OPTI 415/515 Midterm Due: February 23, 2012

Graduate Students do all problems. Undergraduate students choose three problems.

1. Google Earth is improving the resolution of its global maps with data from the SPOT5 satellite. The satellite orbits at an altitude 823 km above the Earth's surface. Assume its camera contains a 1/2" CCD with a sensor size of 6.4 x 4.8 mm and pixel dimensions of 2048 x 1536. Each pixel on the sensor corresponds to a 2.5 m patch on the ground. You can assume a thin lens system. Answer the following:

   - What are the dimensions of the pixels?

     The pixels are 6.4 mm / 2048 by 4.8 mm / 1536, which is equivalent to 3.1 μm square pixels.

   - What magnification is needed?

     The 2.5 m patch on the ground corresponds to the *3.1 μm on the sensor, so the magnification is given by y' / y, where y' is the image height and y is the object height. In this case, the magnification = $3.1x10^{-6}$ / 2.5 = $1.24x10^{-6}$*

   - What is the image distance for the satellite camera?

     The magnification is also equal to the ratio of the image distance so the object distance, so the image distance is given by *$1.24x10^{-6}$(823,000 m)=1.021 m.*

   - What is the focal length of the satellite camera?

     The Gaussian imaging equations tells us that

$$\frac{1}{1.021}+\frac{1}{823000}=\frac{1}{f}\Rightarrow f=1.021\text{m}.$$

     Effectively, the object is at infinity.

- If we want to make the diffraction spot diameter match the pixel size, what is the size of the entrance pupil? (Assume $\lambda = 0.5\ \mu m$)

  The diffraction limited spot size is given by

  $$\text{Spot Size} = 2.44\lambda\frac{f}{D} = 3.1\mu m$$

  Solving for D gives 401 mm. Technically, the working F/# should be used here, but the difference between the F/# and Working F/# is miniscule here since m is so small.

2. Suppose we measured an unknown thick lens system with the reciprocal magnification technique. Using the technique, we determined that the distance from the rear principal plane to the image plane was 100 mm. We also determined that the distance from the front principal plane to the object plane was -25 mm. Answer the following questions:

   - What is the magnification of the lens?

     The object and image distances are -25 mm and 100 mm respectively. The magnification is the ratio of the image to object distance or -4x.

   - What is the focal length of the lens?

     The Gaussian imaging equations tells us that

     $$\frac{1}{100} + \frac{1}{25} = \frac{1}{f} \Rightarrow f = 20mm.$$

   - How far do we need to move the lens to have reciprocal magnification for the same object and image planes?

Here, there are multiple routes to get the answer, but basically use one of the equations on page 35 of the notes and solve for the distance d the lens needs to be moved. In terms of the object distance l_1:

$$l_1 = \frac{d}{1+m_1} \Rightarrow d = -25(1-4) = 75\text{mm}$$

- Where are the nodal points located?

  Since the lens is in air, the nodal points coincide with the principal planes.

3. The wavefront coefficients for an optical system with an exit pupil diameter is 20 mm and the reference sphere radius is 100 mm are W_{040} = 3 μm, W_{220} = 16.6 μm, W_{222} = 13.4 μm. Do the following:

   - Write an expression for the wavefront error.

     $$W(h,\rho,\psi) = 0.003\rho^4 + 0.0166h^2\rho^2 + 0.0134h^2\rho^2 \cos^2(\psi)$$

   - Write an expression for the transverse ray error.

