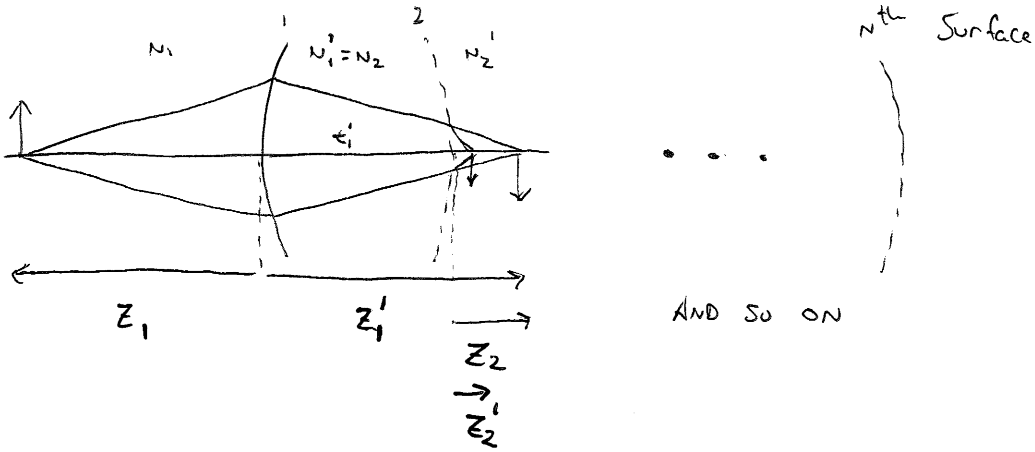
1.3 First Order Properties of an optical system

1.3.1 Gaussian Imaging with multiple surfaces



$$\frac{1}{z_1'} - \frac{1}{z_1} = \frac{N_1' - N_1}{R_1}$$

SOLVE FOR l_1'

$$\frac{1}{z_2'} - \frac{1}{z_2} = \frac{N_2' - N_2}{R_2} \quad \text{but} \quad z_2 = z_1' - t_1' \; ; \; N_2 = N_1'$$

SOLVE FOR z_2' . . .

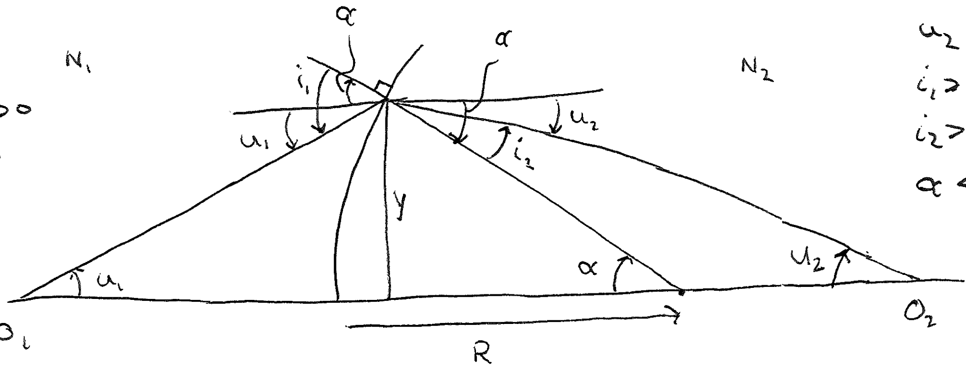
The output (image location) for the 1st surface becomes the input (object location) for the next surface. The process is repeated until the final image plane. Doable, but tedious

1.3.2 ynu raytrace

- Method for tracing rays through a paraxial system. We need two pieces of information for ray tracing. First, how does the ray change direction (refract) at the interface of two materials. Second, how does the ray change height ~~over a region of uniform~~ between interfaces

Counterclockwise $\angle > 0$

clockwise $\angle < 0$



$u_1 > 0$

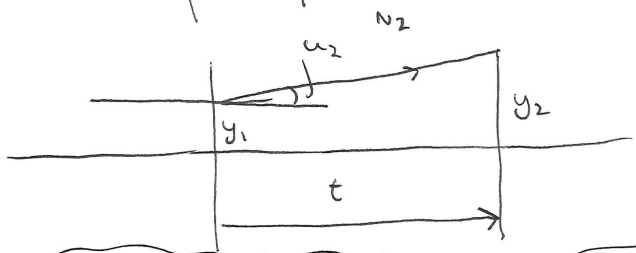
$u_2 < 0$

$i_1 > 0$

$i_2 > 0$

$\alpha < 0$

Paraxial Transfer Equation



$$\tan u_2 \simeq u_2 = \frac{y_2 - y_1}{t}$$

$$\boxed{y_2 = y_1 + u_2 t = y_1 + N_2 u_2 \left(\frac{t}{N_2}\right)}$$ Paraxial Transfer Eq.

