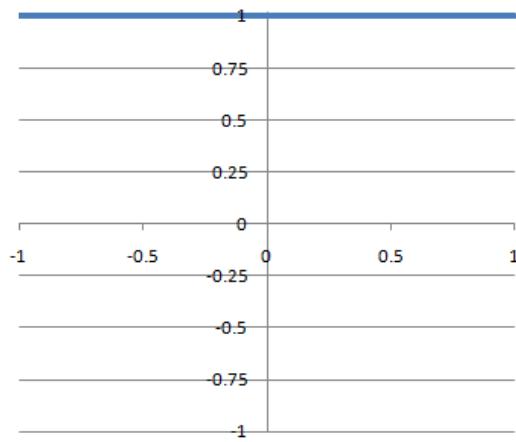
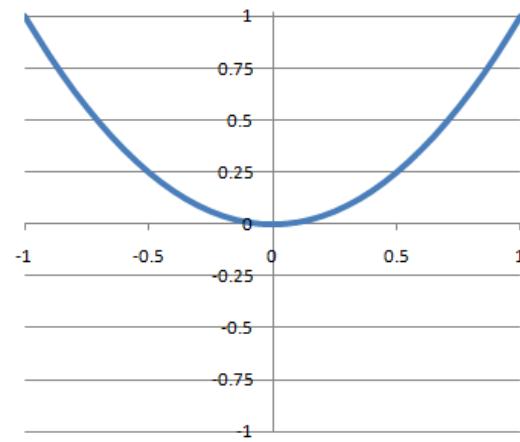


# 1.5.8 Orders of Polynomials

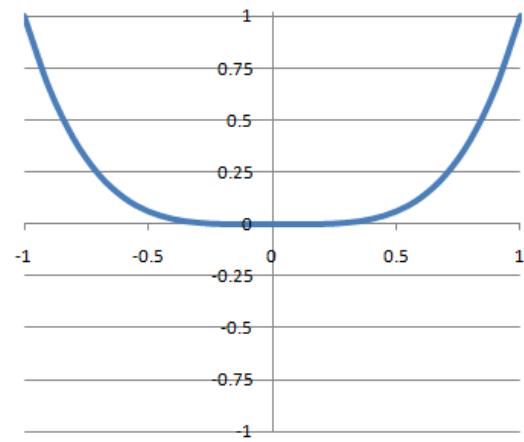
Constant - 0th Order



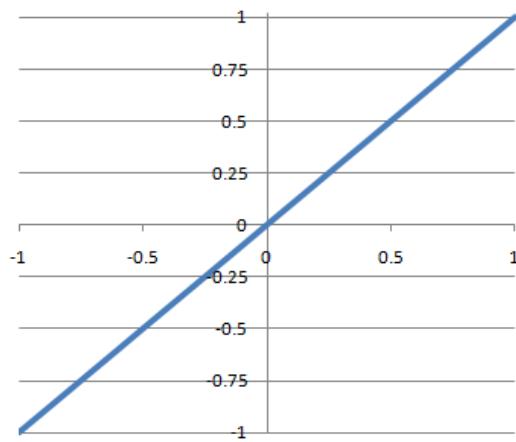
Parabolic - 2nd Order



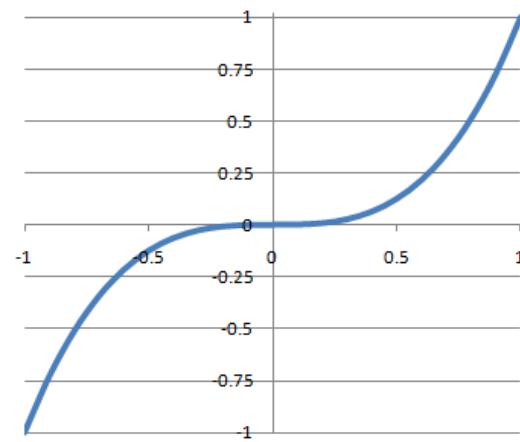
Quartic - 4th Order



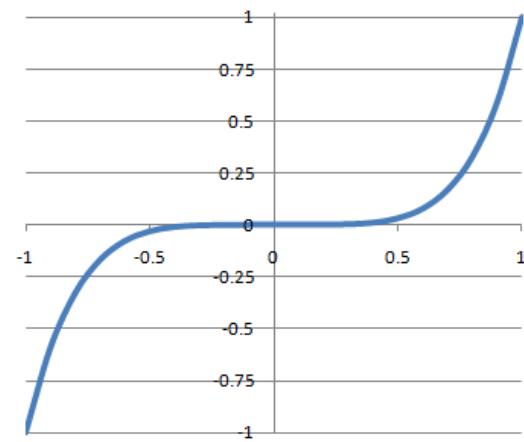
Linear - 1st Order



Cubic - 3rd Order



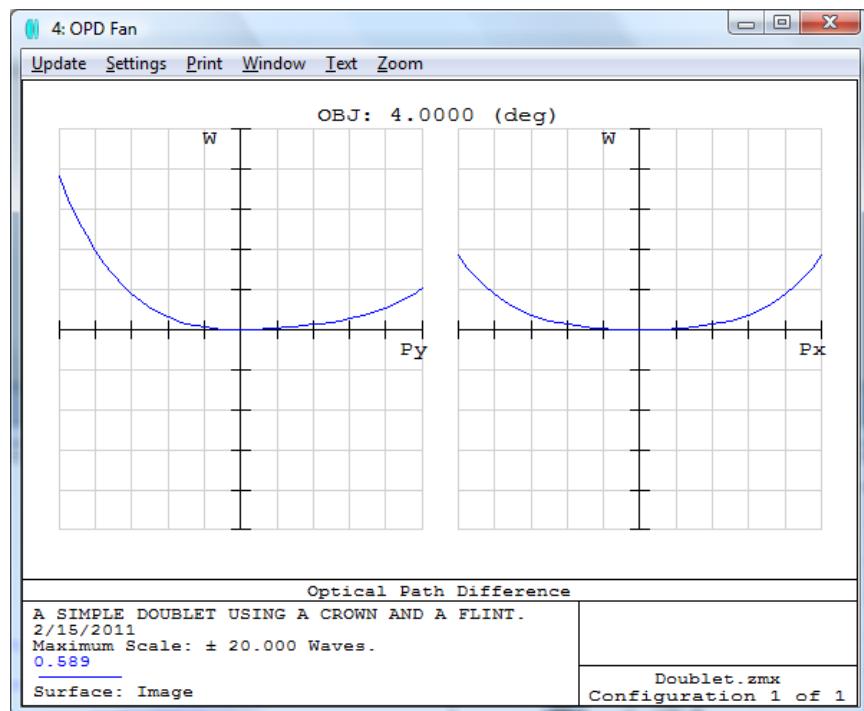
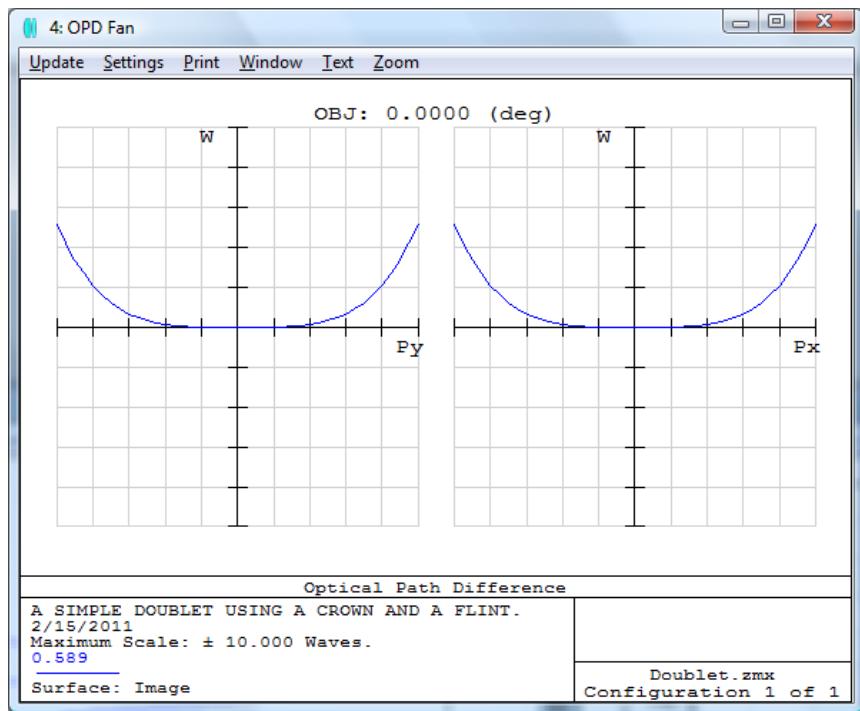
Quintic - 5th Order



