OPTI 415/515 Midterm1 March 1, 2016

Undergraduates and graduates answer all three questions

1. You decide to build your own book scanner to digitize all your old optics textbooks. You have a ½" 1.3 MPix sensor with dimensions 6.4 x 4.8mm (1280 x 1024 pixels). The dimensions of the book page is 366 x 256 mm. Answer the following questions:

   a) If the object distance is 450 mm, what focal length thin lens will you need to ensure the image of the book page fits onto the sensor? Optimize your choice to best fill the sensor.

   The magnification is $m = -6.4/366 = -0.0175$ *in the vertical direction. Trying to fit the width of the page onto the sensor will cause the top and bottom of the page to be cut off. The focal length of the lens is then given by*

   $$f = \frac{mz}{1-m} = 7.74mm.$$

   b) What is the image distance?

   The image distance is given by $z' = (1-m)f$ *=7.875mm.*

   c) A character on the page is 4.5 mm tall. How many pixels on the sensor does this correspond to?

   The pixels are $6.4mm/1280 = 0.005mm$. *The absolute size of the letter on the sensor is* $4.5(0.0175) = 0.07875mm$, *which corresponds to 15.75 pixels.*

   d) If the diameter of the lens is 3.8 mm, what is the working F/#?

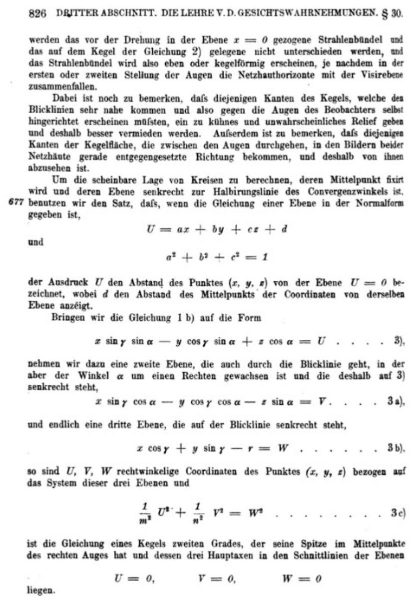
   The f/# is $7.74/3.8 = 2.037$. *The working f/# is* $(1 + 0.0175)f/\# = 2.073$

   e) For a wavelength of 550 nm, what is the size of the Airy disk, if the lens is diffraction limited?

   The Airy disk diameter is $2.44\lambda\, f_W/\# = 2.44(0.55\mu m)2.073 = 2.782\mu m$.

   f) How does the Airy disk size compare to the size of the pixels?

   This is roughly half the size of the pixel.


826 DRITTER ABSCHNITT. DIE LEHRE V. D. GESICHTSWAHRNEHMUNGEN. § 30.

werden das vor der Drehung in der Ebene $x = 0$ gezogene Strahlenbündel und das auf dem Kegel der Gleichung 2) gelegene nicht unterschieden werden, und das Strahlenbündel wird also eben oder kegelförmig erscheinen, je nachdem in der ersten oder zweiten Stellung der Augen die Netzhauthorizonte mit der Visirebene zusammenfallen.

Dabei ist noch zu bemerken, dafs diejenigen Kanten des Kegels, welche den Blicklinien sehr nahe kommen und also gegen die Augen des Beobachters selbst hingerichtet erscheinen müfsten, ein zu kühnes und unwahrscheinliches Relief geben und deshalb besser vermieden werden. Aufserdem ist zu bemerken, dafs diejenigen Kanten der Kegelfläche, die zwischen den Augen durchgehen, in den Bildern beider Netzhäute gerade entgegengesetzte Richtung bekommen, und deshalb von ihnen abzusehen ist.

Um die scheinbare Lage von Kreisen zu berechnen, deren Mittelpunkt fixirt wird und deren Ebene senkrecht zur Halbirungslinie des Convergenzwinkels ist,
677 benutzen wir den Satz, dafs, wenn die Gleichung einer Ebene in der Normalform gegeben ist,

$$U = ax + by + cz + d$$

und

$$a^2 + b^2 + c^2 = 1$$

der Ausdruck U den Abstand des Punktes (x, y, z) von der Ebene $U = 0$ bezeichnet, wobei d den Abstand des Mittelpunkts der Coordinaten von derselben Ebene anzéigt.

