OPTI 435/535 Midterm October 8, 2014

Undergraduate do three problems and Graduate Students do all four problems.

1. The letters of the fine print on a medicine bottle are 1 mm high. Suppose you hold the bottle 125 mm from your eye to read it. Answer the following:

(a) What visual acuity is needed in order to resolve the letters?

Using the small angle approximation, the angular subtense of the letter is

$$\alpha = \frac{1}{125}\,\text{rad} = 27.5\,\text{arcmin}$$

A 20/20 letter subtends 5 arcmin, so this is 5.5x larger, so we would expect the visual acuity to be 20/110.

(b) How many diopters does a person need to accommodate in order to focus on the letters?

The accommodation needed is $\frac{1}{0.125\,\text{m}} = 8\text{D}$.

(c) Based on the plot of accommodation vs. age in the notes, at about what age will this level of accommodation be the maximum a person can do?

Somewhere around 25 years old.

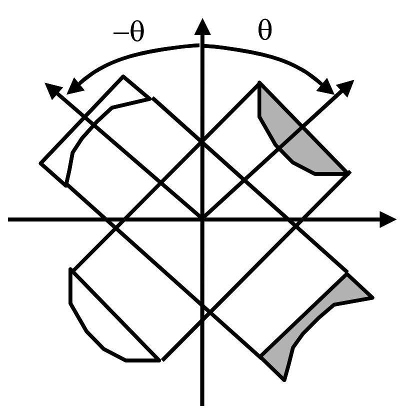
(d) Suppose you only have 2.5 D of accommodation and now move the bottle to a distance of 400 mm to focus on it. What visual acuity is needed in this case

Using the small angle approximation, the angular subtense of the letter is

$$\alpha = \frac{1}{400}\,\text{rad} = 8.6\,\text{arcmin}$$

A 20/20 letter subtends 5 arcmin, so this is 1.7x larger, so we would expect the visual acuity to be 20/34.

2. A Stokes lens is a variable crossed cylinder. It is comprised of a plano-convex cylinder lens of power Φ and a plano-concave lens of power -Φ. The lenses are geared such that if the positive cylinder lens is rotated through an angle θ, then the negative cylinder lens is automatically rotated through an angle -θ.



a) What is the combined prescription of the Stokes lens as a function of θ?

This is an astigmatic decomposition problem. We can *write the two cylinder lenses in terms of J0, J45 and M*

Sph	*Cyl*	*Axis*	*J0*	*J45*	*M*
0	Φ	$90-\theta$	$-\frac{\Phi}{2}cos(180-2\theta)$	$-\frac{\Phi}{2}sin(180-2\theta)$	$\frac{\Phi}{2}$
0	$-\Phi$	$90+\theta$	$\frac{\Phi}{2}cos(180+2\theta)$	$\frac{\Phi}{2}sin(180+2\theta)$	$-\frac{\Phi}{2}$

Summing the columns gives a spherical equivalent M = 0. The crossed cylinder terms are given by

$$J0=\frac{\Phi}{2}\left[cos(180+2\theta)-cos(180-2\theta)\right]=\frac{\Phi}{2}\left[-cos(2\theta)-cos(2\theta)\right]=0$$

and

$$J45=\frac{\Phi}{2}\left[sin(180+2\theta)-sin(180-2\theta)\right]=\frac{\Phi}{2}\left[sin(-2\theta)-sin(2\theta)\right]=-\Phi\, sin\, 2\theta\,.$$

Based on these, the prescription of the lens is given by

$$C_R=2\sqrt{\Phi^2\, sin^2\, 2\theta}=2\Phi\, sin\, 2\theta\,,$$

$$S_R=0-\sqrt{\Phi^2\, sin^2\, 2\theta}=-\Phi\, sin\, 2\theta$$

$$\theta_R=-tan^{-1}\left[\frac{\Phi\, sin\, 2\theta}{-\Phi\, sin\, 2\theta}\right]=45°$$

b) What is the spherical equivalent power of the Stokes lens?

M = 0 in the previous part as is to be expected with a crossed cylinder lens.

3. Measure the wavefront $W(x,y)=-0.002(x^2+y^2)$ with a Shack Hartmann sensor for a 4 mm diameter pupil. Suppose the lenslets of the array have a focal length of 24 mm and a spacing of 1 mm.

(a) What does the *unaberrated* Shack Hartmann pattern look like?