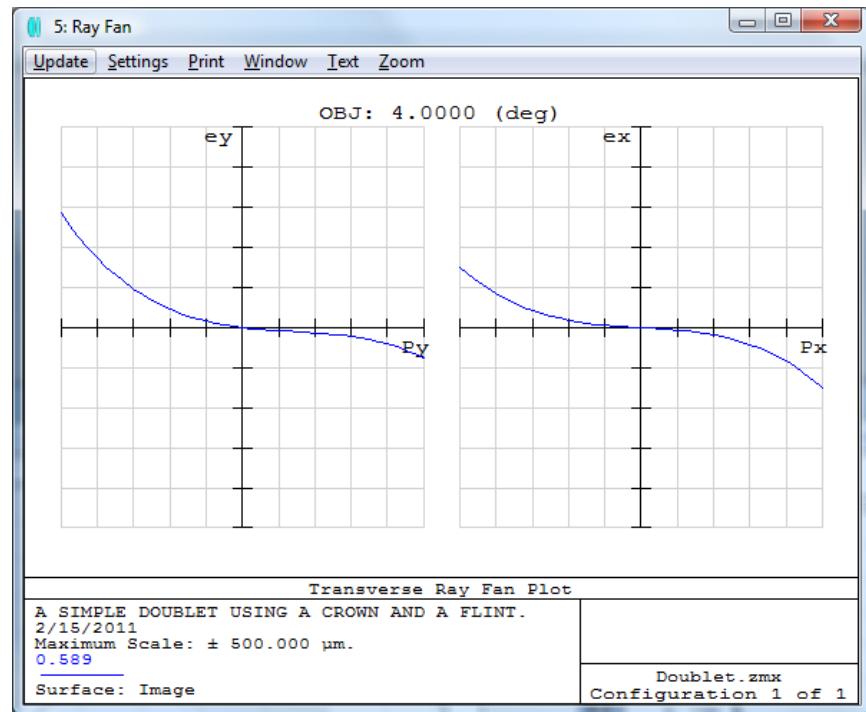
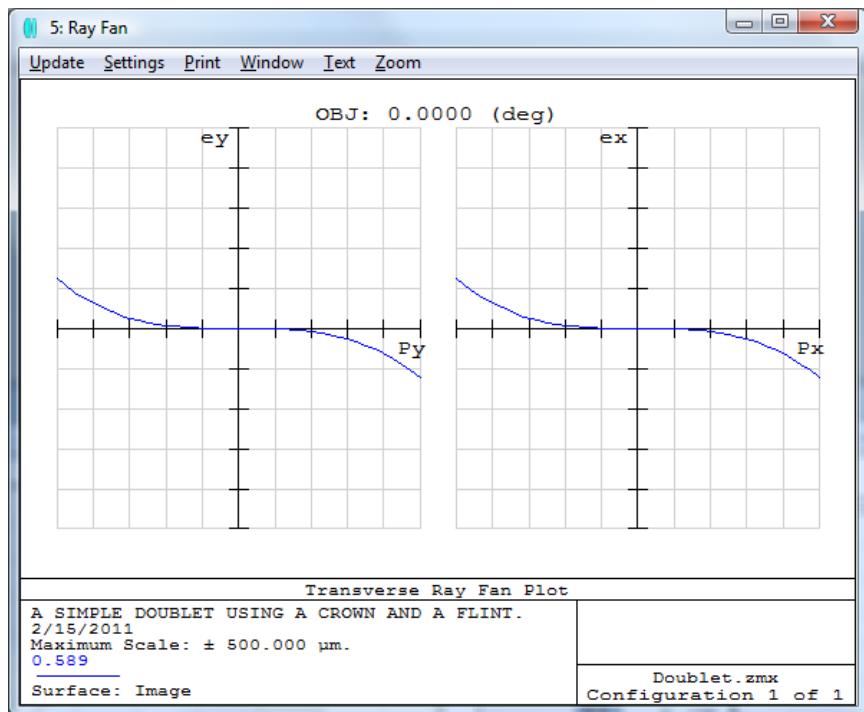
# 1.5.8 Seidel Aberrations

Aberration	$w(0, \rho_y)$	$w(\rho_x, 0)$	$\varepsilon_y(0, \rho_y)$	$\varepsilon_x(\rho_x, 0)$	$h$
Tilt	linear	zero	constant	zero	linear
Defocus	parabolic	parabolic	linear	linear	constant
Spherical	quartic	quartic	cubic	cubic	constant
Coma	cubic	parabolic	parabolic	zero	linear
Astigmatism	parabolic	zero	linear	zero	parabolic
Field Curvature	parabolic	parabolic	linear	linear	parabolic
Distortion	linear	zero	constant	zero	cubic

# Doublet Wavefront Error



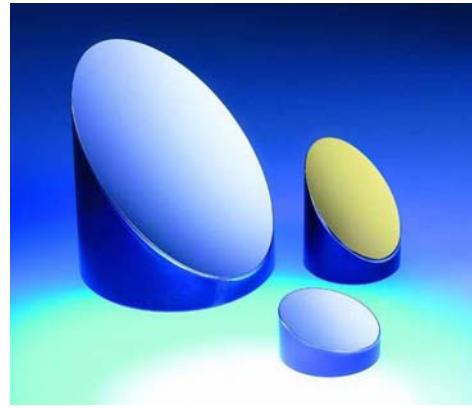
# Doublet Transverse Ray Error



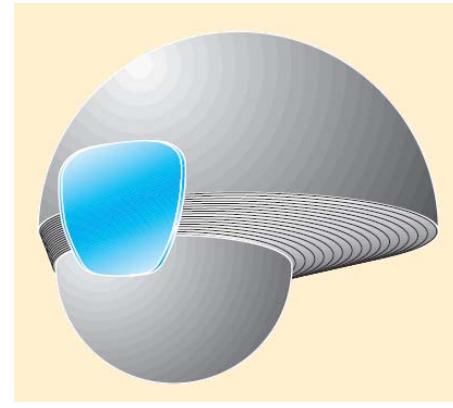
# 1.5.9.1 Non-Rotationally Symmetric Systems



Edmundoptics.com



Edmundoptics.com



Essilor

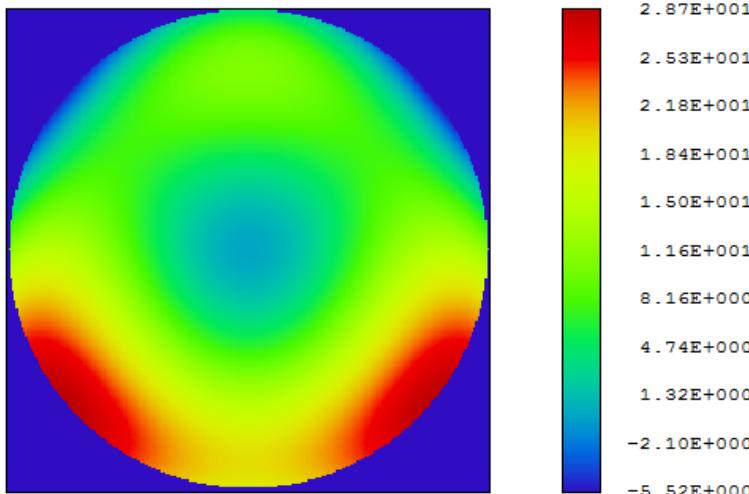


[www.davidmullenasc.com](http://www.davidmullenasc.com)

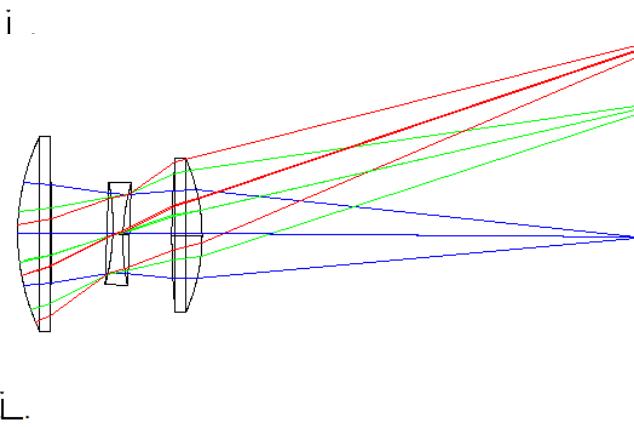
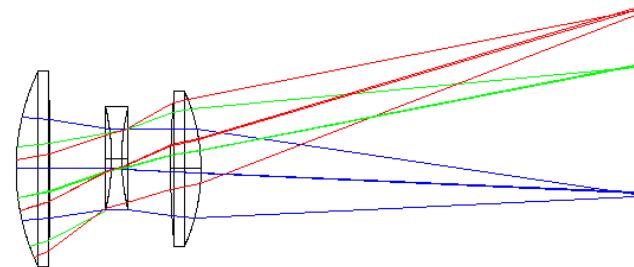
# 1.5.9.1 Non-Rotationally Symmetric Systems



