}$ Autocorrelation of Pupil Function

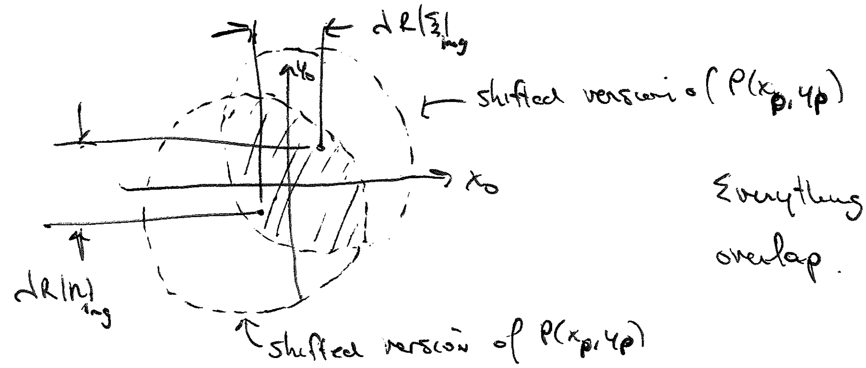
The OTF is also given by

$$OTF(\xi_{img},\eta_{img}) = \frac{\displaystyle\iint_{-\infty}^{\infty} P\left(x_p + \frac{\lambda R\,\xi_{img}}{2},\, y_p + \frac{\lambda R\,\eta_{img}}{2}\right) P^{*}\left(x_p - \frac{\lambda R\,\xi_{img}}{2},\, y_p - \frac{\lambda R\,\eta_{img}}{2}\right) dx_p\, dy_p}{\displaystyle\iint_{-\infty}^{\infty} P_{E1}(x_p,y_p)\, dx_p\, dy_p}$$

← area of pupil ensures $OTF(0,0)=1$

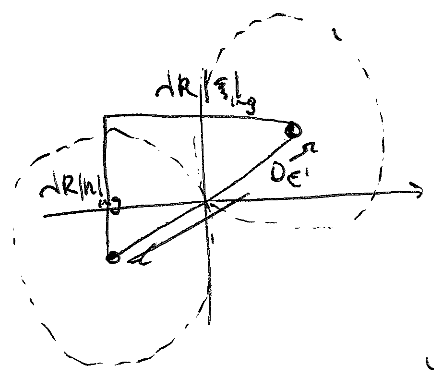
The top integral is called an autocorrelation

Note this integrates two shifted versions of the complex pupil function



Everything is zero except in region of overlap.

What happens if shift is larger?



$$\lambda^2 R^2 |\xi|_{img}^2 + \lambda^2 R^2 |\eta|_{img}^2 = D_{\epsilon'}^2$$

where $D_{\epsilon'}$ = Exit pupil diameter

$$\sqrt{|\xi|_{img}^2 + |\eta|_{img}^2} = \frac{D_{\epsilon'}}{\lambda R}$$

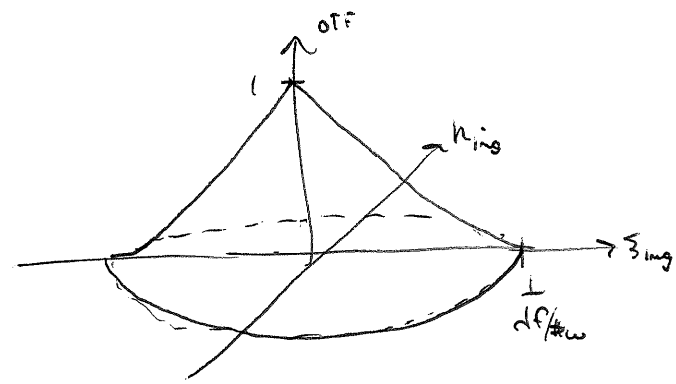
we have already shown $\dfrac{D_{\epsilon'}}{R} = \dfrac{D_\epsilon}{P'E'} = \dfrac{1}{f/\#}$

Radial distance in spatial frequency space $\longrightarrow \sqrt{|\xi|_{img}^2 + |\eta|_{img}^2} = \dfrac{1}{\lambda f/\#}$ For finite conjugate system use $f/\#_w$

This is the cutoff frequency.