Bringen wir die Gleichung 1 b) auf die Form

$$x \sin\gamma \sin\alpha - y \cos\gamma \sin\alpha + z \cos\alpha = U \quad . \ . \ . \ . \ . \quad 3),$$

nehmen wir dazu eine zweite Ebene, die auch durch die Blicklinie geht, in der aber der Winkel α um einen Rechten gewachsen ist und die deshalb auf 3) senkrecht steht,

$$x \sin\gamma \cos\alpha - y \cos\gamma \cos\alpha - z \sin\alpha = V \quad . \ . \ . \ . \ . \quad 3a),$$

und endlich eine dritte Ebene, die auf der Blicklinie senkrecht steht,

$$x \cos\gamma + y \sin\gamma - r = W \quad . \ . \ . \ . \ . \ . \ . \quad 3b),$$

so sind U, V, W rechtwinkelige Coordinaten des Punktes (x, y, z) bezogen auf das System dieser drei Ebenen und

$$\frac{1}{m^2} U^2 + \frac{1}{n^2} V^2 = W^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad 3c)$$

ist die Gleichung eines Kegels zweiten Grades, der seine Spitze im Mittelpunkte des rechten Auges hat und dessen drei Hauptaxen in den Schnittlinien der Ebenen

$$U = 0, \qquad V = 0, \qquad W = 0$$

liegen.

2. Given the wavefront $W(\rho,\theta) = \rho^4 \sin 2\theta + 2\rho^3 \cos 3\theta$, do the following:
   a) Represent the wavefront as a series of Zernike polynomials. There is a table of Zernike polynomials on the next page.

   Since we have a ρ^4 in the first term we should expect to have some $n = 4$ terms for the Zernikes. Similarly, with the second term we have a ρ^3 , so we should expect to have some $n = 3$ terms for the Zernike. The first term also has a $sin2\theta$ dependence, so we should expect to have $m = -2$ terms. The second term has a $cos3\theta$ dependence, so we should expect to have $m = 3$ terms. Taking these all together, the Zernike expansion will have

$$a_{4,-2}Z_4^{-2}(\rho,\theta) + a_{2,-2}Z_2^{-2}(\rho,\theta) + a_{3,3}Z_3^{3}(\rho,\theta) = \rho^4 sin2\theta + 2\rho^3 cos3\theta$$

   Plugging in the definitions gives

$$a_{4,-2}\sqrt{10}(4\rho^4 sin2\theta - 3\rho^2 sin2\theta) + a_{2,-2}\sqrt{6}\rho^2 sin2\theta + a_{3,3}\sqrt{6}\rho^3 cos3\theta$$
$$= \rho^4 sin2\theta + 2\rho^3 cos3\theta$$

   Comparing like terms gives three equations

$$a_{4,-2}4\sqrt{10} = 1$$
$$-a_{4,-2}3\sqrt{10} + a_{2,-2}\sqrt{6} = 0$$

$$a_{3,3}\sqrt{6} = 2$$

   So,

$$a_{4,-2} = \frac{1}{4\sqrt{10}}$$

$$a_{2,-2} = \frac{3}{4\sqrt{6}}$$

$$a_{3,3} = \frac{2}{\sqrt{6}}$$

Unfortunately, I found a typo in the table provide with the test. The trefoil term should be

$$Z_3^3(\rho,\theta) = \sqrt{8}\rho^3 cos3\theta$$

In this case,

$$a_{3,3} = \frac{1}{\sqrt{2}}$$

So I am accepting either answer.

b) Rewrite the wavefront in terms of Cartesian coordinates. HINT: $\sin 2\theta = 2\sin\theta\cos\theta$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

Using the hints

$$W(\rho,\theta) = \rho^4 sin2\theta + 2\rho^3 cos3\theta = 2\rho^4 sin\theta cos\theta + 8\rho^3 cos^3\theta - 6\rho^3 cos\theta$$

and the Cartesian definitions $\rho_x = \rho cos\theta$ *and* $\rho_y = \rho sin\theta$ *gives*

$$W(\rho_x,\rho_y) = 2\rho^2\rho_x\rho_y + 8\rho_x^3 - 6\rho^2\rho_x$$

Finally, using $\rho^2 = \rho_x^2 + \rho_y^2$ *gives*

$$W(\rho_x,\rho_y) = 2\rho_x^3\rho_y + 2\rho_x\rho_y^3 + 8\rho_x^3 - 6\rho_x^3 - 6\rho_x\rho_y^2$$
$$W(\rho_x,\rho_y) = 2\rho_x^3\rho_y + 2\rho_x\rho_y^3 + 2\rho_x^3 - 6\rho_x\rho_y^2$$

c) Write an expression for the transverse rays errors, ε_x and ε_y, of this wavefront.

$$\varepsilon_x = \frac{-R}{nr_{max}}\frac{\partial W(\rho_x,\rho_y)}{\partial\rho_x} = \frac{-R}{nr_{max}}\left(6\rho_x^2\rho_y + 2\rho_y^3 + 6\rho_x^2 - 6\rho_y^2\right)$$
$$\varepsilon_y = \frac{-R}{nr_{max}}\frac{\partial W(\rho_x,\rho_y)}{\partial\rho_y} = \frac{-R}{nr_{max}}\left(2\rho_x^3 + 6\rho_x\rho_y^2 - 12\rho_x\rho_y\right)$$

3. An F/# = 2.2 Retrofocus lens consists of two separated thin lenses. The first lens has a focal length of –100 mm and the second lens has a focal length of 26 mm. The thin lenses are

separated by a distance of 20 mm and the aperture stop is located halfway between the two lenses. The lens is used with a full-frame sensor with dimensions 36 mm x 24 mm in the horizontal and vertical directions, respectively.