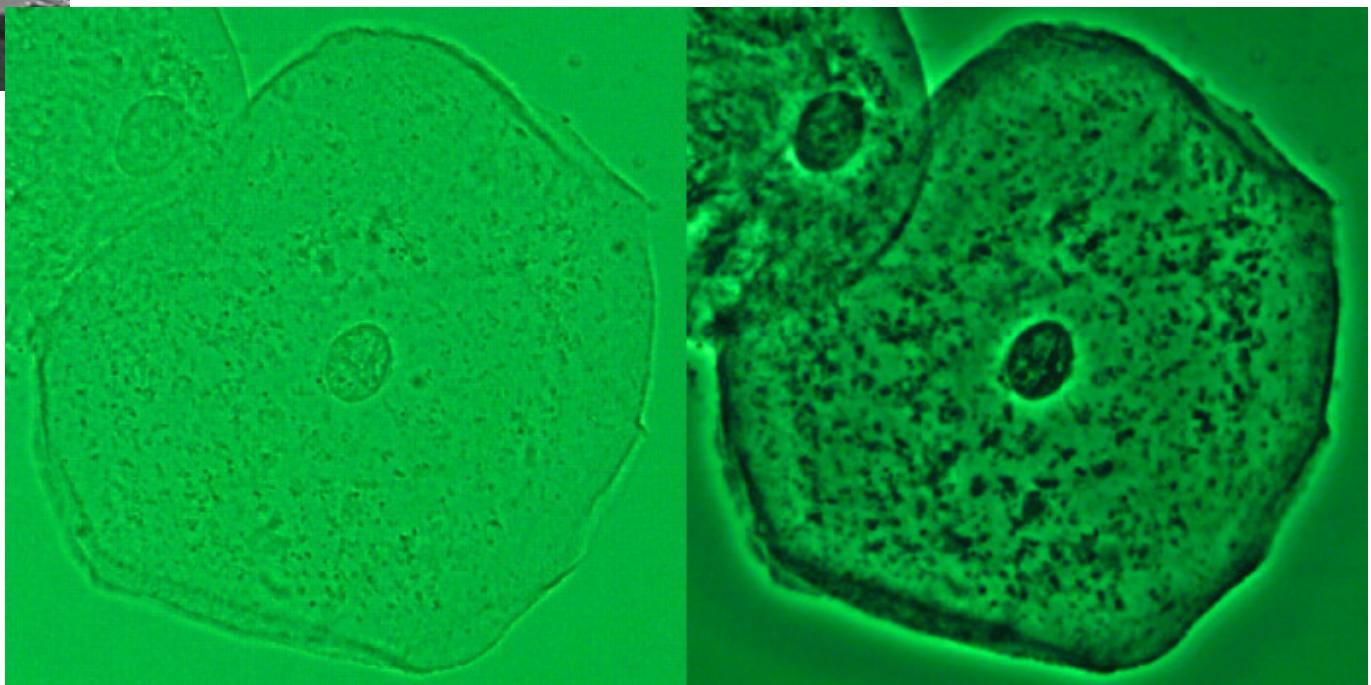
Desmoineseyecare.com



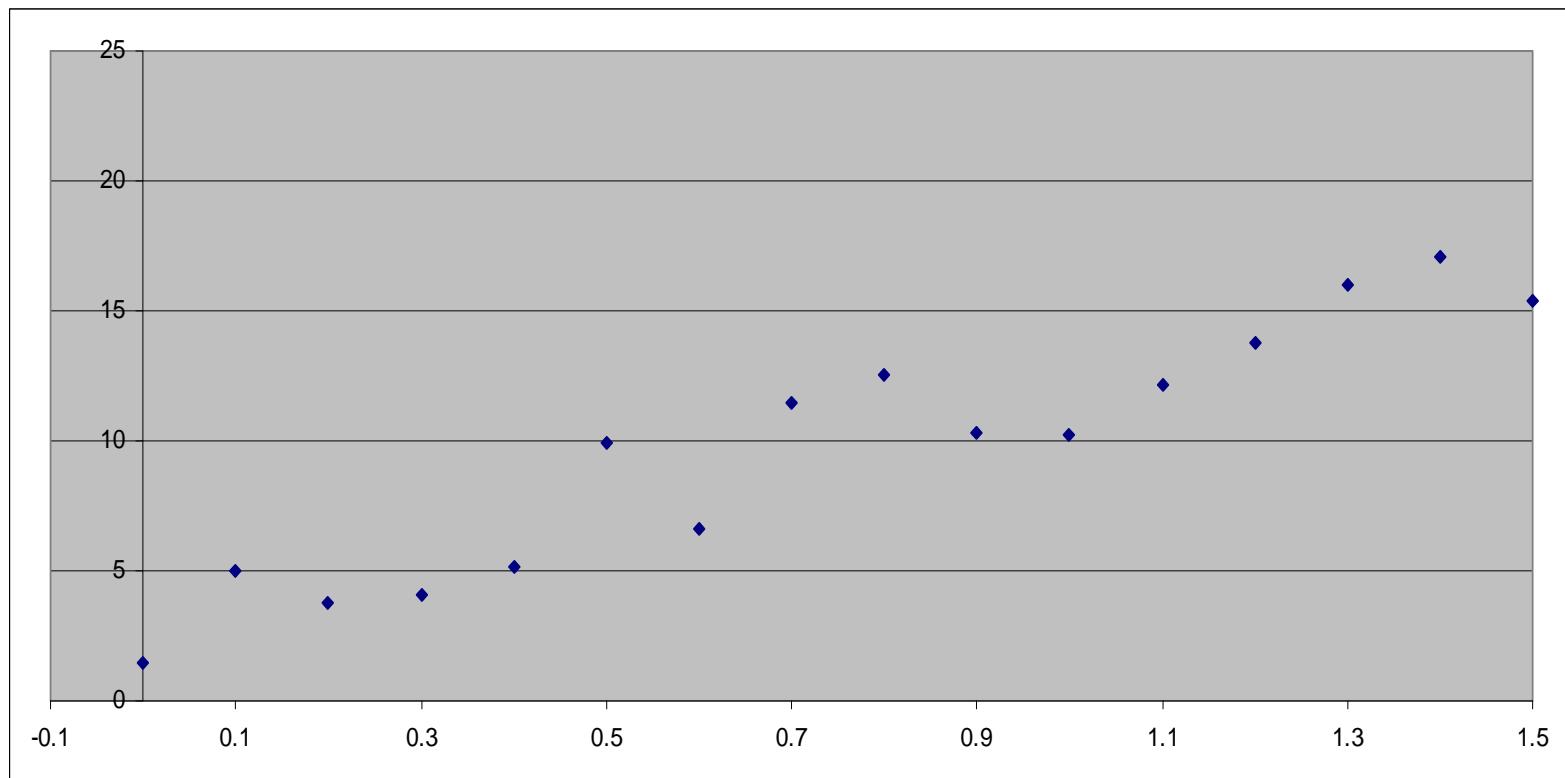
Linhof.de



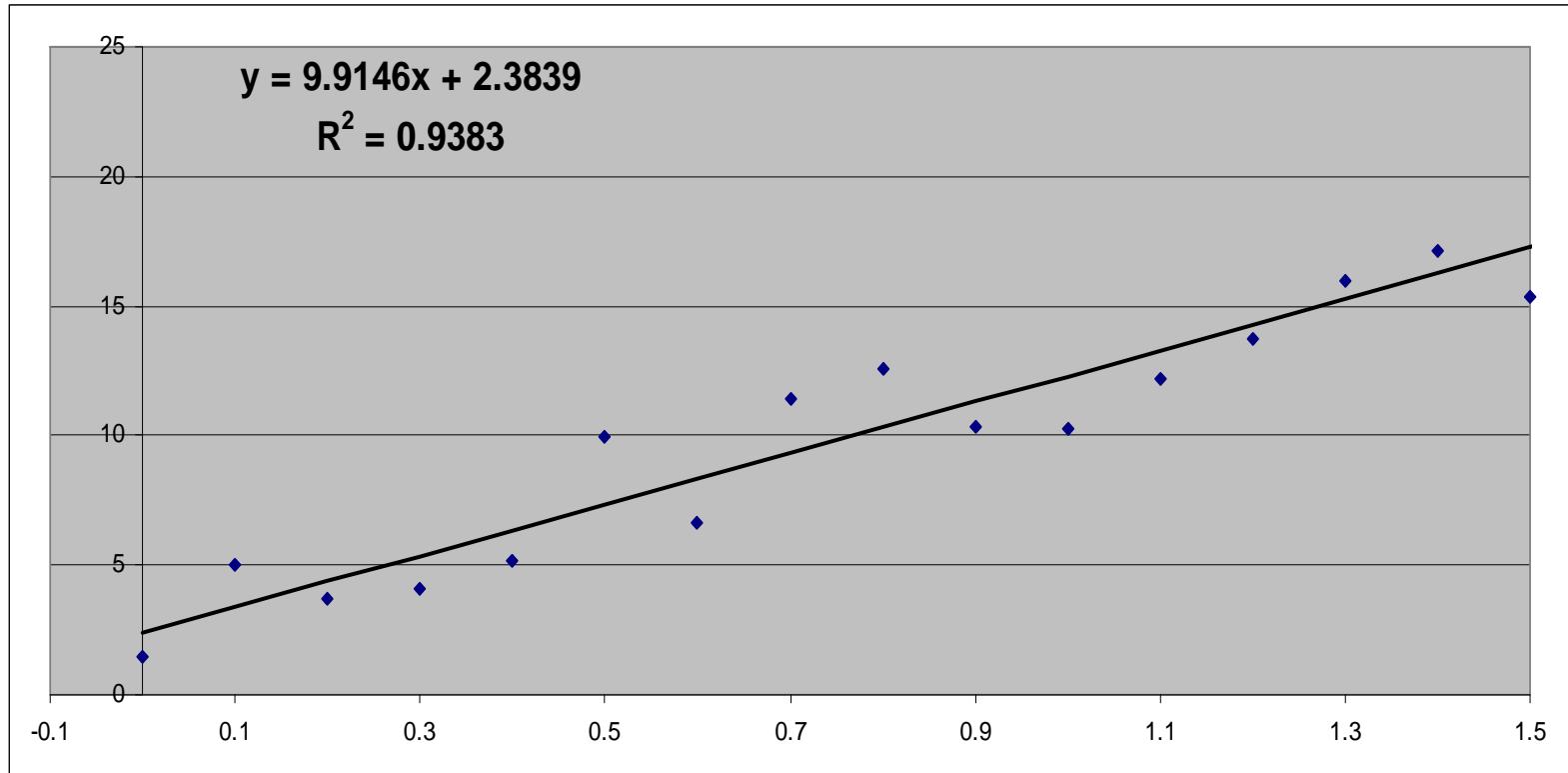
## 1.5.9.3 Zernike Polynomials



# 1D Curve Fitting

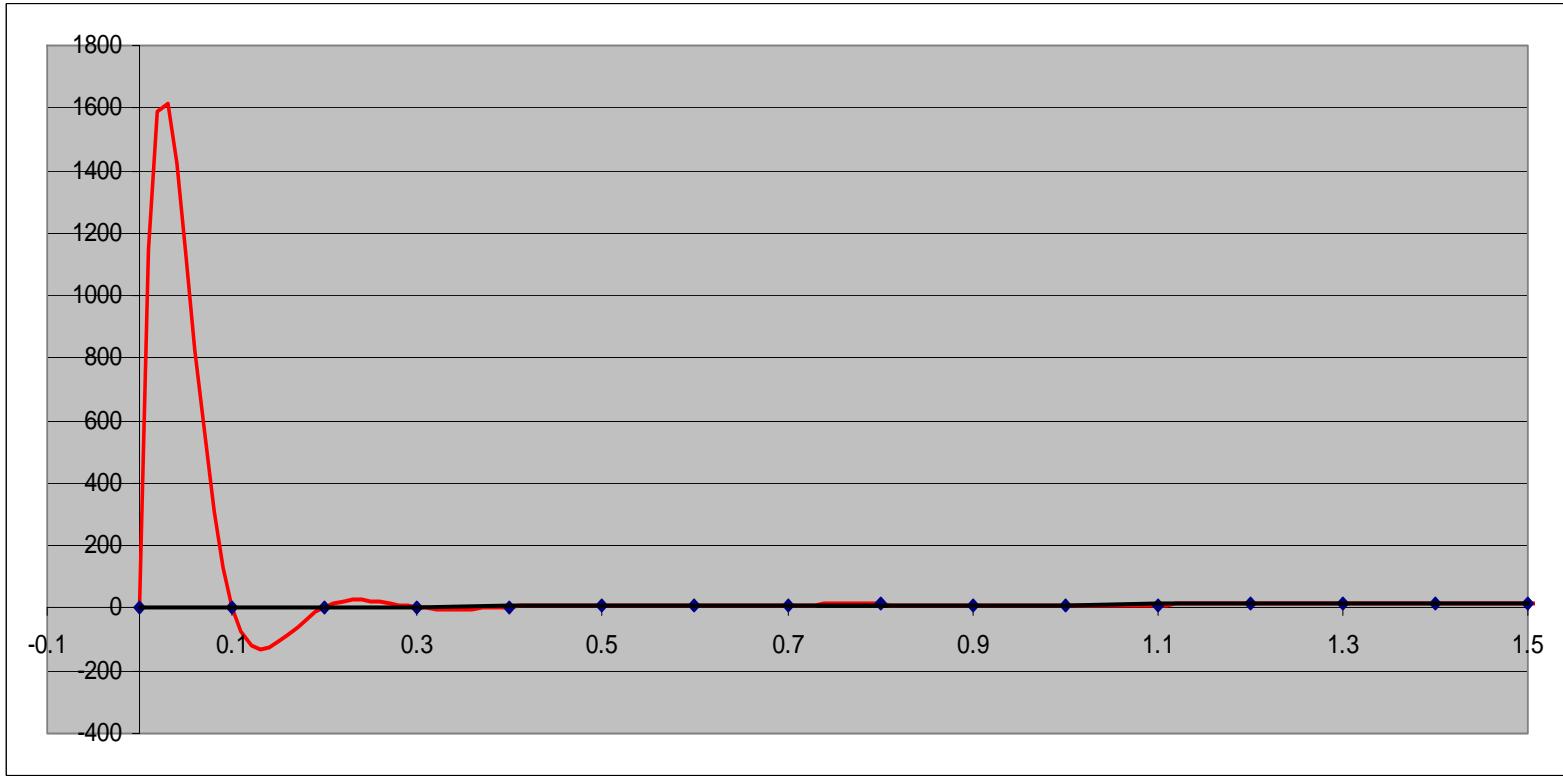


# Low-order Polynomial Fit



In this case, the error is the vertical distance between the line and the data point. The sum of the squares of the error is minimized.