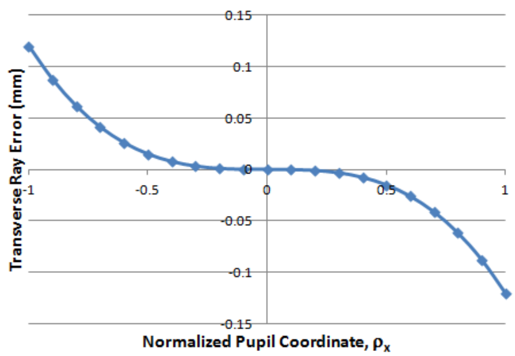
     $$\varepsilon_x = -\left(\frac{100}{10}\right)\left(0.012\left(\rho_x^3 + \rho_x\rho_y^2\right) + 0.0332h^2\rho_x\right)$$

     $$\varepsilon_y = -\left(\frac{100}{10}\right)\left(0.012\left(\rho_x^2\rho_y + \rho_y^3\right) + 0.0332h^2\rho_y + 0.0268h^2\rho_y\right)$$

   - Sketch the transverse ray error in the case where h = 0 (i.e. on-axis).

     $$\varepsilon_x = -0.12\left(\rho_x^3 + \rho_x\rho_y^2\right)$$

     $$\varepsilon_y = -0.12\left(\rho_x^2\rho_y + \rho_y^3\right)$$

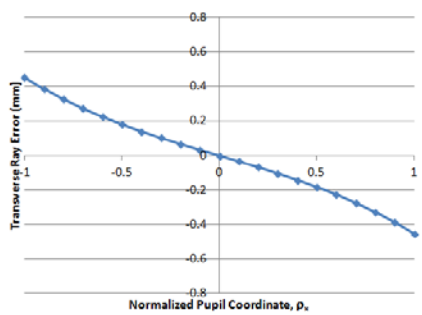


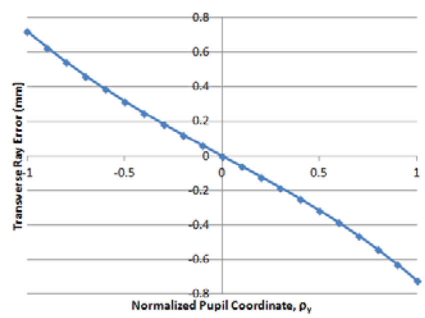
Same for ρ_y direction.

- Sketch the transverse ray error in the case where h = 1 (i.e. full field) along the ρ_x and ρ_y directions.

$$\varepsilon_x = -0.12\left(\rho_x^3 + \rho_x\rho_y^2\right) - 0.332\rho_x$$

$$\varepsilon_y = -0.12\left(\rho_x^2\rho_y + \rho_y^3\right) - 0.332\rho_y - 0.268\rho_y$$





4. In class, we said for non-rotationally symmetric systems, the Seidel aberrations generalize to terms on axis. Suppose we have a wavefront

$$W(\rho, \theta) = W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta$$

Rewrite this wavefront in terms of Zernike polynomials such that

$$W(\rho, \theta) = a_{00}Z_0^0(\rho, \theta) + a_{11}Z_1^1(\rho, \theta) + a_{20}Z_2^0(\rho, \theta) + a_{22}Z_2^2(\rho, \theta) + a_{31}Z_3^1(\rho, \theta)$$

- What are the coefficients a_{00}, a_{11}, a_{20}, a_{22}, a_{31} in terms of W_{20} and W_{22}?

There are two ways to solve for this problem. First, equate the expressions and replace

$$\cos^2\theta = \frac{1}{2}[1+\cos 2\theta].$$

$$W_{20}\rho^2 + \frac{W_{22}}{2}\rho^2 + \frac{W_{22}}{2}\rho^2\cos 2\theta = a_{00}Z_0^0(\rho,\theta) + a_{11}Z_1^1(\rho,\theta) + a_{20}Z_2^0(\rho,\theta) + a_{22}Z_2^2(\rho,\theta) + a_{31}Z_3^1(\rho,\theta).$$

Substitute the definitions of each of the Zernike terms into the right hand side.

$$W_{20}\rho^2 + \frac{W_{22}}{2}\rho^2 + \frac{W_{22}}{2}\rho^2\cos 2\theta = a_{00} + a_{11}2\rho\cos\theta + a_{20}\sqrt{3}(2\rho^2-1) + a_{22}\sqrt{6}\rho^2\cos 2\theta + a_{31}\sqrt{8}(3\rho^3-2\rho)\cos\theta$$