See part (c)

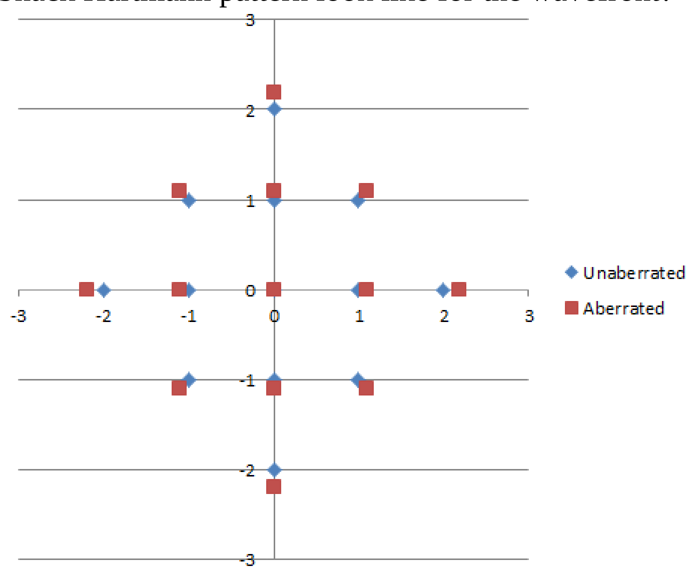
(b) What are the focal spot shifts Δx and Δy for each spot?

The spot displacement are given by

$\Delta x = -f\dfrac{\partial W(x,y)}{\partial x}$ and $\Delta y = -f\dfrac{\partial W(x,y)}{\partial y}$. *The table below summarizes the values. Use symmetry to speed the calculations.*

x	y	Δx	Δy	x+Δx	y+Δy
0	2	0	0.192	0	2.192
-1	1	-0.096	0.096	-1.096	1.096
0	1	0	0.096	0	1.096
1	1	0.096	0.096	1.096	1.096
-2	0	-0.192	0	-2.192	0
-1	0	-0.096	0	-1.096	0
0	0	0	0	0	0
1	0	0.096	0	1.096	0
2	0	0.192	0	2.192	0
-1	-1	-0.096	-0.096	-1.096	-1.096
0	-1	0	-0.096	0	-1.096
1	-1	0.096	-0.096	1.096	-1.096
0	-2	0	-0.192	0	-2.192

(c) What does the Shack Hartmann pattern look like for the wavefront?



4. Given the wavefront $W(x,y) = x^3 - 3xy^2 + x^2$ over a 4 mm diameter pupil:

(a) Rewrite the wavefront in terms of polar coordinates (r, θ).

Using $x = r\cos\theta$ *and* $y = r\sin\theta$, *the wavefront can be rewritten as*

$$W(r,\theta) = r^3\cos^3\theta - 3r^3\cos\theta\sin^2\theta + r^2\cos^2\theta$$

(b) Rewrite the wavefront in terms of normalized polar coordinates (ρ, θ).

Using $\rho = r/2$, $W(r,\theta) = 8\rho^3\cos^3\theta - 24\rho^3\cos\theta\sin^2\theta + 4\rho^2\cos^2\theta$.

(c) Rewrite the wavefront in terms of a linear combination of Zernike polynomials $Z_n^m(\rho,\theta)$. Give explicit expressions for the relevant expansion coefficients $a_{n,m}$.

Rewrite the wavefront as

$$W(r,\theta) = 8\rho^3\left(cos^3\theta - 3\left(cos\theta - cos^3\theta\right)\right) + 4\rho^2 cos^2\theta .$$

Using

$$cos^2\theta = \frac{1}{2}\left[1 + cos\,2\theta\right] \text{ and } cos^3\theta = \frac{1}{4}\left[cos\,3\theta + 3cos\,\theta\right],$$

$$W(r,\theta) = 8\rho^3 cos\,3\theta + 2\rho^2 + 2\rho^2 cos\,2\theta .$$

Examining a list of Zernike polynomials, we would expect the following terms:

$$W(r,\theta) = 8\rho^3 cos\,3\theta + 2\rho^2 + 2\rho^2 cos\,2\theta = a_{33}\sqrt{8}\rho^3 cos\,3\theta + a_{20}\sqrt{3}\left(2\rho^2 - 1\right) + a_{00} + a_{22}\sqrt{6}\rho^2 cos\,2\theta .$$

Note the a_{00} term is needed to cancel the constant in the a_{20} term. Comparing like terms:

$$8\rho^3 cos\,3\theta = a_{33}\sqrt{8}\rho^3 cos\,3\theta$$

$$2\rho^2 = a_{20}\sqrt{3}\left(2\rho^2\right)$$

$$2\rho^2 cos\,2\theta = a_{22}\sqrt{6}\rho^2 cos\,2\theta$$

$$0 = a_{20}\sqrt{3}(-1) + a_{00} .$$

Solving these gives $a_{33} = \sqrt{8}$; $a_{20} = \frac{1}{\sqrt{3}}$; $a_{22} = \frac{2}{\sqrt{6}}$; $a_{00} = 1$.