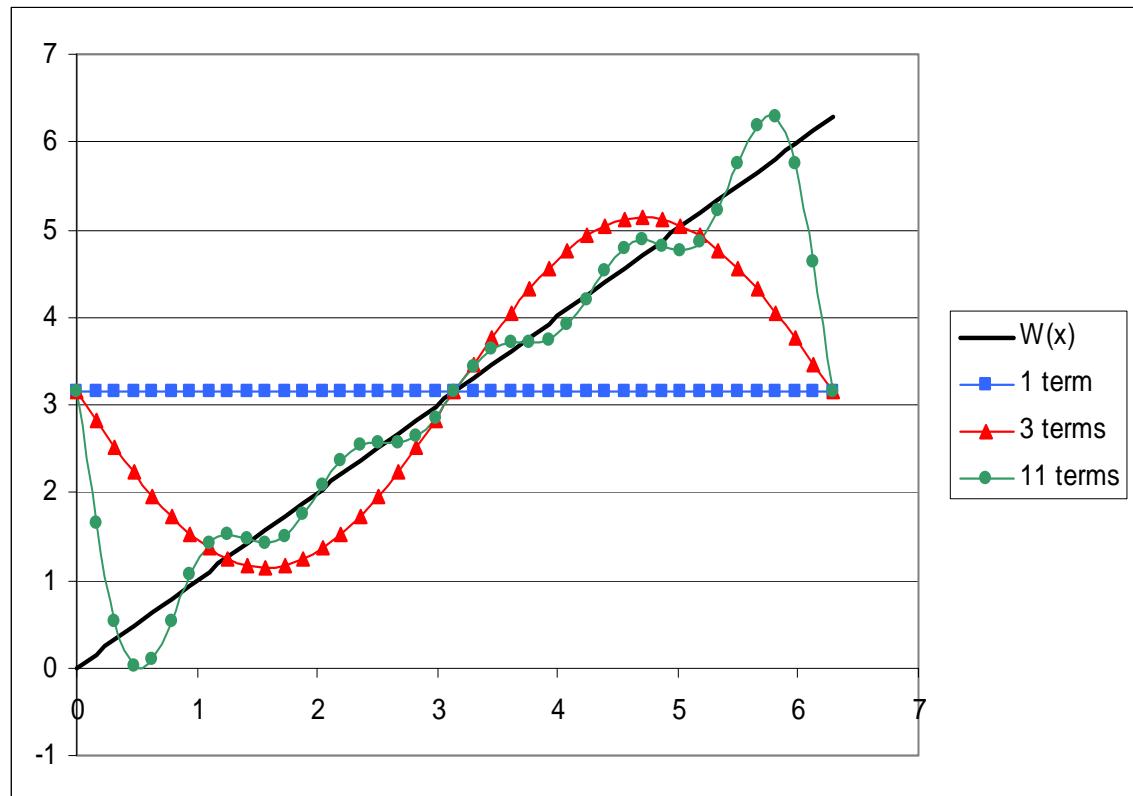
# High-order Polynomial Fit



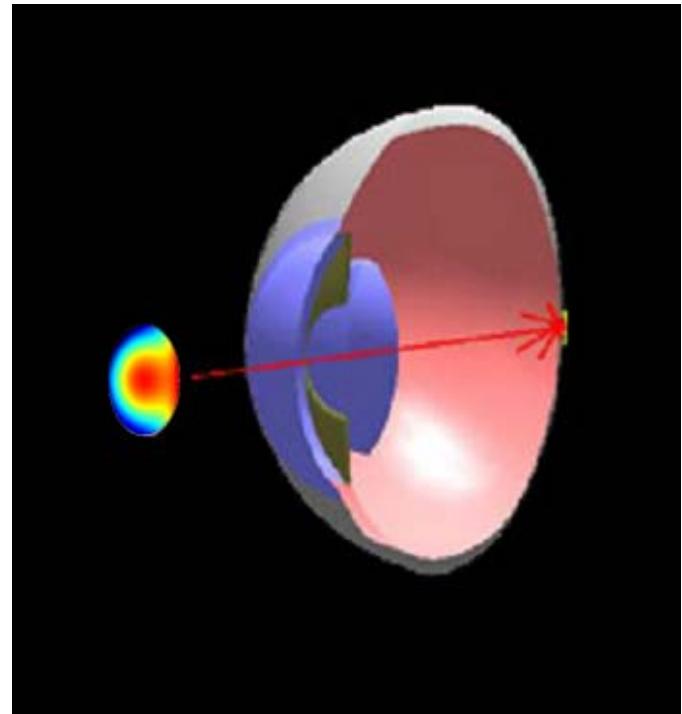
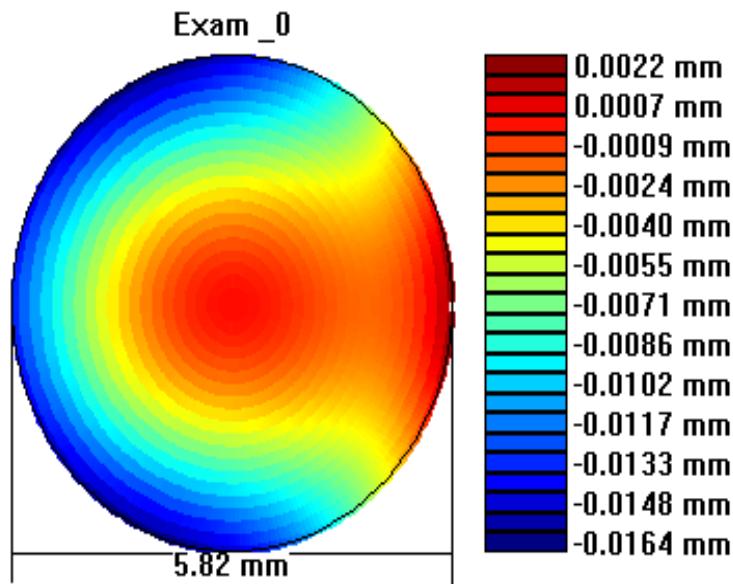
$$y = a_0 + a_1x + a_2x^2 + \dots + a_{16}x^{16}$$

# Fit to $W(x) = x$

$j$	$a_j$
0	$\pi$
1	-2
2	0
3	-1
4	0
5	-0.6666
6	0
7	-0.5
8	0
9	-0.4
10	0

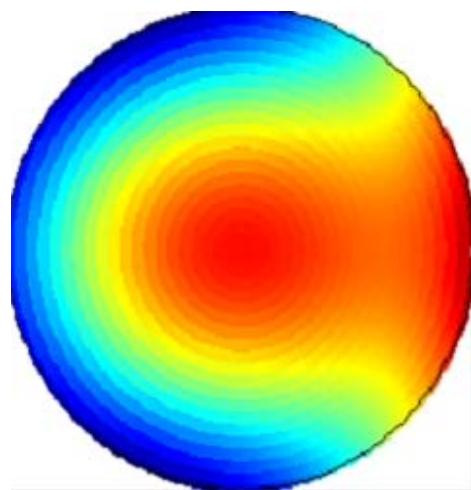


# Extension to Two Dimensions



In many cases, wavefronts take on a complex shape defined over a circular region and we wish to fit this surface to a series of simpler components.

