1. (25 Points) Suppose the cornea is a conic section with radius R and conic constant K.

   (a) What is difference in corneal sag between this surface and a spherical surface with the same radius?

   *The sag of a sphere is given by* $z = R - \sqrt{R^2 - r^2}$ .
   *The sag of a conic is given by* $z = \frac{1}{K+1}\left[R - \sqrt{R^2 - (K+1)r^2}\right]$.
   *Subtract the two to get the difference is sag.*

   (b) Show that the axial power $\phi_a$ of the cornea is given by $\phi_a = \frac{(n_k - 1)}{\sqrt{R^2 - Kr^2}}$

$$\textit{If } z = \frac{1}{K+1}\left[R - \sqrt{R^2 - (K+1)r^2}\right], \textit{ then } \frac{dz}{dr} = \frac{r}{\sqrt{R^2 - (K+1)r^2}}, \textit{ and}$$

$$1 + \left(\frac{dz}{dr}\right)^2 = \frac{R^2 - (K+1)r^2}{R^2 - (K+1)r^2} + \frac{r^2}{R^2 - (K+1)r^2} = \frac{R^2 - Kr^2}{R^2 - (K+1)r^2}$$

   *Axial power is given by*

$$\phi_a = \frac{(n_k - 1)dz/dr}{r\sqrt{1 + (dz/dr)^2}} = \frac{(n_k - 1)r\sqrt{R^2 - (K+1)r^2}}{r\sqrt{R^2 - (K+1)r^2}\sqrt{R^2 - Kr^2}} = \frac{(n_k - 1)}{\sqrt{R^2 - Kr^2}}$$

   (c) Show that the instantaneous power $\phi_i$ of the cornea is given by $\phi_i = \frac{(n_k - 1)R^2}{\left[R^2 - Kr^2\right]^{3/2}}$

   *The instantaneous power is given by*

$$\phi_i = \frac{d(r\phi_a)}{dr} = r\frac{d\phi_a}{dr} + \phi_a = \frac{(n_k - 1)Kr^2}{\left[R^2 - Kr^2\right]^{3/2}} + \frac{(n_k - 1)}{\sqrt{R^2 - Kr^2}}\frac{R^2 - Kr^2}{R^2 - Kr^2}$$

$$\phi_i = \frac{(n_k - 1)R^2}{\left[R^2 - Kr^2\right]^{3/2}}$$

2. (25 Points) The local planetarium is showing a Laser Metallica show. They have three lasers to create the special effects. The wavelengths of these lasers are 480 nm, 540 nm and 630 nm and each laser has the same maximum power.

   (a) What are the chromaticity coordinates of each laser?

*The spectrum for the blue laser is given by* $\mathrm{P}(\lambda)=\mathrm{B}\delta(\lambda-480)$ *where B is the power of the laser. The Tristimulus values are given by*

$$\mathrm{X}=\int\left[\mathrm{B}\delta(\lambda-480)\right]\bar{\mathrm{x}}(\lambda)d\lambda=\mathrm{B}\bar{\mathrm{x}}(480)$$
$$\mathrm{Y}=\int\left[\mathrm{B}\delta(\lambda-480)\right]\bar{\mathrm{y}}(\lambda)d\lambda=\mathrm{B}\bar{\mathrm{y}}(480)$$
$$\mathrm{Z}=\int\left[\mathrm{B}\delta(\lambda-480)\right]\bar{\mathrm{z}}(\lambda)d\lambda=\mathrm{B}\bar{\mathrm{z}}(480)$$

*The chromaticity coordinates are*

$$\mathrm{x}=\frac{\mathrm{X}}{\mathrm{X}+\mathrm{Y}+\mathrm{Z}}=\frac{\mathrm{B}\bar{\mathrm{x}}(480)}{\mathrm{B}\left[\bar{\mathrm{x}}(480)+\bar{\mathrm{y}}(480)+\bar{\mathrm{z}}(480)\right]}=\frac{0.09564}{0.09564+0.13902+0.8129501}=0.0913$$

$$\mathrm{y}=\frac{\mathrm{Y}}{\mathrm{X}+\mathrm{Y}+\mathrm{Z}}=\frac{0.13902}{0.09564+0.13902+0.8129501}=0.1327$$