Collect like terms

$$\left(W_{20} + \frac{W_{22}}{2}\right)\rho^2 + \frac{W_{22}}{2}\rho^2\cos 2\theta = (a_{00} - \sqrt{3}a_{20}) + (2a_{11} - 2\sqrt{8}a_{31})\rho\cos\theta + 2\sqrt{3}a_{20}\rho^2 + \sqrt{6}a_{22}\rho^2\cos 2\theta + 3\sqrt{8}a_{31}\rho^3\cos\theta$$

Comparing both sides gives series of simultaneous equations

$$(a_{00} - \sqrt{3}a_{20}) = 0$$
$$(2a_{11} - 2\sqrt{8}a_{31}) = 0$$
$$2\sqrt{3}a_{20} = \left(W_{20} + \frac{W_{22}}{2}\right)$$
$$\sqrt{6}a_{22} = \frac{W_{22}}{2}$$
$$3\sqrt{8}a_{31} = 0$$

From inspection, this gives

$$a_{00} = \frac{1}{2}\left(W_{20} + \frac{W_{22}}{2}\right)$$
$$a_{11} = 0$$
$$a_{20} = \frac{1}{2\sqrt{3}}\left(W_{20} + \frac{W_{22}}{2}\right)$$
$$a_{22} = \frac{W_{22}}{2\sqrt{6}}$$
$$a_{31} = 0$$

Alternatively, we can use the orthogonal properties of the Zernike polynomials to calculate each of the coefficients via

$$a_{nm} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 W(\rho,\theta) Z_n^m(\rho,\theta)\rho d\rho d\theta$$

So,

$$a_{00} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \left(W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta\right)\rho d\rho d\theta = \frac{1}{2}\left(W_{20} + \frac{W_{22}}{2}\right)$$

$$a_{11} = \frac{2}{\pi}\int_0^{2\pi}\int_0^1 \left(W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta\right)\rho^2\cos\theta d\rho d\theta = 0$$

$$a_{20} = \frac{\sqrt{3}}{\pi}\int_0^{2\pi}\int_0^1 \left(W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta\right)\left(2\rho^3 - \rho\right)d\rho d\theta = \frac{1}{2\sqrt{3}}\left(W_{20} + \frac{W_{22}}{2}\right)$$

$$a_{22} = \frac{\sqrt{6}}{\pi}\int_0^{2\pi}\int_0^1 \left(W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta\right)\rho^3\cos 2\theta d\rho d\theta = \frac{W_{22}}{2\sqrt{6}}$$

$$a_{31}\ \frac{\sqrt{8}}{\pi}\int_0^{2\pi}\int_0^1 \left(W_{20}\rho^2 + W_{22}\rho^2\cos^2\theta\right)\left(3\rho^4 - 2\rho^2\right)\cos\theta d\rho d\theta = 0$$

- For what values of θ is W(ρ,θ) a maximum when ρ = 1 and W_{22} >0?

For ρ = 1 and W_{22} >0, the peaks occurs when $\cos^2\theta$ = 1. This occurs when θ = 0, ±π…

Note, the following trig relationship may be useful: $\cos^2\theta = \frac{1}{2}\left[1 + \cos 2\theta\right]$. Also, the first few Zernike polynomials are given by

Zernike Polynomials: Table in Polar Coordinates

j	n	m	$Z_n^m(\rho,\theta)$
0	0	0	1
1	1	-1	$2\ \rho \sin\theta$
2	1	1	$2\ \rho \cos\theta$
3	2	-2	$\sqrt{6}\ \rho^2 \sin 2\theta$
4	2	0	$\sqrt{3}\ (2\rho^2-1)$
5	2	2	$\sqrt{6}\, \rho^2 \cos 2\theta$
6	3	-3	$\sqrt{8}\, \rho^3 \sin 3\theta$
7	3	-1	$\sqrt{8}\ (3\rho^3-2\rho) \sin\theta$
8	3	1	$\sqrt{8}\ (3\rho^3-2\rho) \cos\theta$
9	3	3	$\sqrt{8}\, \rho^3 \cos 3\theta$