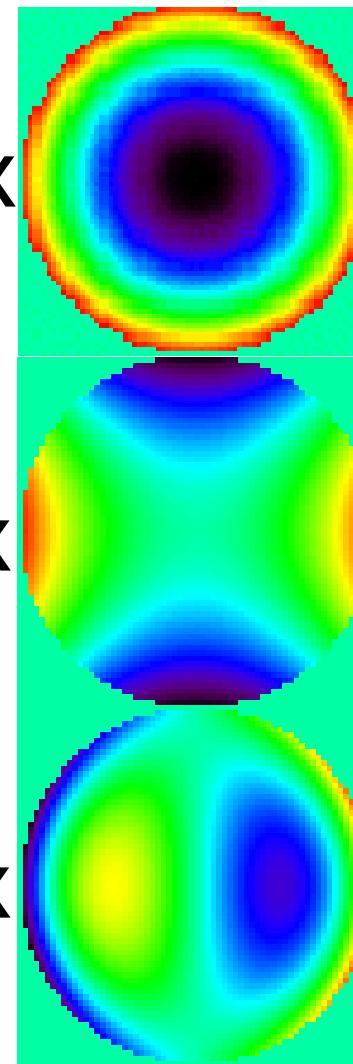
# Wavefront Fitting



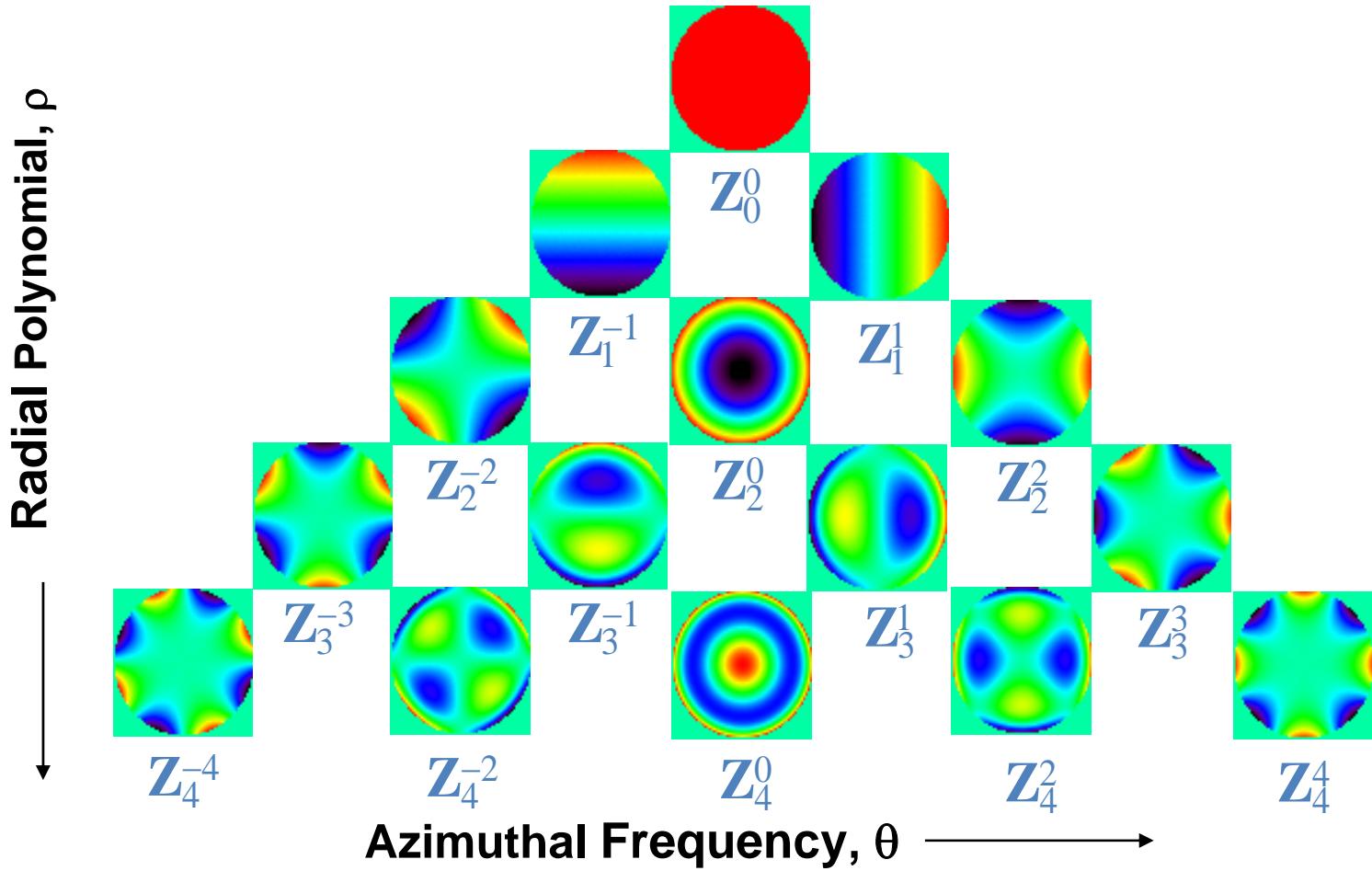
-0.003 x

= + 0.002 x

+ 0.001 x



# Zernike Polynomials



# Zernike Polynomials - Single

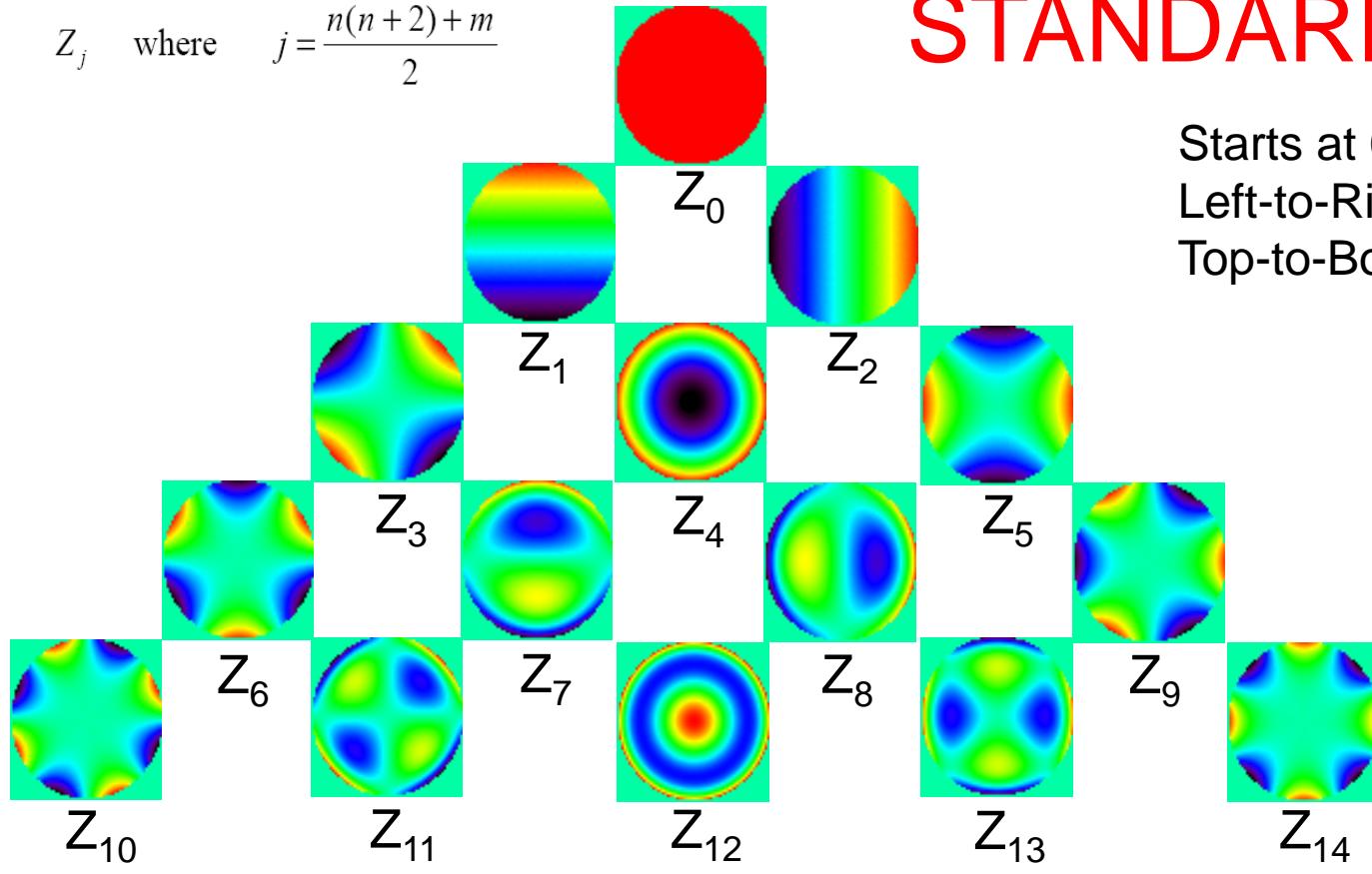
Index

ANSI

STANDARD

$$Z_j \quad \text{where} \quad j = \frac{n(n+2)+m}{2}$$

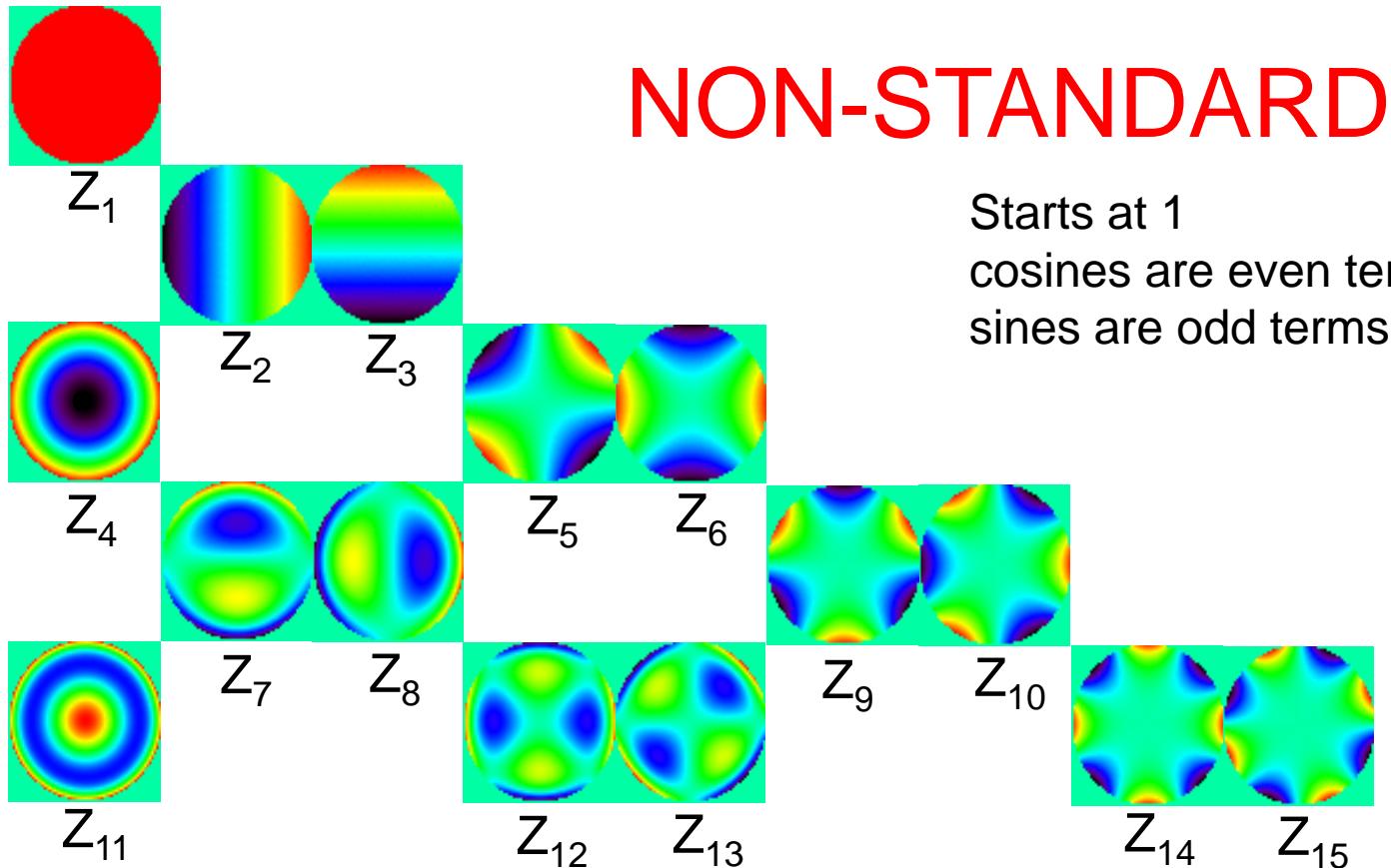
Radial Polynomial,  $\rho$



Starts at 0  
Left-to-Right  
Top-to-Bottom

Azimuthal Frequency,  $\theta$

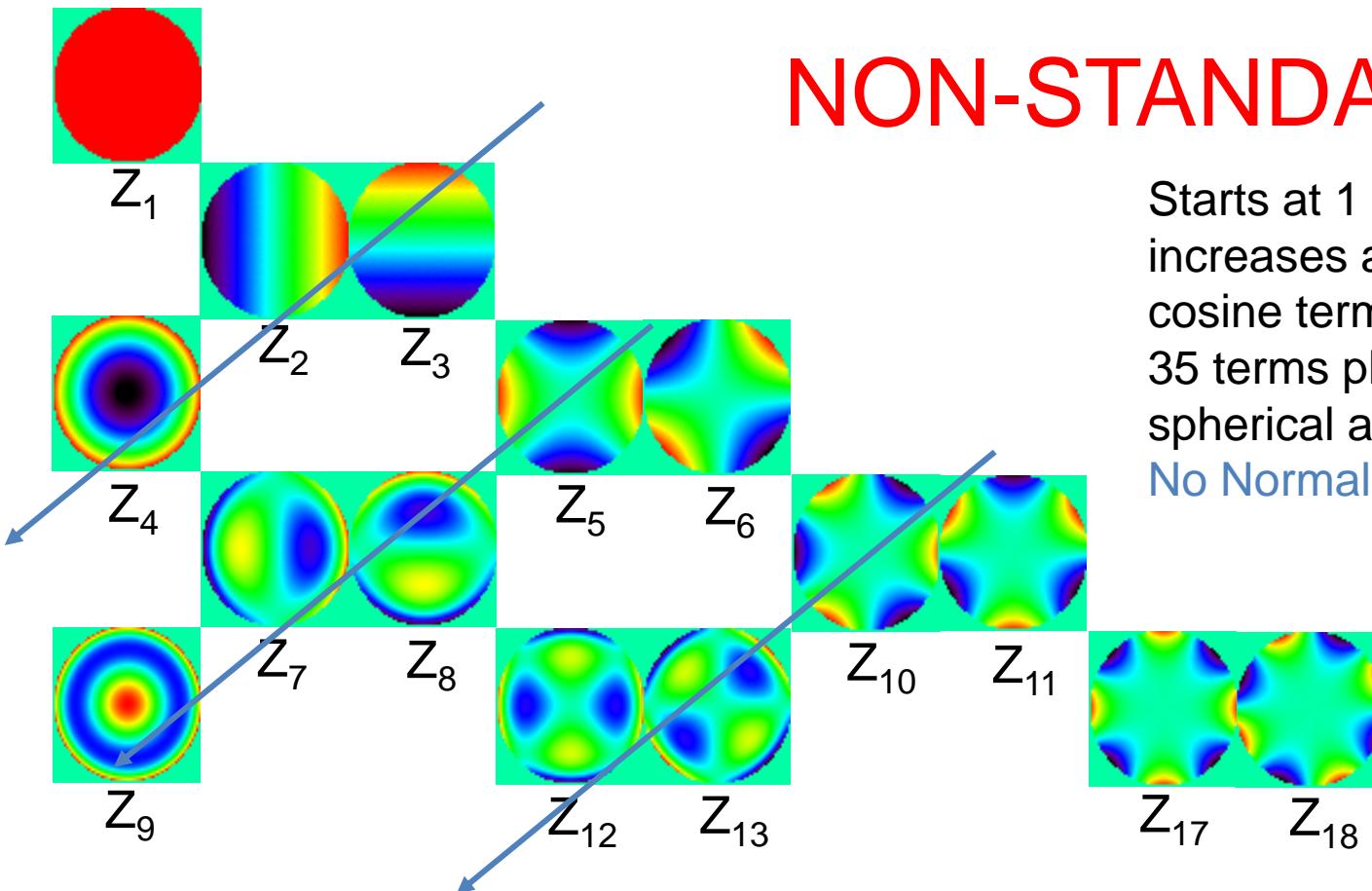
# Other Single Index Schemes



Noll, RJ. Zernike polynomials and atmospheric turbulence. J Opt Soc Am 66; 207-211 (1976).

Also Mahajan and Zemax “Standard Zernike Coefficients”

# Other Single Index Schemes



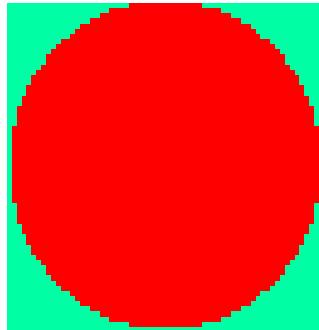
## NON-STANDARD

Starts at 1  
increases along diagonal  
cosine terms first  
35 terms plus two extra  
spherical aberration terms.  
**No Normalization!!!**

Zemax “Zernike Fringe Coefficients”

Also, Air Force or University of Arizona

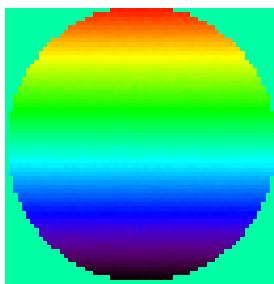
# Zeroth Order Zernike Polynomials



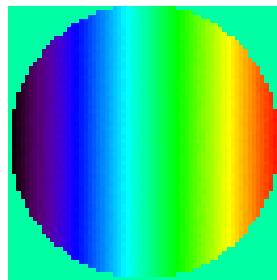
This term is called Piston and is usually ignored.  
The surface is constant over the entire circle, so  
no error or variance exists.

$$Z_0^0$$

# First Order Zernike Polynomials



$$Z_1^{-1}$$



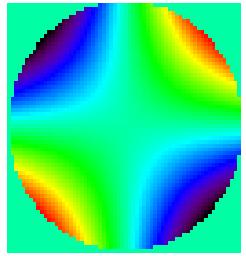
$$Z_1^1$$

These terms represent a tilt in the wavefront.

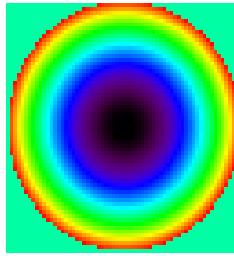
Combining these terms results in a general equation for a plane, thus by changing the coefficients, a plane at any orientation can be created. This rotation of the pattern is true for the sine/cosine pairs of Zernikes