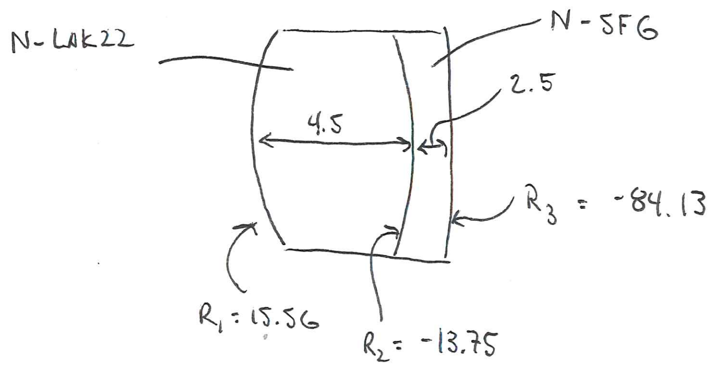
To trace rays through an optical system repeat the refraction and transfer process for each surface and intervening space.

NOTE: FOR REFLECTIVE SURFACES $N' = -N$

We can use a spread sheet to perform the paraxial raytracing

SHOW EXCEL SPREADSHEET EXAMPLE

Example lens is achromatic doublet from Edmund Optics 45793



$\lambda = 880$ nm

for N-LAK22 $N = 1.6408$

for N-SF6 $N = 1.7801$

at design wavelength

To determine where image is formed in the example

$$y_{IMAGE} = y_3 + N_3' u_3' \left(\frac{t_3'}{N_3'}\right)$$

want $y_{IMAGE} = 0$

$$\boxed{t_3' = \frac{-N_3' y_3}{N_3' u_3'}} = \frac{-y_3}{u_3'} \text{ in AIR}$$

From the drawing

$$\boxed{i_1 = u_1 - \alpha} \qquad i_2 - u_2 = -\alpha \Rightarrow \boxed{i_2 = u_2 - \alpha}$$

In paraxial approximation, we assume $u_1, u_2, i_1, i_2, \alpha$ all $\ll 1$

So $$\sin i = i - \frac{i^3}{3!} + \dots \cong i$$

Snell's law $$N_1 \sin i_1 = N_2 \sin i_2$$

becomes $\boxed{N_1 i_1 = N_2 i_2}$

NOTE: when we derived traverse magnification, we used $N_1 u_1 = N_2 u_2$. The ray in this case passed through the ~~axy~~ vertex of the surface so $\alpha = 0$ and $i_1 = u_1$ and $i_2 = u_2$.

Plugging in the expression for i_1 and i_2

$$N_1 u_1 - N_1 \alpha = N_2 u_2 - N_2 \alpha \Rightarrow N_2 u_2 = N_1 u_1 + (N_2 - N_1)\alpha$$

Again from our drawing and using the small angle approximation

$$\tan(-\alpha) \cong -\alpha = \frac{y}{R}$$

So $$\alpha = \frac{-y}{R}$$

$$N_2 u_2 = N_1 u_1 - \frac{N_2 - N_1}{R} y =$$

$$\boxed{N_2 u_2 = N_1 u_1 - \Phi y}$$ Paraxial refraction equation

In our example, what are ~~the~~ the two rays corresponding to y_a, nu_a and y_b, nu_b?

- If the aperture stop is at surface 1 ~~other~~ and the diameter of the aperture is 2 mm, then $ray_a \Rightarrow$ marginal ray and $ray_b \Rightarrow$ chief ray.

What if stop located somewhere else?

<u>Ray Scaling</u> - Once two independent rays are traced through the system, a new ray can be formed as a linear combination of the two rays. In other words

$$y_c = A\, y_a + B\, y_b$$
$$nu_c = A\, nu_a + B\, nu_b$$

$$\begin{pmatrix} y_a & y_b \\ nu_a & nu_b \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} y_c \\ nu_c \end{pmatrix} \qquad A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} y_a & y_b \\ nu_a & nu_b \end{pmatrix}^{-1} \begin{pmatrix} y_c \\ nu_c \end{pmatrix} \qquad A^{-1} = \frac{1}{\text{Det}\, A} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$$

$$\begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{y_a nu_b - y_b nu_a} \begin{pmatrix} nu_b & -y_b \\ -nu_a & y_a \end{pmatrix} \begin{pmatrix} y_c \\ nu_c \end{pmatrix}$$

$$\boxed{A = \frac{y_c nu_b - y_b nu_c}{y_a nu_b - y_b nu_a}} \qquad \boxed{B = \frac{-y_c nu_a + y_a nu_c}{y_a nu_b - y_b nu_a}}$$

In our doublet example, suppose ~~the stop is at surface 3. The first marginal ray,~~ we want a ray with $nu_c = 0.1$ and $y_c = 0$ at surface 3. ~~At this point we don't have enough information to determine the required ... so let's just assume nu_c = ... for now.~~

The ray scaling coefficients are

subscript 3 means at surface 3

$$A = \frac{-0.1\, \bar{y}_{b3}}{y_{a3}\, nu_{b3} - \bar{y}_{b3}\, nu_{a3}} \qquad B = \frac{0.1\, y_{a3}}{y_{a3}\, nu_{b3} - \bar{y}_{b3}\, nu_{a3}}$$