*Similar calculations are made for the other wavelengths.*
*For the green laser*

$$\mathrm{x}=\frac{0.2904}{0.2904+0.9540+0.0203}=0.2296$$
$$\mathrm{y}=\frac{0.9540}{0.2904+0.9540+0.0203}=0.7543$$

*For the red laser*

$$\mathrm{x}=\frac{0.6424}{0.6424+0.2650+0.00005}=0.7079$$
$$\mathrm{y}=\frac{0.2650}{0.6424+0.2650+0.00005}=0.2920$$

(b) The three laser spots are overlapped to create a new color. If the blue laser provides 1 Watt, what powers do the red and green lasers need to put out to create a white spot? Assume the equal energy white ($x_w$ = 0.333, $y_w$ = 0.333).

*The equal energy white point has X = Y = Z = C, where C is a constant. If the combined spectrum of the three lasers is* $P(\lambda)=\delta(\lambda-480)+G\delta(\lambda-540)+R\delta(\lambda-630)$, *where G and R are the powers of the green and red laser, respectively, then*

$$X=\bar{x}(480)+G\bar{x}(540)+R\bar{x}(630)=C$$
$$Y=\bar{y}(480)+G\bar{y}(540)+R\bar{y}(630)=C$$
$$Z=\bar{z}(480)+G\bar{z}(540)+R\bar{z}(630)=C$$

*Solving these equations gives G = 0.4598 and R = 0.9234 (Also, C = 0.8223).*

3. (25 Points) The Tscherning ellipse describes the solution for zero astigmatism for a thin spectacle lens and is given by

$$\phi_1^2(n+2)-\phi_1\left[\frac{2}{q'}(n^2-1)+\Phi(n+2)\right]+n\left[\Phi+\frac{n-1}{q'}\right]^2=0$$

where n is the spectacle lens refractive index, q' is the separation between the lens and the center of rotation of the eye, $\phi_1$ is the power of the front surface of the lens and $\Phi$ is the total power of the lens. Assuming n = 1.494, q' = 0.027 m and $\Phi$ = +4.00 diopters:

(a) What are the two solutions for the front surface power $\phi_1$?

*Plugging in the known values gives the following quadratic equation:*

$$3.494\phi_1^2-105.238\phi_1+742.704=0\,.$$

*Solving for $\phi_1$ gives $\phi_1$ = 11.29 D and $\phi_1$ = 18.83 D. In terms of the surface radii, these are 43.766 mm and 26.231 mm, respectively.*

(b) What are the two back surface powers?

*For a thin lens, $\Phi = \phi_1 + \phi_2$, so the back surface powers are -7.29 D and -14.83 D, respectively.*

4. (25 Points) Suppose we want to design a new accommodating IOL based on a fluidic lens. The fluidic lens is shown in the figure below. It consists of one rigid surface and one flexible surface which form a hollow chamber. The space between the two lenses is filled with silicone oil (refractive index = 1.52). There is a reservoir off the side of the side of the lens which contains additional silicone oil. When the reservoir is compressed by the ciliary muscle extra silicone oil is pumped into the chamber causing the radius of the flexible surface to change from R1 to R2 and the thickness of the lens changes from t1 to t2. Assume that the thicknesses of the flexible and rigid surfaces are negligible.