$$\begin{aligned} & a_{1-1} Z_1^{-1}(\rho, \theta) + a_{11} Z_1^1(\rho, \theta) \\ &= a_{1-1} \rho \sin \theta + a_{11} \rho \cos \theta \\ &= a_{1-1} \frac{y}{r_{\max}} + a_{11} \frac{x}{r_{\max}} \end{aligned}$$

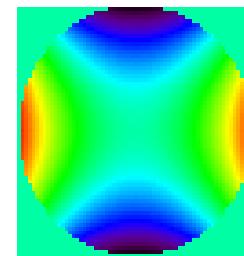
# Second Order Zernike Polynomials



$$Z_2^{-2}$$



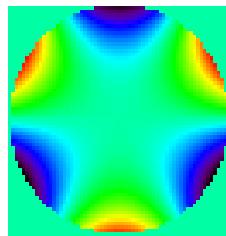
$$Z_2^0$$



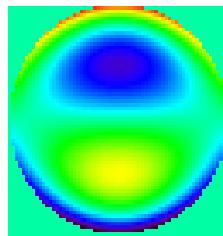
$$Z_2^2$$

$Z(2,0)$  is a paraboloid, so it represents defocus.  $Z(2,2)$  and  $Z(2,-2)$  are saddle shaped surfaces rotated 45 degrees to one another. Much like tilt, appropriate weighting of these two terms can create a saddle shape rotated through any angle.

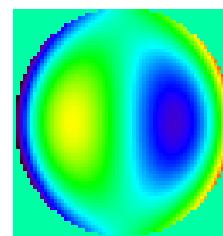
# Third Order Zernike Polynomials



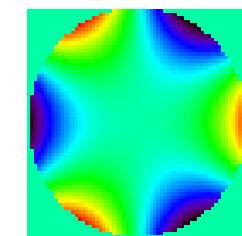
$$Z_3^{-3}$$



$$Z_3^{-1}$$



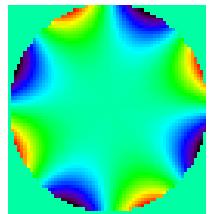
$$Z_3^1$$



$$Z_3^3$$

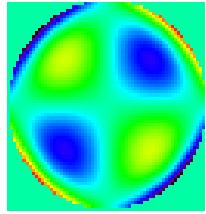
The inner two terms are coma and the outer two terms are trefoil.  
These terms represent asymmetric.

# Fourth Order Zernike Polynomials



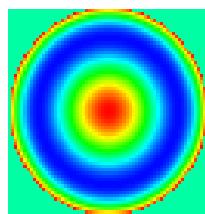
$$Z_4^{-4}$$

Quadroil



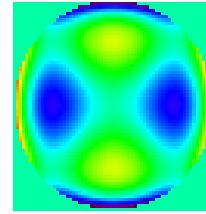
$$Z_4^{-2}$$

4<sup>th</sup> order  
Astigmatism



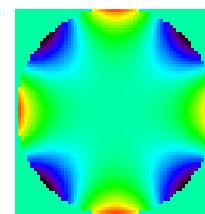
$$Z_4^0$$

Spherical  
Aberration



$$Z_4^2$$

4<sup>th</sup> order  
Astigmatism



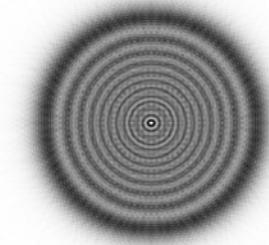
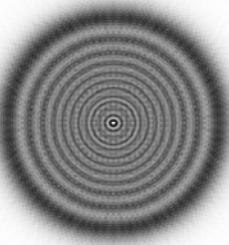
$$Z_4^4$$

Quadroil

These terms represent more complex shapes of the wavefront.

## 1.5.10 Through-Focus PSF and the Star Test

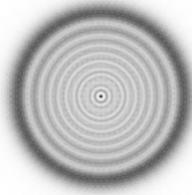
Defocus



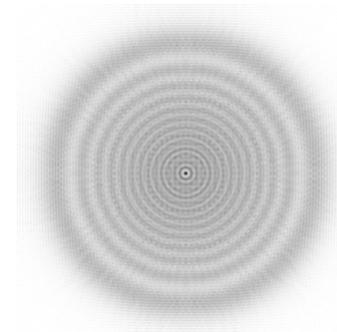
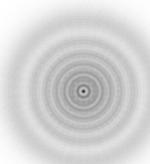
Paraxial  
Focus

# Spherical Aberration

Seidel Spherical Aberration

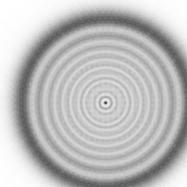
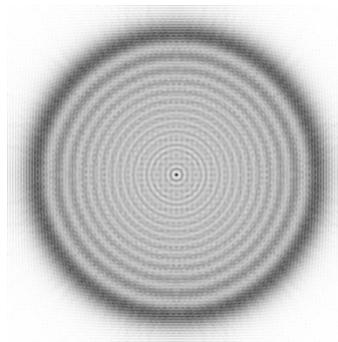


.