Calculate $y_{a3}\, nu_{b3} - \bar{y}_{b3}\, nu_{a3}$ on spread sheet

$$y_{a3}\, nu_{b3} - \bar{y}_{b3}\, nu_{a3} = 0.1$$

$$A = -\bar{y}_{b3} = -0.4186 \qquad B = y_{a3} = 0.841838$$

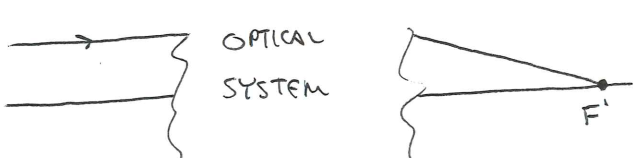
Calculate y_c, nu_c on spread sheet

1.3.3 Cardinal Points

We are typically interested in finding 6 cardinal points of an optical system: Front and Rear Focal Points; Front and Rear Nodal Points; Front and Rear Principal Points. Knowledge of the locations of these points allows us to simplify multielement complex optical systems down to a black box where the Principal Points (and their corresponding planes) allow us to determine the imaging properties of the system

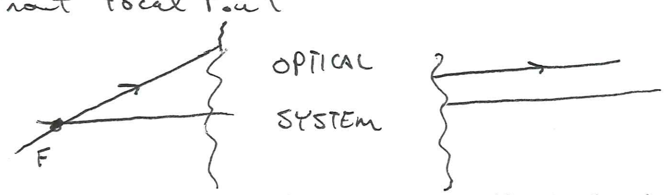
Definitions

Rear Focal Point



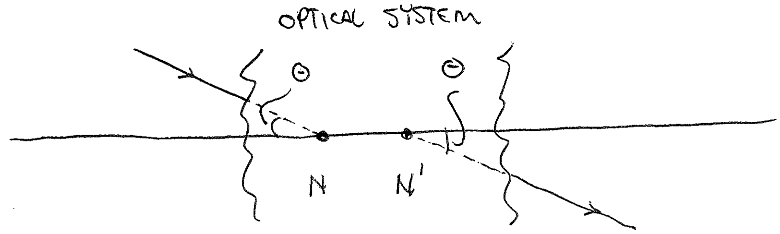
where a collimated ray in object space intersects the optical axis in image space

Front Focal Point



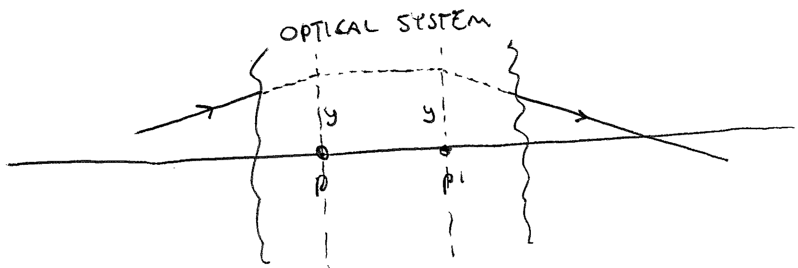
A ray passing through the Front Focal Point in object space, emerges parallel to optical axis in image space

FRONT / REAR NODAL POINTS



A ray appearing to pass through the front Nodal Point at an angle θ, appears to emerge from the rear nodal point at the same angle θ.

FRONT / REAR PRINCIPAL PLANES (POINTS)



The Principal Points define a pair of planes perpendicular to the optical axis. A ray appearing to strike the front PRINCIPAL plane at a height y gets mapped to the Rear PRINCIPAL plane at the same height for the emerging ray. NOTE: The angle of the emerging ray is typically different than the incident ray.

How do we find the Cardinal Points from our ynu raytrace?

<u>Rear Focal Point</u> - Trace a ray with $nu_1 = 0$ through the system. The incident ray height is arbitrary ($y_1 = 1$ is a nice choice). The Rear Focal Point is the point where the ray emerging from the last surface crosses the optical axis. NOTE: you may need to project this ray backwards to find an intersection.

In our doublet example, we have already traced this ray. with $y_a = 1$ and $nu_a = 0$. We also calculate $t_3' = 21.04498$ mm to get $y_{a\,image} = 0$. This means F' is located 21.04498 mm behind the last surface. NOTE: EDMUND website calls this Back Focal Length (BFL). Don't confuse with Rear Focal Length we defined in a previous class. Later we will call the Back Focal Distance

FRONT FOCAL POINT - Trace two independent rays through the system. Use ray scaling to create a ray with $nu_{C\,IMAGE} = 0$. Again $y_{C\,IMAGE}$ is arbitrary so choose $y_{C\,IMAGE} = 1.0$.

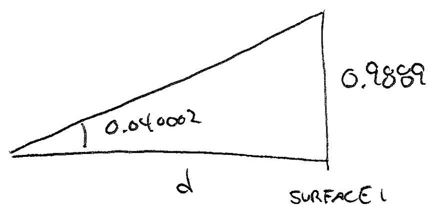
For the doublet example, the ray scaling equations are

$$A = \frac{nu_{b\,IMAGE}}{0.1} \qquad B = \frac{-nu_{a\,IMAGE}}{0.1}$$

NOTE: DENOMINATOR IS THE SAME AS PREVIOUS EXAMPLE

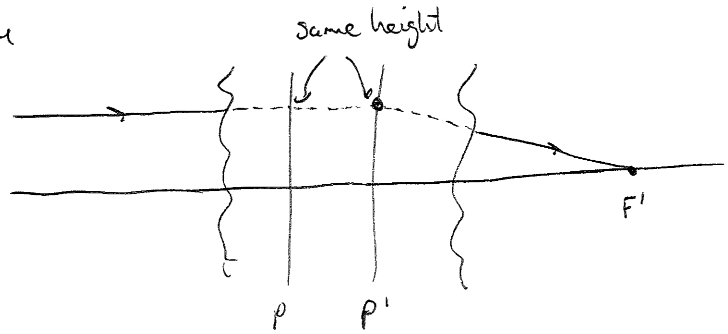
These values correspond to a ray with $y_{c1} = 0.98897$ $\quad nu_{c1} = 0.040002$ $= u_{c1}$

Where does this ~~strike~~ cross the optical axis?

