OPTI 415/515 Midterm Due: February 24, 2011

Graduate Students do all problems. Undergraduate students choose three problems.

1. Your new job at an unnamed government agency requires you to design a spy camera. The camera is designed to record the keystrokes on a computer keyboard and will be mounted on a ceiling 2.3 m above the keyboard and desktop. The camera contains a 1/3" CCD with a sensor size of 4.8 x 3.6 mm and pixel dimensions of 1024 x 768. The dimensions of the keyboard are 440 mm x 150 mm. The letter keys on the keyboard are 18 mm square. You can assume a thin lens system. Answer the following:

   - What magnification is needed to image the full keyboard onto the sensor?

     The magnification is given by $m = \frac{y'}{y} = \frac{-4.8}{440} = -0.011$

   - What is the focal length of the lens needed for the spy camera?

     The object distance is -2300 mm, so $f_E = \frac{mz}{1-m} = 25.025\,\mathrm{mm}$

   - What is the distance from the lens to the camera sensor?

     The image distance is $z' = (1-m)f_E = 25.3\,\mathrm{mm}$

   - What are the dimensions of a pixel on the sensor?

     The width of the sensor is 4.8 mm and there are 1024 pixels, so each pixel is 4.7 μm square.

   - How many pixels correspond to the width of a single letter key in the image?

     *On the image plane, the absolute size of a letter key is 0.011 * 18 mm = 198 μm. If we divide this dimension by the size of a pixel, we get 198 / 4.7 = 42.13 pixels per letter key.*

- If we want to make the diffraction spot diameter match the pixel size, what is the size of the entrance pupil? (Assume λ = 0.55 μm)

  Setting the Airy Disk Diameter to the width of a pixel gives $\frac{2.44\lambda f_E}{D_E} = 4.7\ \mu\text{m}$.

  Solving for D_E gives 7.16 mm.

2. Suppose we have an unknown thick lens system and we want to determine its Cardinal points. I set up a 25 mm tall target a distance 199.459 mm from the front vertex of the lens. The image is formed at a plane 295 mm from the rear vertex of the lens. The image is inverted and has a size of 36.975 mm. Keeping the object and image plane fixed, the lens is shifted axially 100.541 mm. At this point, the targeted is again imaged on the image plane. Assume the object and image space are in air. Answer the following questions:

   - What is the magnification of the lens in its initial position?

     The magnification is the ratio of the image height to the object height. Since the image is inverted, the magnification must be negative. m = -36.975/25 = -1.479.

   - What is the size of the image when the lens is in its final position?

     *This technique is called reciprocal magnification, so 1/m = -0.676.The image size is therefore -0.676*25 mm = 16.9 mm.*

   - How far from the front vertex is the front principal plane?

     From the notes, $L = \frac{d}{1+m} = \frac{100.541}{-0.479} = -209.898\ \text{mm}$. *The front principal plane is -199.459+209.898 = 10.439 mm from the front vertex.*

   - How far from the rear vertex is the rear principal plane?

From the notes, $L' = mL = -1.479(-209.898) = 310.439\ \text{mm}$

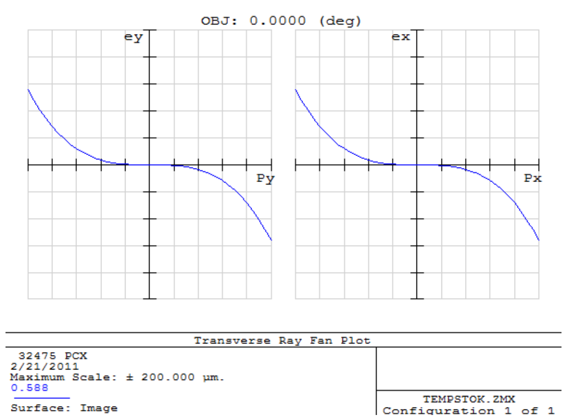
- What is the effective focal length of the lens?

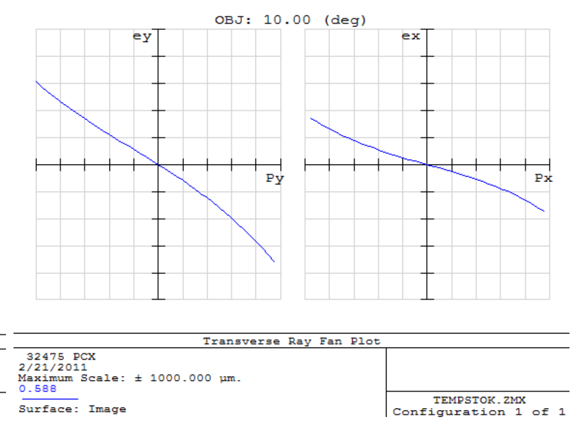
From the notes, $f_E = \dfrac{md}{m^2 - 1} = 125.227\ \text{mm}$.

- Where are the nodal points located?

Since the system is in air, the nodal points are located at the principal planes.