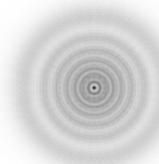


Marginal  
Focus

Paraxial  
Focus



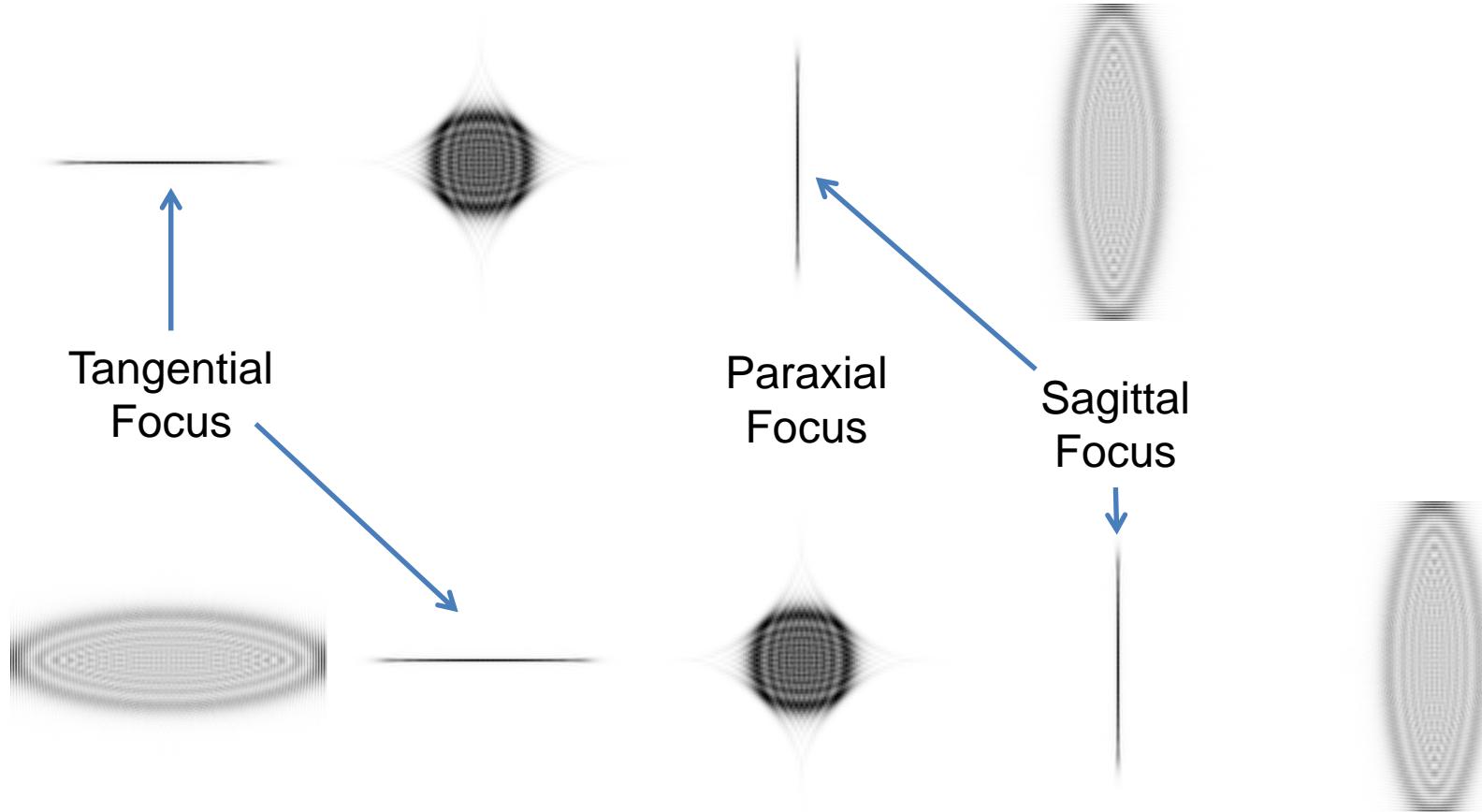
.