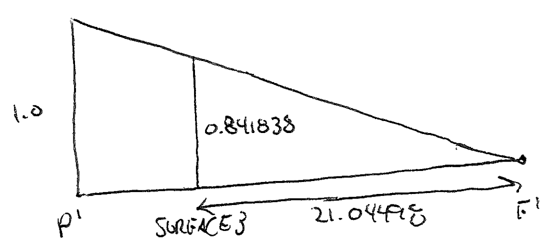


$$d = \frac{0.98897}{0.040002}$$

REAR PRINCIPAL POINT (PLANE) - ~~Determine where~~ Trace ray with $nu_1 = 0$ (e.g. the ray we used to find the rear focal point). Determine location of the intersection the object space ray and it's corresponding image space ray



In the doublet example, use similar triangles



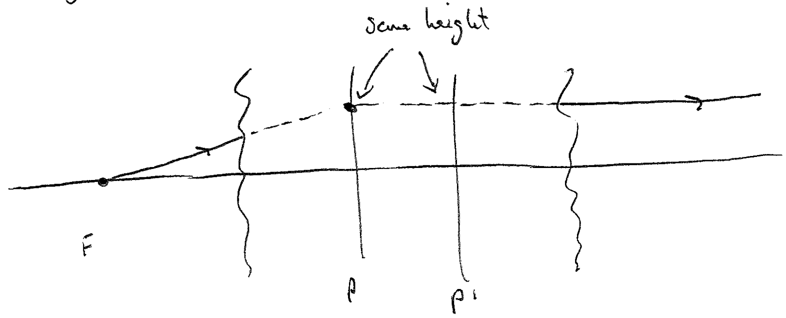
$\overline{P'F'}$ IS REAR FOCAL LENGTH f_R'

$$\frac{y_3}{t_3'} = \frac{1}{P'F'}$$

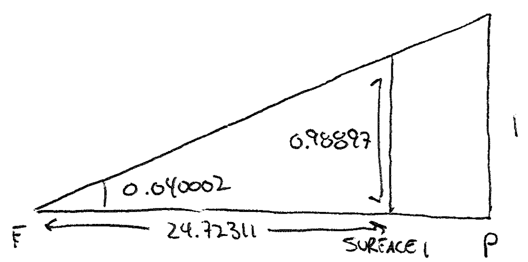
$$\overline{P'F'} = \frac{21.04498}{0.841838} = 24.999 \text{ mm}$$

~~P'F' IS REAR FOCAL LENGTH f_R'~~

FRONT PRINCIPAL PLANE (POINT) - TRACE RAY WHERE $nu_{IMAGE} = 0$ (e.g. The ray we used to find the front focal point). Determine where the object and image portions of the ray intersect.



In the doublet example, use similar triangles



$$\frac{0.98897}{24.72311} = \frac{1}{-\overline{PF}}$$

$$\overline{PF} = -24.999 \text{ mm}$$

$\overline{PF}$ is FRONT FOCAL LENGTH f_F

FRONT/REAR NODAL POINTS

In general, the nodal points are shifted relative to the principal points. The amount of shift is given by

$$\overline{PN} = \overline{P'N'} = f_F + f_R' = (n' - n) f_E$$ where n' is image space index, n is object space index

In many cases $n' = n$, so $\overline{PN} = \overline{P'N'} = 0$

In our doublet example, the lens is in air, so the Nodal Points coincide with the Principal Points.

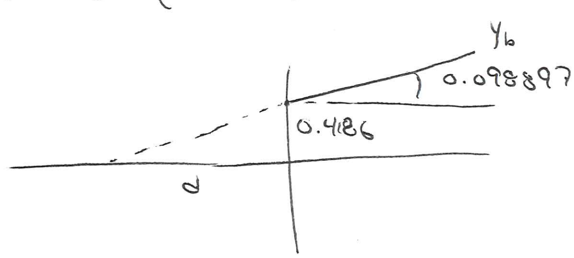
1.3.4 ENTRANCE AND EXIT PUPILS

The ~~pupil~~ entrance pupil is the image of the aperture stop formed in object space by all of the optical surfaces preceding it. The exit pupil is the image of the aperture stop in image space formed by all of the optical surfaces following the aperture stop. Finally, the entrance pupil is conjugate to the exit pupil meaning that if a object is placed at the entrance pupil, then the image is at the exit pupil.

The positions of the entrance and exit pupils can be determined by where the chief ray appears to cross the optical axis in object and image space. The height of the marginal ray at these crossings determines the size of the pupil.

SHOW SLIDES (1-3)

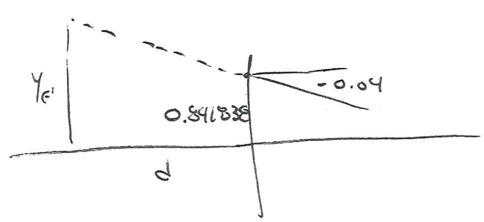
For our doublet example, entrance pupil is at the first surface and has a diameter of 2 mm.