(a) If we want the power of the flexible surface to be 20 diopters in the eye, what is R1?

$$\frac{1.52-1.336}{R1}=20D\Rightarrow R1=9.2mm$$

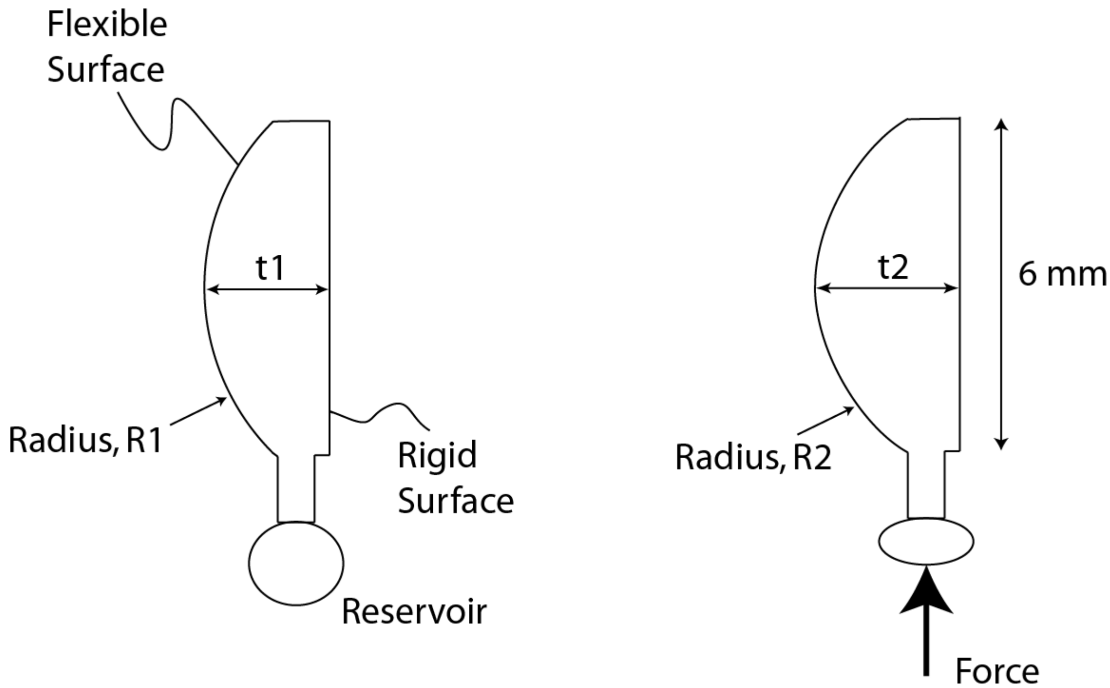
(b) To get accommodation, we want the flexible surface to increase its power to 23 diopters when the reservoir is compressed. What is R2 in this case?

$$\frac{1.52-1.336}{R2}=23D\Rightarrow R1=8.0mm$$

(c) If the thickness of the uncompressed lens is t1 = 2 mm, what is the thickness t2 of the lens in the compressed state? Assume a parabolic shape for the flexible surface.

*To keep the surface sag fixed at the edge of the lens (r = 3 mm), the following must be true:*

$$\frac{r^2}{2R1} - t1 = \frac{r^2}{2R2} - t2 \Rightarrow t2 = \frac{r^2}{2}\left(\frac{1}{R1} - \frac{1}{R2}\right) + t1 = 2.073\text{mm}$$



