OPTI 415/515 Midterm1
Undergraduates and graduates answer all three questions

1. Suppose you have a wavefront given by $W(h, \rho, \psi)=W_{020} \rho^{2}+W_{040} \rho^{4}$ in a system where the reference sphere radius is 100 mm and the exit pupil diameter is 20 mm .
(a) Write an expression for the transverse ray error.

We can just combine the transverse ray errors from defocus and astigmatism, so

$$
\begin{aligned}
& \varepsilon_{x}\left(\rho_{x}, \rho_{y}\right)=-\frac{2 R}{r_{\max }} W_{020} \rho_{x}-\frac{4 R}{r_{\max }} W_{040}\left(\rho_{x}^{3}+\rho_{x} \rho_{y}^{2}\right) \\
& \varepsilon_{y}\left(\rho_{x}, \rho_{y}\right)=-\frac{2 R}{r_{\max }} W_{020} \rho_{y}-\frac{4 R}{r_{\max }} W_{040}\left(\rho_{x}^{2} \rho_{y}+\rho_{y}^{3}\right)
\end{aligned}
$$

where $R=100 \mathrm{~mm}$ is the reference sphere radius and $r_{\max }=10 \mathrm{~mm}$ (i.e. half the exit pupil diameter).
(b) For $\rho_{x}=0$ and $\rho_{y}=\frac{\sqrt{2}}{2}$, what value of $W_{020}$ makes $\varepsilon_{y}=0$ ?

Find $W_{020}$ when

$$
\varepsilon_{y}\left(0, \frac{\sqrt{2}}{2}\right)=-\frac{2 R}{r_{\max }} W_{020} \frac{\sqrt{2}}{2}-\frac{4 R}{r_{\max }} W_{040}\left(\frac{\sqrt{2}}{2}\right)^{3}=0
$$

Solving this gives $W_{020}=-W_{040}$.
2. We have an optical system in air with power $\Phi=0.02 \mathrm{~mm}^{-1}$. For an object distance of $\mathrm{z}=-300$ mm , answer the following:
(a) What is the image distance?

The Gaussian imaging equation is
$\frac{1}{z^{\prime}}-\frac{1}{-300}=0.02$
Solving gives $z^{\prime}=60 \mathrm{~mm}$.
(b) What is the transverse magnification?

The transverse magnification is $m=\frac{z^{\prime}}{z}=-\frac{60}{300}=-0.2$.
(c) How large can the object be to fit onto a $1 / 2$ " CCD array?

A 11/2" CCD array is $6.4 \times 4.8 \mathrm{~mm}$. Based on Part (b), the object can be $5 \times$ larger and be reduced by the optical system to fit on the detector. This suggests an object that is $32 \times 24 \mathrm{~mm}$.
3. The figure below shows the marginal and chief rays for a Cooke triplet in air. The surface numbers are labeled in the figure. Surface 3 is the aperture stop of the lens. The image plane is located 79.083 mm from the last lens surface (surface 7). The table on Page 3 shows the ray heights and angles for the marginal and chief rays for an object at infinity. Answer the following questions base on this data.

(a) In the figure, label the marginal ray and the chief ray.

See figure. The Marginal Ray goes through the edge of the aperture stop and the Chief Ray goes through the center of the aperture stop.
(b) Where is the rear principal plane located?

The rear principal plane $P^{\prime}$ is found by projecting the incident marginal ray forward and the exiting marginal ray backward to find their intersection.


The height of the marginal ray at the first surface is 17.41365 mm and the height of the marginal ray at the last surface is 13.77008 mm . Based on similar triangles

$$
\frac{17.41365}{79.083-\overline{V_{7} P^{\prime}}}=\frac{13.77008}{79.083} \Rightarrow \overline{V_{7} P^{\prime}}=-20.925 \mathrm{~mm}
$$

where $\overline{V_{7} P^{\prime}}$ is the distance from the vertex of surface 7 to the rear principal point $P^{\prime}$. The minus sign indicates the principal point is to the left of the last surface.
(c) What is the effective focal lens of the lens?

Since the object is at infinity, the rear focal point $F^{\prime}$ is at the image plane. The effective focal length is $f_{E}=\overline{P^{\prime} F^{\prime}}=79.083-(-20.925)=100.008 \mathrm{~mm}$.
(d) What is the Half Field of View (HFOV) of the system?

The HFOV is related to the incident angle of the chief ray. From the raytrace, the chief ray angle at the first surface is $\bar{u}_{1}=0.343969$ and the HFOV is given by

$$
\theta_{1 / 2}=\tan ^{-1}\left(\bar{u}_{1}\right)=18.98^{\circ}
$$

(e) Where is the Entrance Pupil located?

The entrance pupil is located where the incident chief ray appears to cross the optical axis.


Based on the triangle

$$
0.343969=\frac{5.35085}{\overline{V_{1} E}} \Rightarrow \overline{V_{1} E}=+15.556 \mathrm{~mm}
$$

where $\overline{V_{1} E}$ is the distance from the first surface vertex to the entrance pupil.
(f) If the Entrance Pupil diameter is 34.827 mm , what is the $\mathrm{F} / \#$ of the system?

$$
F / \#=\frac{f_{E}}{34.827}=2.87
$$

|  |  |  | Surface |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Object |  | 1 |  | 2 |  | Stop |  | 4 |  | 5 |  | 6 |  | 7 |  | Image |
| y (marginal) |  |  | 17.41365 |  | 16.14704 |  | 13.6 |  | 13.10948 |  | 12.92113 |  | 13.83049 |  | 13.77008 |  | 0 |
| nu (marginal) |  | 0 |  | $-0.34045$ |  | -0.30467 |  | -0.30467 |  | -0.19263 |  | 0.081266 |  | -0.01624 |  | -0.17412 |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $y$ (chief) |  |  | -5.35085 |  | -3.68194 |  | 0 |  | 0.709081 |  | 1.145645 |  | 6.413543 |  | 7.996781 |  | 34.4 |
| nu (chief) |  | 0.343969 |  | 0.448582 |  | 0.440423 |  | 0.440423 |  | 0.446483 |  | 0.470768 |  | 0.425553 |  | 0.333865 |  |