Surface 3

$$d = \frac{0.4186}{0.098897} = 4.233 \text{ mm}$$

left of surface 3



Surface 3

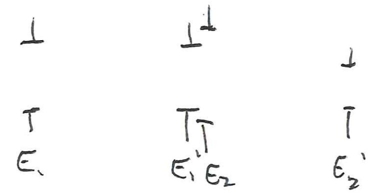
$$Y_{E'} = 0.841838 + 0.04(4.233)$$

$$Y_{E'} = 1.011 \text{ mm}$$

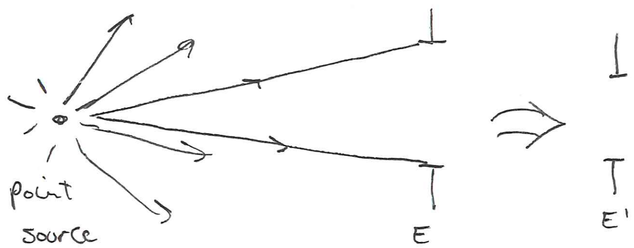
SHOW EXCEL SPREAD SHEET

Cascading Optical Systems

Typically you want to match the entrance and exit pupils of cascaded systems

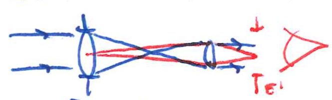


The entrance pupil can be thought of as a port that captures light from the object scene. The larger the port, the more light that gets through the system.



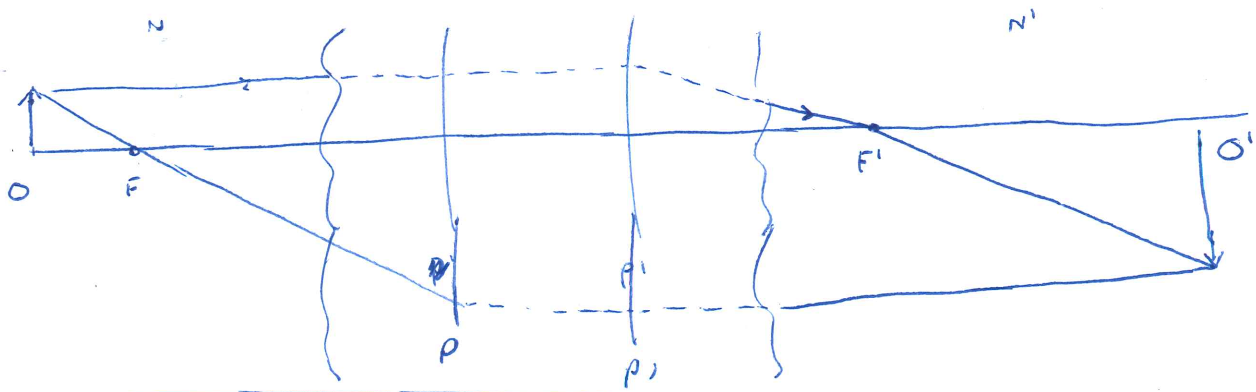
The exit pupil (for a well-corrected optical system) is just a 1:1 mapping from the entrance pupil.

KEPLARIAN TELESCOPE EXAMPLE



1.3.5 EXTENSION OF GAUSSIAN IMAGING TO THICK SYSTEMS

Knowledge of the Cardinal Points allow us to extend the gaussian imaging equation to thick lenses and multielement systems.



$z' = \overline{P'O'}$

$z = \overline{PO}$

$$\boxed{\frac{1}{\overline{P'O'}} - \frac{1}{\overline{PO}} = \Phi = \frac{N'}{\overline{P'F'}} = \frac{-N}{\overline{PF}}}$$

Gaussian Imaging Eq.

$f_R' = \overline{P'F'}$ $\quad f_F = \overline{PF}$

$\boxed{1.3.6}$ Transverse and Longitudinal Magnification

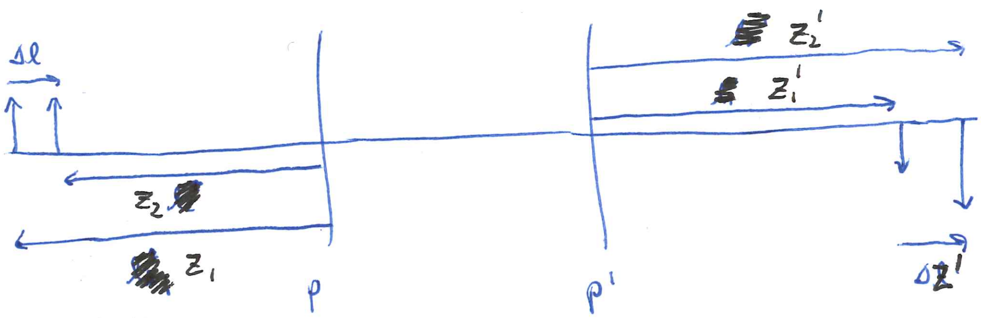
We already showed for a single surface that the transverse magnification m is

$$m = \frac{N z'}{N' z}$$

For thick systems $z = \overline{PO}$ and $z' = \overline{P'O'}$ and the same definition holds