The answers are based on the ynu raytrace on the next page.

a) What is the Back Focal Distance (BFD)?

The a-ray has zero incident slope, so it passes through the rear focal point. The height of this ray at the last surface is 1.2 mm and its slope following the last surface is -0.03615. The BFD is given by

$$BFD = \frac{-(1.2)}{-0.03615} = 33.19\ mm$$

b) What is the effective focal length f_E?

The effective focal length is the same as the rear focal length and is given by

$$f_E = f'_R = \frac{-1}{-0.03615} = 27.66\ mm$$

c) Where is the rear principal plane P' located relative to the last surface?

Based on the first two parts, the rear principal plane is located $33.19 - 27.66 = 5.53\ mm$ *to the right of the last surface.*

d) What is the entrance pupil diameter (EPD) of the lens?

Since this is an F/2.2 lens, the EPD is given by

$$EPD = \frac{f_E}{F/\#} = 12.57\ mm$$

e) What are the full fields of view in the horizontal and vertical directions?

The last two rays in the show the chief rays in the vertical and horizontal directions. For the vertical direction, the incident slope of the chief ray is 0.433846. The full field of view in this direction is given by

$$FFOV(v) = 2 * tan^{-1}(0.433846) = 46.9°$$

Similarly, in the horizontal direction

$$FFOV(h) = 2 * tan^{-1}(0.650769) = 66.1°$$

	O		1		2		3		O'			
f			-100				26					
-phi			0.01		0		-0.03846					
t				10		10		33.19149				
ya			1		1.1		1.2		0			
nua		0		0.01		0.01		-0.03615			fE	27.65957
											Fnum	2.2
yb			0		10		20		27.65957		EPD	12.57253
nub		1		1		1		0.230769				
yc			-10		0		10		30.42553		A	-10
nuc		1.1		1		1		0.615385			B	1.1
ybar (v)			-3.94406		0		3.944056		12		A	0.394406
nubar(v)		0.433846		0.394406		0.394406		0.242711			FFOV(v)	46.90685
ybar (h)			-5.91608		0		5.916084		18		A	0.591608
nubar(h)		0.650769		0.591608		0.591608		0.364067			FFOV(h)	66.10968

j	n	m	$Z_n^m(\rho,\theta)$
0	0	0	1
1	1	–1	$2\rho\sin\theta$
2	1	1	$2\rho\cos\theta$
3	2	–2	$\sqrt{6}\rho^2\sin 2\theta$
4	2	0	$\sqrt{3}(2\rho^2 - 1)$
5	2	2	$\sqrt{6}\rho^2\cos 2\theta$
6	3	–3	$\sqrt{6}\rho^3\sin 3\theta$
7	3	–1	$\sqrt{8}(3\rho^3 - 2\rho)\sin\theta$
8	3	1	$\sqrt{8}(3\rho^3 - 2\rho)\cos\theta$
9	3	3	$\sqrt{6}\rho^3\cos 3\theta$
10	4	–4	$\sqrt{10}\rho^4\sin 4\theta$
11	4	–2	$\sqrt{10}(4\rho^4 - 3\rho^2)\sin 2\theta$
12	4	0	$\sqrt{5}(6\rho^4 - 6\rho^2 + 1)$
13	4	2	$\sqrt{10}(4\rho^4 - 3\rho^2)\cos 2\theta$
14	4	4	$\sqrt{10}\rho^4\cos 4\theta$
15	5	–5	$\sqrt{12}\rho^5\sin 5\theta$
16	5	–3	$\sqrt{12}(5\rho^5 - 4\rho^3)\sin 3\theta$
17	5	–1	$\sqrt{12}(10\rho^5 - 12\rho^3 + 3\rho)\sin\theta$
18	5	1	$\sqrt{12}(10\rho^5 - 12\rho^3 + 3\rho)\cos\theta$
19	5	3	$\sqrt{12}(5\rho^5 - 4\rho^3)\cos 3\theta$
20	5	5	$\sqrt{12}\rho^5\cos 5\theta$
21	6	–6	$\sqrt{14}\rho^6\sin 6\theta$
22	6	–4	$\sqrt{14}(6\rho^6 - 5\rho^4)\sin 4\theta$
23	6	–2	$\sqrt{14}(15\rho^6 - 20\rho^4 + 6\rho^2)\sin 2\theta$
24	6	0	$\sqrt{7}(20\rho^6 - 30\rho^4 + 12\rho^2 - 1)$
25	6	2	$\sqrt{14}(15\rho^6 - 20\rho^4 + 6\rho^2)\cos 2\theta$
26	6	4	$\sqrt{14}(6\rho^6 - 5\rho^4)\cos 4\theta$
27	6	6	$\sqrt{14}\rho^6\cos 6\theta$