1. The wavelengths 480nm and 580nm are complementary colors for a white point with chromaticity coordinates (0.3155, 0.3211). This means these two colors can be mixed to get neutral (gray) color. If the spectrum entering the eye is written as

$$P(\lambda) = B[A\delta(\lambda - 480) + (1 - A)\delta(\lambda - 580)]$$

Find the constants A and B which makes the chromaticity coordinates equal to the white point and has a value of Y = 100. Hint: Use the chromaticity coordinates (x,y) to find A, then use the Tristimulus values to find B.

wavelength	xbar	ybar	zbar		wavelength	xbar	ybar	zbar
380	0.001368	0.000039	0.00645		590	1.0263	0.757	0.0011
390	0.004243	0.00012	0.02005		600	1.0622	0.631	0.0008
400	0.01431	0.000396	0.06785		610	1.0026	0.503	0.00034
410	0.04351	0.00121	0.2074		620	0.85445	0.381	0.00019
420	0.13438	0.004	0.6456		630	0.6424	0.265	5E-05
430	0.2839	0.0116	1.3856		640	0.4479	0.175	0.00002
440	0.34828	0.023	1.74706		650	0.2835	0.107	0
450	0.3362	0.038	1.77211		660	0.1649	0.061	0
460	0.2908	0.06	1.6692		670	0.0874	0.032	0
470	0.19536	0.09098	1.28764		680	0.04677	0.017	0
480	0.09564	0.13902	0.81295		690	0.0227	0.00821	0
490	0.03201	0.20802	0.46518		700	0.011359	0.004102	0
500	0.0049	0.323	0.272		710	0.00579	0.002091	0
510	0.0093	0.503	0.1582		720	0.002899	0.001047	0
520	0.06327	0.71	0.07825		730	0.00144	0.00052	0
530	0.1655	0.862	0.04216		740	0.00069	0.000249	0
540	0.2904	0.954	0.0203		750	0.000332	0.00012	0
550	0.43345	0.99495	0.00875		760	0.000166	0.00006	0
560	0.5945	0.995	0.0039		770	8.31E-05	0.00003	0
570	0.7621	0.952	0.0021		780	4.15E-05	1.5E-05	0
580	0.9163	0.87	0.00165					

The tristimulus values are given by

$$X = B\int [A\delta(\lambda - 480) + (1 - A)\delta(\lambda - 580)]\bar{x}(\lambda)d\lambda = B[0.09564A + 0.9163(1 - A)]$$

$$Y = B\int [A\delta(\lambda - 480) + (1 - A)\delta(\lambda - 580)]\bar{y}(\lambda)d\lambda = B[0.13902A + 0.87(1 - A)]$$

$$Z = B\int [A\delta(\lambda - 480) + (1 - A)\delta(\lambda - 580)]\bar{z}(\lambda)d\lambda = B[0.81295A + 0.00165(1 - A)]$$

The x chromaticity coordinate is given by

$$x = \frac{B[0.09564A + 0.9163(1-A)]}{B[(0.09564 + 0.13902 + 0.81295)A + (0.9163 + 0.87 + 0.00165)(1-A)]} = 0.3155$$

Solving for A gives A = 0.6.

The Y tristimulus value is now

$$Y = B[0.13902 \times 0.6 + 0.87 \times 0.4] = 100$$

which gives B = 231.797.

2. Write $\rho_x^2 + 2\rho_x\rho_y - \rho_y^2$ in terms of Zernike polynomials, where $\rho^2 = \rho_x^2 + \rho_y^2$. The following trig identities may be useful:

$$\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta \\ \sin 2\theta &= 2\sin\theta\cos\theta \\ \cos 3\theta &= 4\cos^3\theta - 3\cos\theta \\ \sin 3\theta &= 3\sin\theta - 4\sin^3\theta \end{aligned}$$

Convert to polar coordinates

$$\rho^2\cos^2\theta + 2\rho^2\cos\theta\sin\theta - \rho^2\sin^2\theta$$

$$\rho^2\cos 2\theta + \rho^2\sin 2\theta$$

$$\frac{1}{\sqrt{6}}\left[Z_2^2(\rho,\theta) + Z_2^{-2}(\rho,\theta)\right]$$

3. An IOL manufacturer makes IOLs in a range of powers from 10D to 30D in steps of 0.5D. The SRK formula is used to calculate the required IOL power for a given eye. The SRK formula is

$$\phi_{IOL} = A - 0.9K - 2.5L\,,$$

where ϕ_{IOL} is the IOL power in diopters, *A* is a constant provided by the manufacturer, *K* is the corneal power in diopters and *L* is the axial length of the eye in mm. The manufacturer suggests an *A*-constant of 118 be used. The corneal power is measured to be $K = 43\text{D}$ and the axial length of the eye is $L = 24\,\text{mm}$.

(a) What is the closest available IOL power to the one predicted by the SRK formula?

$$\phi_{IOL} = 118 - 0.9 \times 43 - 2.5 \times 24 = 19.3\,\text{D}\,.$$

The closest available lens is 19.5 D.

(b) If the cornea is considered a thin lens, where does the cornea alone form an image if no IOL is present? The object is at infinity.

$$\frac{1.336}{z_1'} = 0.043 \Rightarrow z_1' = 31.07 \text{ mm}$$

The image formed by the cornea is 31.07 mm behind the cornea.

(c) Where does the IOL need to sit relative to the cornea in order to form a sharp image on the retina for an object at infinity?

If d is the distance between the cornea and the IOL, then

$$\frac{1.336}{24-d} - \frac{1.336}{31.07-d} = 0.0195 \Rightarrow d = 5.24 \text{ mm or } d = 49.83 \text{ mm}.$$

Only the first answer makes sense, so the IOL is 5.24 mm behind the cornea.