$$m = \frac{N\, \overline{P'O'}}{N'\, \overline{PO}}$$

For longitudinal magnification, we are interested in how far the image plane shifts when we shift the object plane



$$\Delta z = z_2 - z_1$$

$$\Delta z' = z_2' - z_1'$$

Already showed the gaussian imaging eq. can be written in terms of transverse magnification

$$z_1 = -f_F\left(\frac{1-m_1}{m_1}\right)$$

$$z_1' = f_R'\,(1-m_1)$$

$$\Delta z = -f_F\left(\frac{1-m_2}{m_2} - \frac{1-m_1}{m_1}\right)$$

$$\Delta z = -f_F\left(\frac{m_1 - m_2}{m_1 m_2}\right)$$

Similarly

$$z_2 = -f_F\left(\frac{1-m_2}{m_2}\right)$$

$$z_2' = f_R'\,(1-m_2)$$

$$\Delta z' = f_R'\,\big(1-m_2-(1-m_1)\big)$$

$$\Delta z' = f_R'\,(m_1 - m_2) \Rightarrow m_1 - m_2 = \frac{\Delta z'}{f_R'}$$

$$\Delta z = -f_F \left(\frac{\Delta z'}{f_R' m_1 m_2} \right)$$

Recall $f_E = \dfrac{f_R'}{N'} = \dfrac{-f_F}{N}$

$$\boxed{\Delta z' = \frac{N'}{N} m_1 m_2 \Delta z}$$

Also for small Δz, $m_1 \simeq m_2 \Rightarrow \Delta z' = \dfrac{N'}{N} m^2 \Delta z$

Local longitudinal magnification $\boxed{\bar{m} = \left(\dfrac{N'}{N}\right) m^2}$

1.3.7 Lagrange invariant, Etendue, Throughput, $A\Omega$ Product

The Lagrange invariant ~~(optical invariant)~~ is a quantity that is constant throughout the optical system.

Paraxial Refraction Eq.

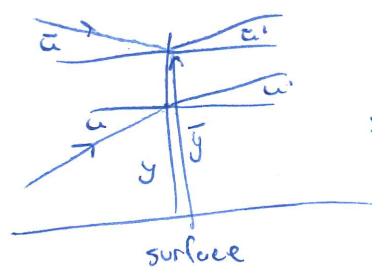
$$N'\bar{u}' = N\bar{u} - \phi\bar{y} \qquad N'u' = Nu - \phi y$$

primes mean after surface

chief ray $\bar{y}, N\bar{u}$

marginal ray y, Nu

$$\phi = \frac{N\bar{u} - N'\bar{u}'}{\bar{y}} = \frac{Nu - N'u'}{y}$$



Quantity is same before and after surface

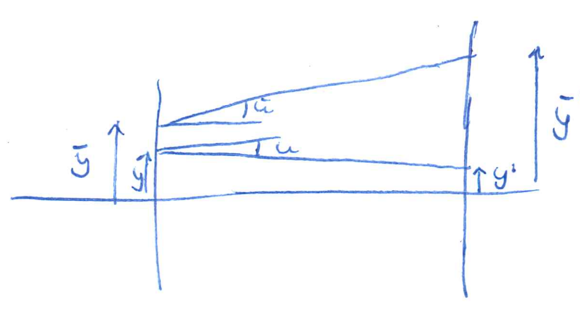
$$N\bar{u}y - N'\bar{u}'y = Nu\bar{y} - N'u'\bar{y}$$

$$\boxed{N\bar{u}y - Nu\bar{y} = N'\bar{u}'y - N'u'\bar{y}}$$

Paraxial Transfer Eq.

$$\bar{y}' = \bar{y} + N\bar{u}\left(\frac{t}{N}\right) \qquad y' = y + Nu\left(\frac{t}{N}\right)$$

$$\frac{t}{N} = \frac{\bar{y}' - \bar{y}}{N\bar{u}} = \frac{y' - y}{Nu}$$



$$\boxed{N\bar{u}y - Nu\bar{y} = N\bar{u}y' - Nu\bar{y}'}$$

Quantity is same after transfer

Define Lagrange invariant Ж (Cyrillic "Zhe")

$$\boxed{\text{Ж} = n\bar{u}y - nu\bar{y}}$$

Special cases

@ image plane $y=0$ $\text{Ж} = -nu\bar{y}$

@ Pupil plane $\bar{y}=0$ $\text{Ж} = n\bar{u}y$

We already saw this in our ynu raytrace

Show Excel spreadsheet and propagate $y_a n u_b - y_b n u_a$ forward + backwards
Handy way of verifying you did spreadsheet correctly.