3. Given the following transverse ray diagrams, estimate the wavefront coefficients W_{040}, W_{220}, W_{222}. Note the scales on the plots. A maximum scale of 200 μm for example means that the ex and ey axes (ε_x and ε_y in our notation) range from -200 to +200 μm. Also, the exit pupil diameter is 20 mm and the reference sphere radius is 100 mm.





On axis, each square has a height of 200 μm / 5 = 40 μm. The plot looks like a third order polynomial, so we should expect spherical aberration. At $\rho_y = 1$, $\varepsilon_y = -120$ μm. For spherical aberration

$$\varepsilon_y(0, \rho_y) = -\frac{4R}{r_{max}} W_{040} \rho_y^3 \Rightarrow \varepsilon_y(0,1) = -40 W_{040} = -120\ \mu\text{m}$$

This gives $W_{040} = 3\ \mu m$.

Off axis, each square in the plot has a height of 1000 μm / 5 = 200 μm. The transverse ray error is approximately linear, but with different slopes in the two directions. We should expect astigmatism and field curvature in this case. At $\rho_y = 1$, $\varepsilon_y = -600$ μm. At $\rho_x = 1$, $\varepsilon_x = -300$ μm. For astigmatism and field curvature,

$$\varepsilon_y\left(0,\rho_y;h=1\right)=-\frac{2R}{r_{max}}\left(W_{220}+W_{222}\right)\rho_y \Rightarrow \varepsilon_y\left(0{,}1;h=1\right)=-20\left(W_{220}+W_{222}\right)=-600\ \mu\text{m}$$

$$\varepsilon_x\left(\rho_x,0;h=1\right)=-\frac{2R}{r_{max}}W_{220}\rho_x \Rightarrow \varepsilon_x\left(1{,}0;h=1\right)=-20W_{220}=-300\ \mu\text{m}\,.$$

Based on these approximations, W_{220} = 15 μm and W_{222} = 15 μm. The actual values are W_{220} = 16.6 μm and W_{222} = 13.4 μm, but we would need the raw transverse ray error data to determine these.

4. In class, we said for non-rotationally symmetric systems, the Seidel aberrations generalize to terms on axis. Suppose we have a wavefront

$$W(\rho,\theta)=W_{22}\rho^2\cos^2\theta+W_{31}\rho^3\cos\theta$$

Rewrite this wavefront in terms of Zernike polynomials such that

$$W(\rho,\theta)=a_{00}Z_0^0(\rho,\theta)+a_{11}Z_1^1(\rho,\theta)+a_{20}Z_2^0(\rho,\theta)+a_{22}Z_2^2(\rho,\theta)+a_{31}Z_3^1(\rho,\theta)$$

What are the coefficients a_{00}, a_{11}, a_{20}, a_{22}, a_{31} in terms of W_{22} and W_{31}? Note, the following trig relationship may be useful: $\cos^2\theta=\frac{1}{2}\left[1+\cos 2\theta\right]$. Also, the first few Zernike polynomials are given by

Zernike Polynomials: Table in Polar Coordinates

j	n	m	$Z_n^m(\rho,\theta)$
0	0	0	1
1	1	-1	$2\ \rho \sin\theta$
2	1	1	$2\ \rho \cos\theta$
3	2	-2	$\sqrt{6}\ \rho^2 \sin 2\theta$
4	2	0	$\sqrt{3}\ (2\rho^2-1)$
5	2	2	$\sqrt{6}\,\rho^2 \cos 2\theta$
6	3	-3	$\sqrt{8}\,\rho^3 \sin 3\theta$
7	3	-1	$\sqrt{8}\ (3\rho^3-2\rho) \sin\theta$
8	3	1	$\sqrt{8}\ (3\rho^3-2\rho) \cos\theta$
9	3	3	$\sqrt{8}\,\rho^3 \cos 3\theta$

Writing out the Zernike expansion, we get

$$W(\rho,\theta) = a_{00} + a_{11} 2\rho\cos\theta + a_{20}\sqrt{3}\left(2\rho^2 - 1\right) + a_{22}\sqrt{6}\rho^2\cos 2\theta + a_{31}\sqrt{8}\left(3\rho^3 - 2\rho\right)\cos\theta .$$

Comparing like terms, we get the following

$$a_{20} 2\sqrt{3} = \frac{W_{22}}{2}; \quad a_{22}\sqrt{6} = \frac{W_{22}}{2}; \quad a_{31} 3\sqrt{8} = W_{31}.$$

We also require that the piston and tilt terms cancel

$$a_{00} = a_{20}\sqrt{3}; \quad a_{11} = a_{31}\sqrt{8}$$

So, the final result is

$$a_{00}\frac{W_{22}}{4}; \quad a_{11} = \frac{W_{31}}{3}; \quad a_{20} = \frac{W_{22}}{4\sqrt{3}}; \quad a_{22} = \frac{W_{22}}{2\sqrt{6}}; \quad a_{31} = \frac{W_{31}}{3\sqrt{8}}$$