| λ | White | Color | IllumC | xbar | ybar | zbar |
|---|---|---|---|---|---|---|
| 380 | 0.153 | 0.118 | 31.3 | 0.001368 | 0.000039 | 0.006450001 |
| 390 | 0.245 | 0.179 | 45 | 0.004243 | 0.00012 | 0.02005001 |
| 400 | 0.409 | 0.283 | 60.1 | 0.01431 | 0.000396 | 0.06785001 |
| 410 | 0.671 | 0.343 | 76.5 | 0.04351 | 0.00121 | 0.2074 |
| 420 | 0.84 | 0.359 | 93.2 | 0.13438 | 0.004 | 0.6456 |
| 430 | 0.878 | 0.35 | 106.7 | 0.2839 | 0.0116 | 1.3856 |
| 440 | 0.883 | 0.327 | 115.4 | 0.34828 | 0.023 | 1.74706 |
| 450 | 0.886 | 0.298 | 117.8 | 0.3362 | 0.038 | 1.77211 |
| 460 | 0.887 | 0.267 | 116.9 | 0.2908 | 0.06 | 1.6692 |
| 470 | 0.888 | 0.239 | 117.6 | 0.19536 | 0.09098 | 1.28764 |
| 480 | 0.888 | 0.209 | 117.7 | 0.09564 | 0.13902 | 0.8129501 |
| 490 | 0.888 | 0.182 | 114.6 | 0.03201 | 0.20802 | 0.46518 |
| 500 | 0.887 | 0.163 | 106.5 | 0.0049 | 0.323 | 0.272 |
| 510 | 0.887 | 0.146 | 97.2 | 0.0093 | 0.503 | 0.1582 |
| 520 | 0.887 | 0.124 | 92 | 0.06327 | 0.71 | 0.07824999 |
| 530 | 0.887 | 0.106 | 93.1 | 0.1655 | 0.862 | 0.04216 |
| 540 | 0.887 | 0.102 | 97 | 0.2904 | 0.954 | 0.0203 |
| 550 | 0.886 | 0.107 | 99.9 | 0.4334499 | 0.9949501 | 0.008749999 |
| 560 | 0.887 | 0.106 | 100 | 0.5945 | 0.995 | 0.0039 |
| 570 | 0.888 | 0.112 | 97.2 | 0.7621 | 0.952 | 0.0021 |
| 580 | 0.887 | 0.141 | 92.9 | 0.9163 | 0.87 | 0.001650001 |
| 590 | 0.886 | 0.198 | 88.5 | 1.0263 | 0.757 | 0.0011 |
| 600 | 0.887 | 0.279 | 85.2 | 1.0622 | 0.631 | 0.0008 |
| 610 | 0.889 | 0.394 | 84 | 1.0026 | 0.503 | 0.00034 |
| 620 | 0.891 | 0.522 | 83.7 | 0.8544499 | 0.381 | 0.00019 |
| 630 | 0.891 | 0.628 | 83.6 | 0.6424 | 0.265 | 5E-05 |
| 640 | 0.89 | 0.696 | 83.4 | 0.4479 | 0.175 | 0.00002 |
| 650 | 0.889 | 0.742 | 83.8 | 0.2835 | 0.107 | 0 |
| 660 | 0.889 | 0.766 | 83.5 | 0.1649 | 0.061 | 0 |
| 670 | 0.888 | 0.78 | 82 | 0.0874 | 0.032 | 0 |
| 680 | 0.888 | 0.791 | 79.8 | 0.04677 | 0.017 | 0 |
| 690 | 0.888 | 0.798 | 76.2 | 0.0227 | 0.00821 | 0 |
| 700 | 0.888 | 0.804 | 72.5 | 0.01135916 | 0.004102 | 0 |
| 710 | 0.886 | 0.807 | 68.8 | 0.005790346 | 0.002091 | 0 |
| 720 | 0.886 | 0.807 | 64.9 | 0.002899327 | 0.001047 | 0 |
| 730 | 0.885 | 0.813 | 61.2 | 0.001439971 | 0.00052 | 0 |
| 740 | 0.884 | 0.813 | 58.4 | 0.000690079 | 0.0002492 | 0 |
| 750 | 0.883 | 0.808 | 56.2 | 0.000332301 | 0.00012 | 0 |
| 760 | 0.882 | 0.814 | 55.2 | 0.000166151 | 0.00006 | 0 |
| 770 | 0.88 | 0.785 | 55.3 | 8.30753E-05 | 0.00003 | 0 |
| 780 | 0.879 | 0.752 | 55.3 | 4.15099E-05 | 0.00001499 | 0 |