The optical invariant I is a generalization when rays besides the marginal and chief ray are used

$$\boxed{I_{ij} = n u_i y_j - n u_j y_i}$$

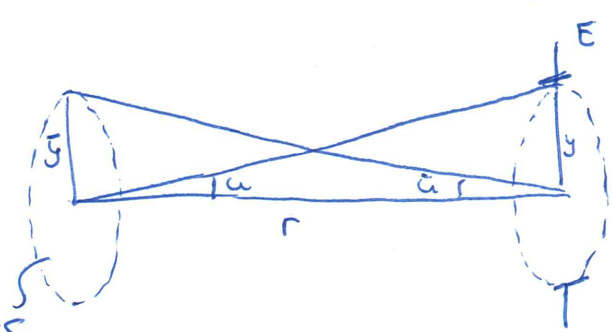
Recall our ray scaling coefficients

$$y_c = A y_a + B y_b \qquad u_c = A u_a + B u_b$$

$$A = \frac{I_{cb}}{I_{ab}} \qquad B = \frac{I_{ac}}{I_{ab}}$$

The throughput, etendue or $A\Omega$ product are related to the square of the Lagrange invariant

$\cancel{\pi^2 A\Omega = \pi^2 \text{Ж}^2}$



$\mathbb{H}_E = n\bar{u}y \qquad \mathbb{H}_S = -nu\bar{y}$

Area of Entrance Pupil $A = \pi y^2$

• Solid angle of source as seen from ~~exit~~ entrance pupil

$$\Omega = \frac{\pi \bar{y}^2}{r^2}$$

$$A\Omega = \frac{\pi^2 y^2 \bar{y}^2}{r^2} \qquad u = \frac{y}{r} \qquad \bar{u} = \frac{-\bar{y}}{r}$$

$$A\Omega = \pi^2 \bar{u}y \cdot u\bar{y}$$

$$n^2 A\Omega = \pi^2 (n\bar{u}y)(nu\bar{y})$$

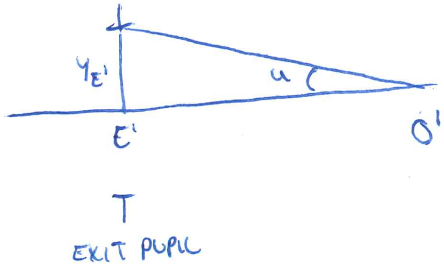
$$n^2 A\Omega = \pi^2 (\mathbb{H}_E)(\mathbb{H}_S) \qquad \text{but } \mathbb{H}_E = \mathbb{H}_S$$

$$\boxed{n^2 A\Omega = \pi^2 \mathbb{H}^2}$$

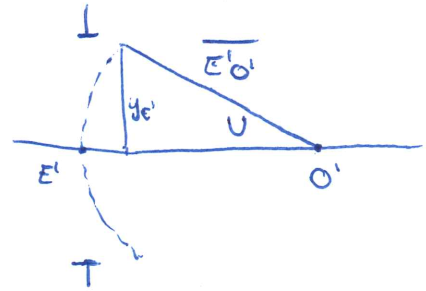
Increasing $|\mathbb{H}|$ increase the amount of light getting into the entrance pupil.

1.3.8 F-Number, Working F-Number and Numerical Aperture

In the paraxial picture



In non-paraxial picture



$$u = \frac{y_{E'}}{\overline{E'O'}} \qquad \sin U = \frac{y_{E'}}{\overline{E'O'}}$$

so $u = \sin U$ relates paraxial angle u to real marginal ray angle U

Numerical aperture $= |n' \sin U'| = |n'u'|$ where prime denotes image space

We can also define Numerical Aperture in object space

What does high NA mean? Large angle $U \Rightarrow$ bigger pupil = more light

What is max value of NA? $NA_m = n'$ since $|\sin U|_{max} = 1$

ASIDE: MICROSCOPE OBJECTIVES

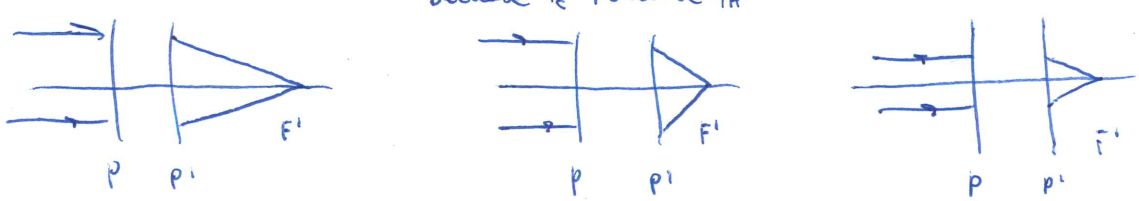
SHOW SLIDES COMMENT ON TUBE LENGTH / NA

F-NUMBER

$$F/\# = \frac{f_E}{D_{EP}}$$ where D_{EP} is diameter of entrance pupil

This value describes the cone of light in image space for an object at infinity.

Decrease f_E / Decrease F/# — Decrease D_{EP} / Increase F/#



SHOW SLIDE OF CAMERA LENS

SEQUENCE OF F/#s

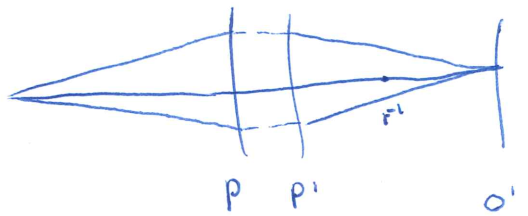
F/#	22	16	11	8	5.6	4	2.8	2
D_{EP} (mm)	1.59	2.19	3.18	4.38	6.25	8.75	12.5	17.5
πD_{EP}^2	7.94	15.07	31.77	60.26	-	-	-	-