For a diffraction limited system

$$OTF(\xi_{img}, \eta_{img}) = \begin{cases} \frac{2}{\pi}\left[\cos^{-1}\left(\sqrt{\xi_{img}^2 + \eta_{img}^2}\,\lambda f/\#_w\right) - \sqrt{\xi_{img}^2 + \eta_{img}^2}\,\lambda f/\#_w \sqrt{1 - (\xi_{img}^2 + \eta_{img}^2)(\lambda f/\#_w)^2}\right] & \text{for } \sqrt{\xi_{img}^2 + \eta_{img}^2} \le \frac{1}{\lambda f/\#_w} \\ 0 & \text{otherwise} \end{cases}$$



$\boxed{1.6.5.5}$ LINE SPREAD FUNCTION [Book 3.6.5]

Suppose we have an ~~perfect~~ object $o(x_{obj}, y_{obj})$. A "perfect" image would be given by $o(x_{img}, y_{img})$ where m is the magnification.
$x_{img} = m x_{obj}$ $y_{img} = m y_{obj}$ i.e. image plane coordinates

$$o(x_{img}, y_{img}) \underset{\text{convolve}}{*} PSF(x_{img}, y_{img}) = i(x_{img}, y_{img})$$

← Treat $o(x_1, y_1)$ as a bunch of points and replace each one with a point spread function

$$\begin{array}{ccccc} o(x_{img}, y_{img}) & * & PSF(x_{img}, y_{img}) & = & i(x_{img}, y_{img}) \\ \Big\downarrow \text{F.T.} & & \Big\downarrow \text{F.T.} & & \Big\uparrow \text{F.T.}^{-1} \\ O(\xi_{img}, \eta_{img}) & \underset{\text{multiply}}{\times} & OTF(\xi_{img}, \eta_{img}) & = & I(\xi_{img}, \eta_{img}) \end{array}$$

Convert back to points

Represent object as sines and cosines

Degrade contrast and phase shift

Image represented as sines and cosines.

Recall some properties of Fourier transforms and delta functions

$$\mathcal{F}\{\delta(x)\} = 1$$

$$\mathcal{F}\{1\} = \delta(\xi)$$

$$\delta(x) f(x) = f(0)\,\delta(x)$$

Suppose the object is infinitely thin line

$$o(x_{img}, y_{img}) = \delta(x_{img}) \cdot 1$$

$$i(x_{img}, y_{img}) = \delta(x_{img}) \cdot 1 * PSF(x_{img}, y_{img})$$

$$I(\xi_{img}, \eta_{img}) = 1 \cdot \delta(\eta_{img}) \cdot OTF(\xi_{img}, \eta_{img}) = OTF(\xi_{img}, 0)$$

By Fourier transforming the image of a infinitely thin line, we can get a cross-section through the OTF.

In the real world, it is impossible to create an infinitely thin line for our object. Let's use a narrow slit instead

$o(x_{img}, y_{img}) = \mathrm{rect}\left(\frac{x_{img}}{d}\right)$ where d is slit width

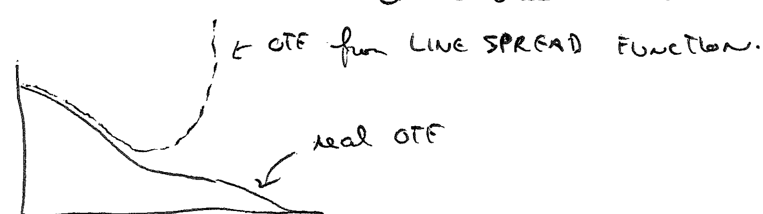
$i(x_{img}, y_{img}) = \mathrm{rect}\left(\frac{x_{img}}{d}\right) * PSF(x_{img}, y_{img})$

$I(\xi_{img}, \eta_{img}) = d\,\mathrm{sinc}(d\xi_{img})\,\delta(\eta_{img})\,OTF(\xi_{img}, \eta_{img})$

$$\boxed{OTF(\xi_{img}, 0) = \frac{I(\xi_{img}, 0)}{d\,\mathrm{sinc}(d\xi_{img})}}$$

Can get cross-section through OTF by Fourier transforming blurred image of slit and dividing by sinc() function. Obviously, orienting slit at different angles gives different slices through OTF.

The main problem with this technique is that it blows up when sinc() = 0. This occurs when $\xi = \frac{N}{d}$ where N is a non-zero integer.



Can use edge instead of line in similar fashion.

$\boxed{1.6.5.6}$ Siemens Star is a test pattern with a range of spatial frequencies and orientations to rapidly assess system performance.

Show slide examples.