Zernike Spherical Aberration  $Z(4,0)$

# Astigmatism

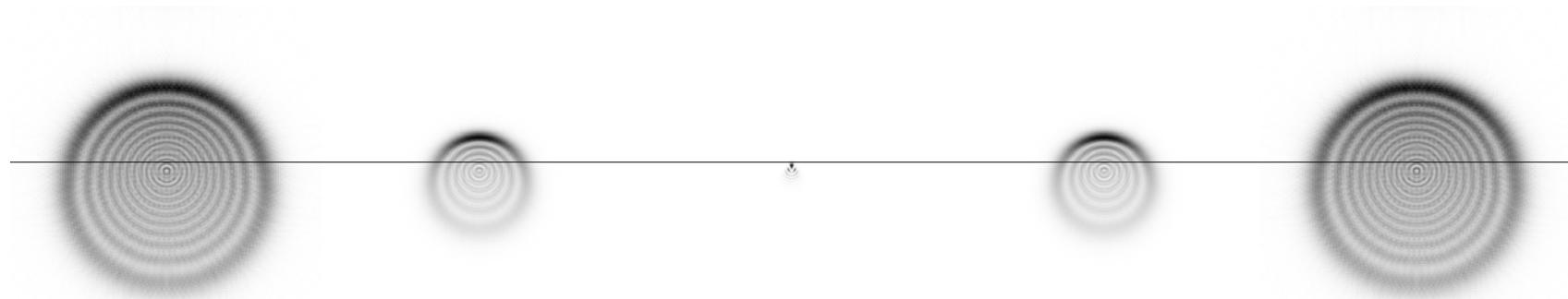
Seidel Astigmatism



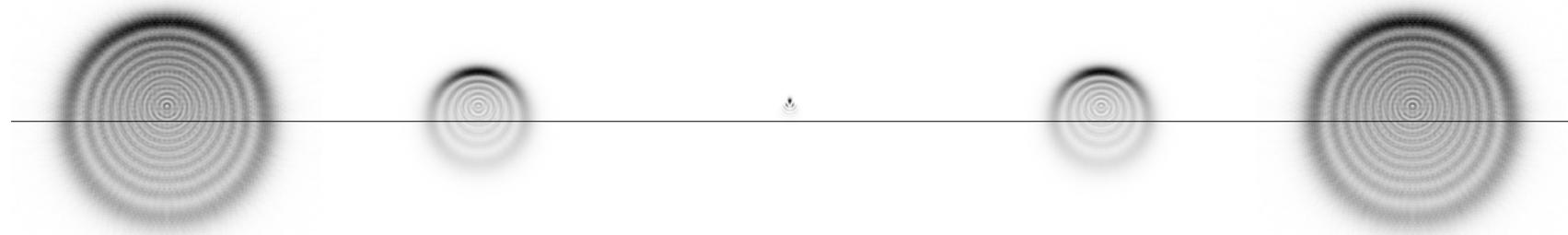
Zernike Astigmatism  $Z(2,2)$

# Coma

Seidel Coma



Paraxial  
Focus

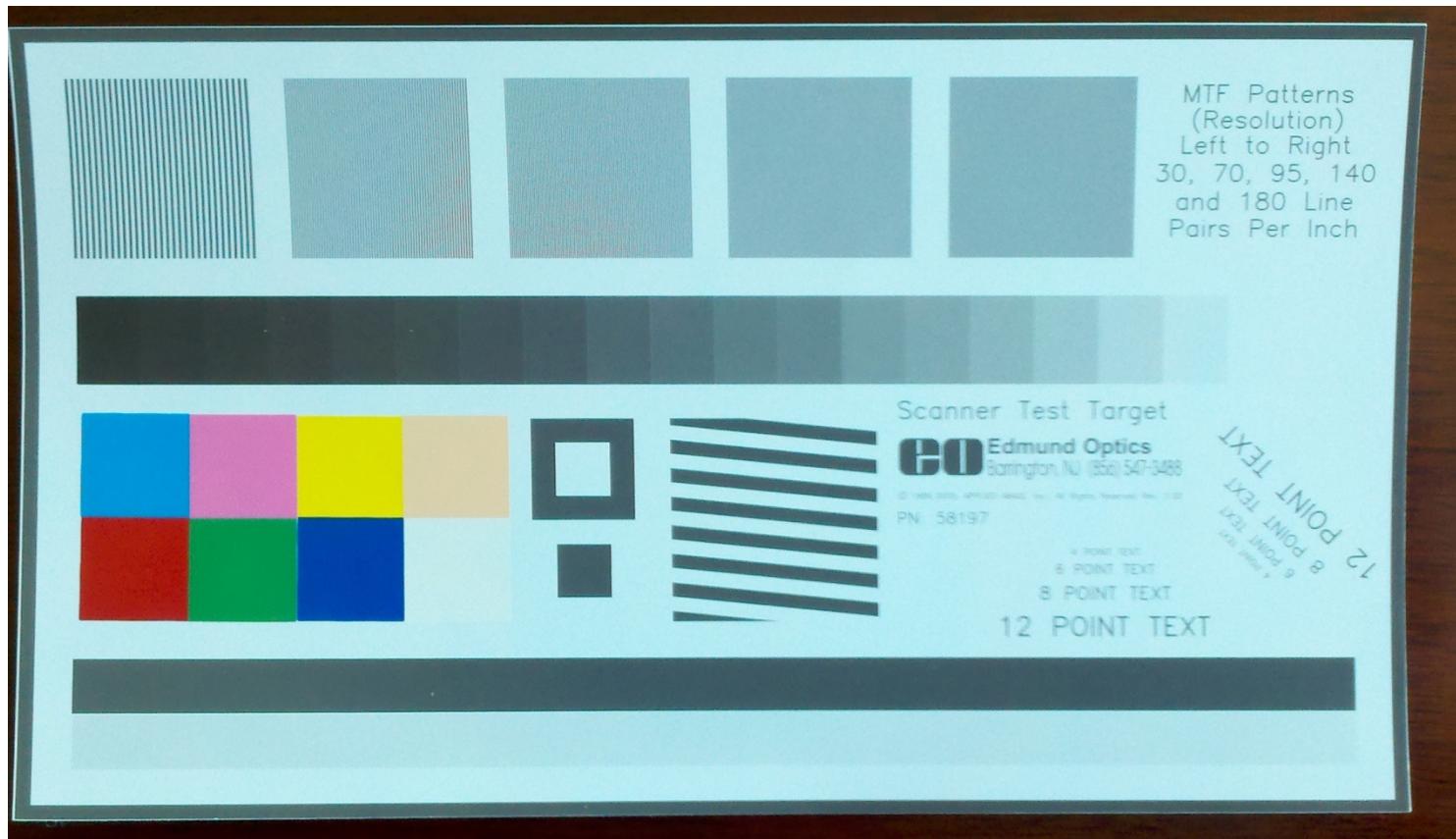


Zernike Coma  $Z(3,-1)$

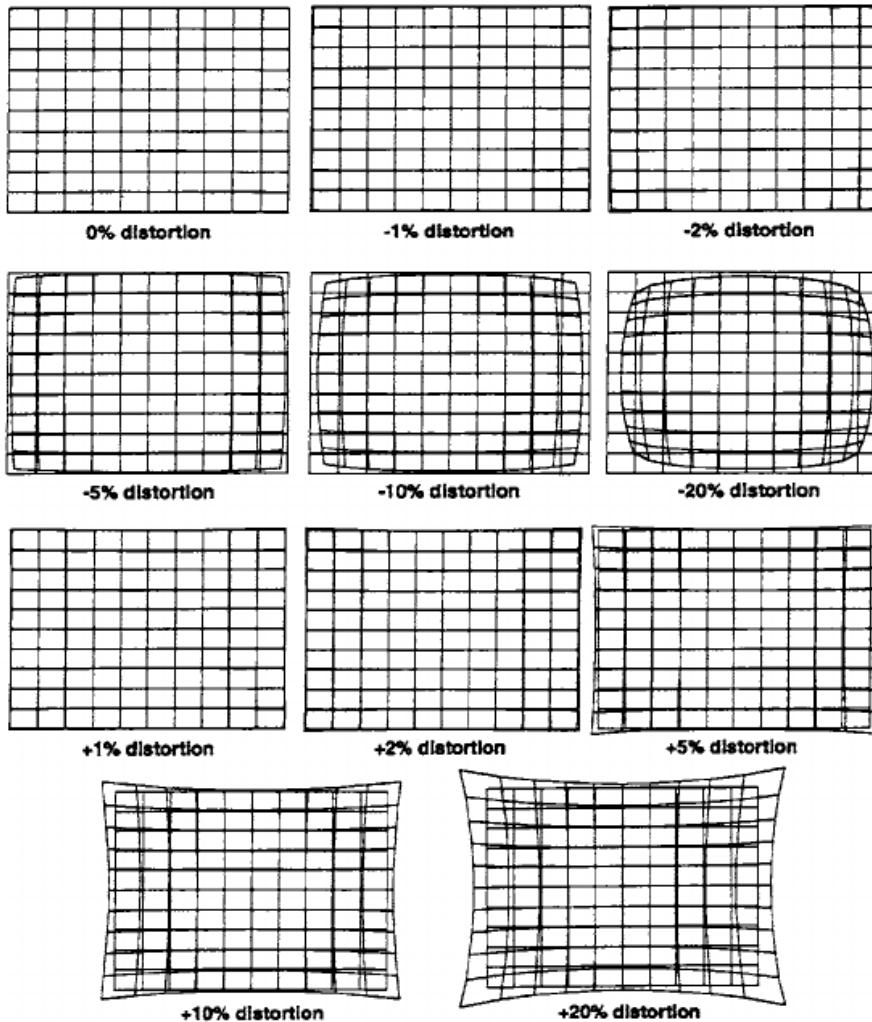
## 1.5.11 Barrel Distortion



# Pincushion Distortion



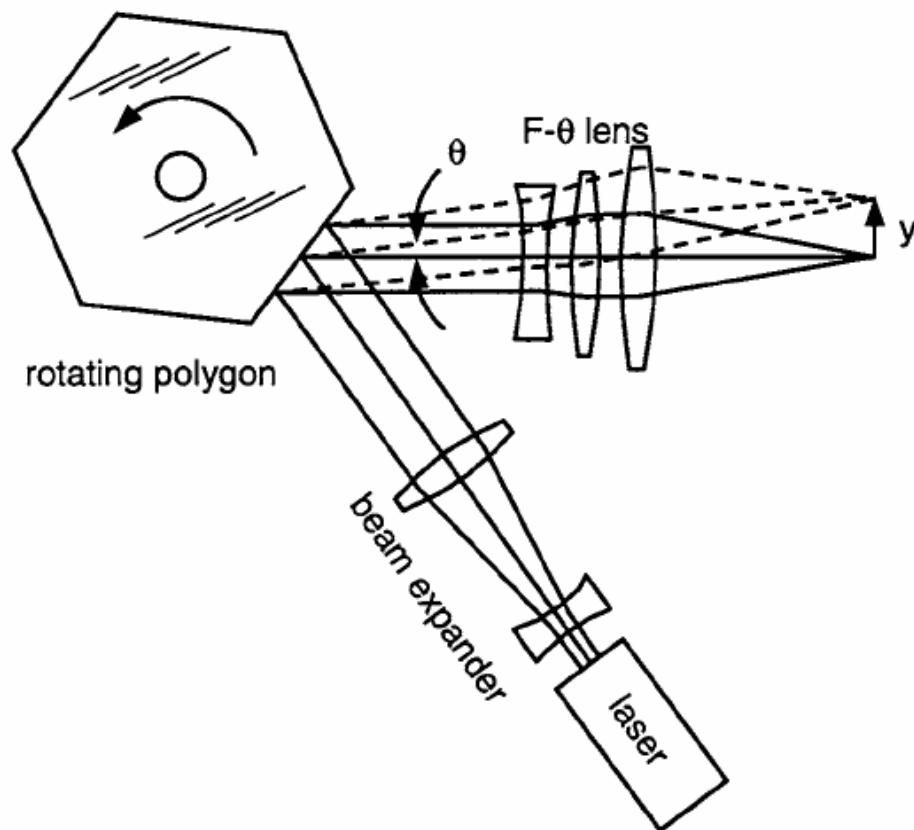
# Percent Distortion



# Anamorphic Distortion

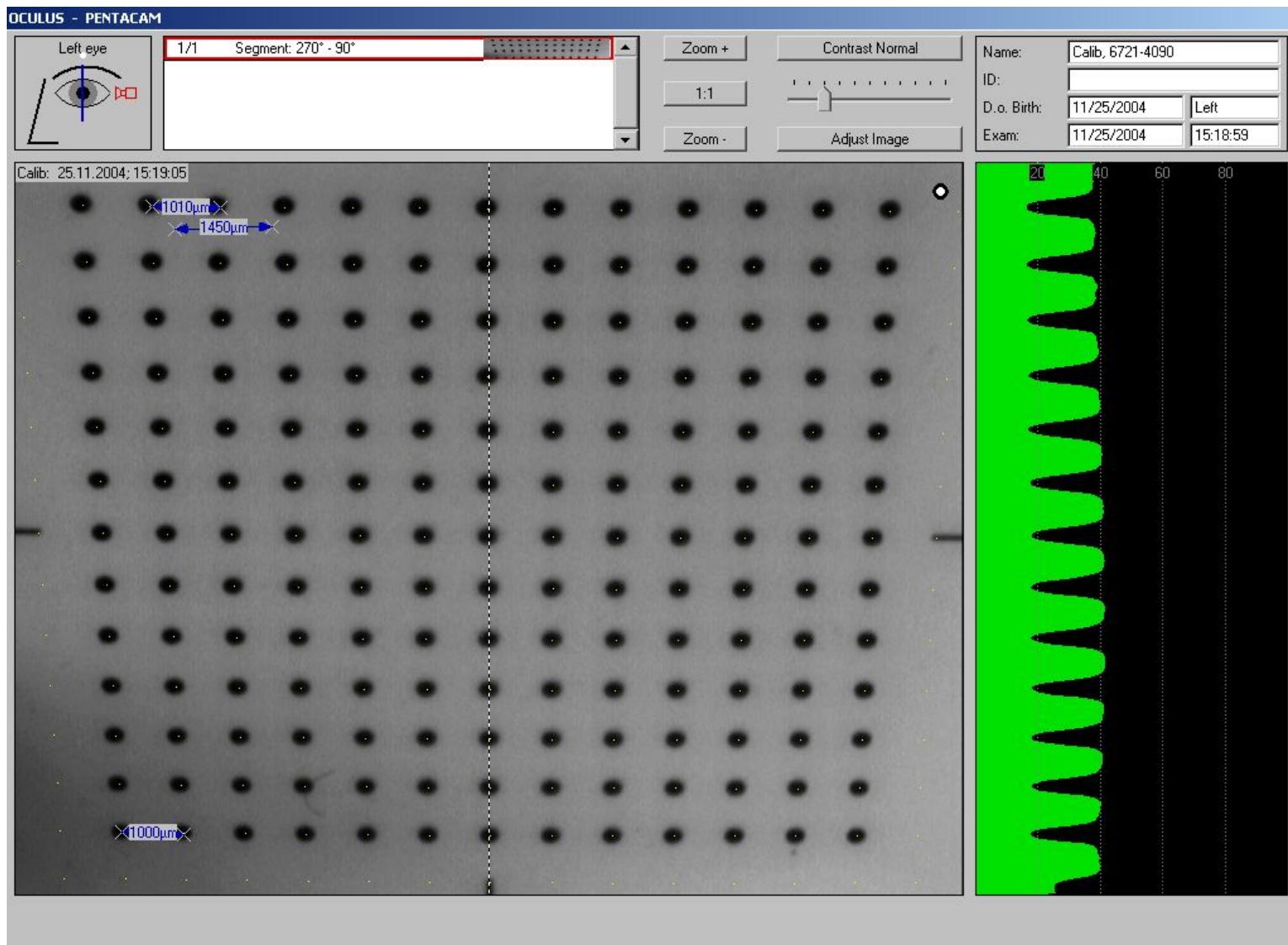


# F-θ Lens



Fischer, Optical System Design

# Keystone Distortion



# Keystone Correction



[www.lonestardigital.com/perspective\\_correction.htm](http://www.lonestardigital.com/perspective_correction.htm)

# Scheimpflug Imaging



<http://users.gsinet.net/pjwhite/tilt-shift.htm>