In general for fixed f_E, halving f/#, quadruples entrance pupil area

If we assume a thin lens with stop at the lens then

$$f/\# \simeq \frac{1}{2\,NA}$$ NA in image space

Often we are working a finite conjugates, so working f/# more appropriate

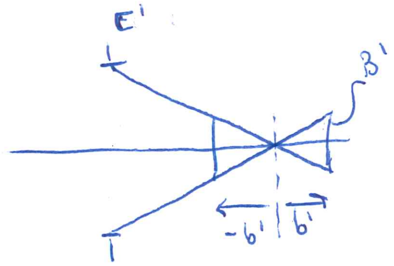


$$\text{Working } f/\# = (1-m)\frac{f_E}{D_{EP}}$$

~~we say~~ "Fast" optical systems are systems with low F/#

$\boxed{1.3.9}$ Depth of Field / Depth of Focus

Depth of Focus DOF is an image space concept



~~DOF ≈ ±b'~~

suppose, we can tolerate some blur size B'

then we say DOF = ± b'

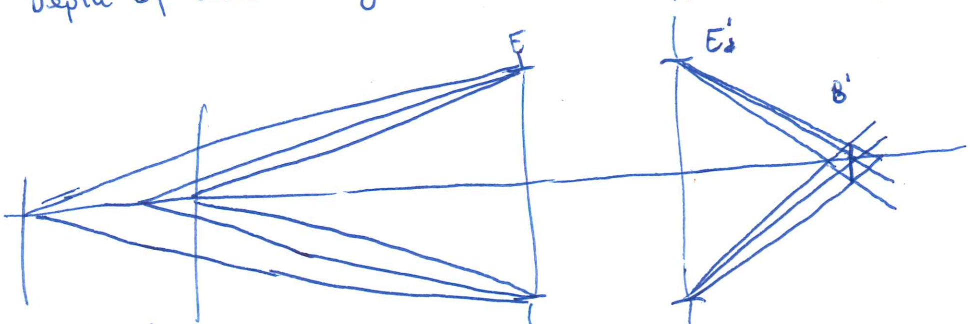
$$DOF = \pm B' f/\# = \pm \frac{B'}{2NA}$$

B' might be the dimensions of a pixel or some other value which is application specific

For the microscope objectives, the higher the NA, the shallower the DoF

For the camera lens, the higher the F/#, the larger the DOF

Depth of Field is just the DOF mapped to object space

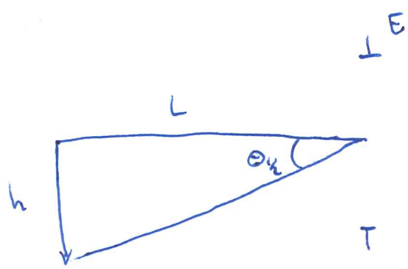


Depth of Field

SHOW CAMERA LENS DOF

1.3.10 Field of view

Field of View FOV (sometimes FFOV for full field of view) is the maximum angular size of the object as seen from the entrance pupil. HFOV is half field of view. Sometimes FOV is used loosely, so verify FFOV or HFOV



HFOV = $\theta_{1/2}$ FFOV = $2\theta_{1/2}$

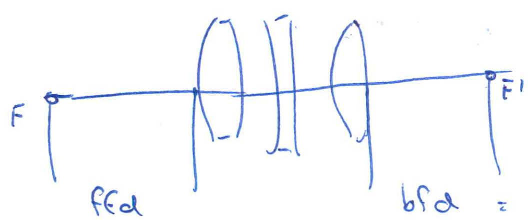
$$\tan\theta_{1/2} = \frac{h}{L}$$

in paraxial picture

$$\bar{u} = \tan\theta_{1/2} = \frac{h}{L}$$

1.3.11 Front and Back Focal Distances

Distance from respective focal point to the vertex of the first/last optical surface.



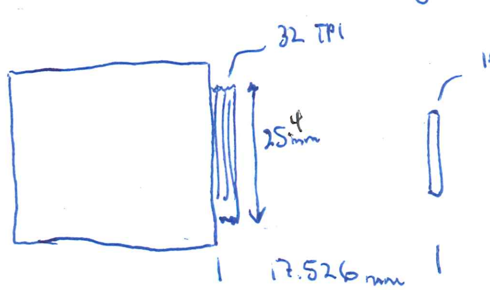
bfd = back focal distance

ffd
front focal distance

NOTE: EDMUND page called this back focal length in achromat example. Do not CONFUSE with rear focal length f_R'

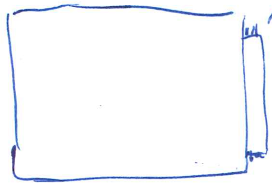
1.3.11.1 Standard flange distances for cameras

C-mount   originate with TV and movie cameras



CS-mount   same threads as C-mount but flange focal distance is 12.526 mm   GO TO PTGREY AND EDMUND WEBSITES

T-MOUNT



M42 X 0.75 Thread

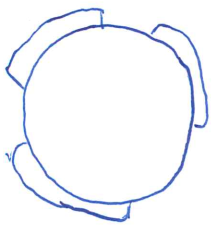
metric 42 mm Diameter

0.75 mm pitch for threads

F-MOUNT (NIKON LENS FAMILY)

Bayonet Mount

flange distance 46.5 mm



44mm throat diameter

EF-MOUNT (CANON LENS FAMILY

throat diameter 54mm

flange distance 44mm