OPTI 415/515 Midterm 2 Due: April 5, 2012

Undergraduate and Graduate Students do all three problems.

1. An optical system has a focal length f = 100 mm and an entrance pupil diameter D_E = 25 mm. The object distance L = -1000 mm and the wavelength λ = 0.5 μm. Answer the following:

   - Where is the image formed?

     The Gaussian imaging equation gives

     $$\frac{1}{L'}+\frac{1}{1000}=\frac{1}{100}\Rightarrow L'=111.111\,mm$$

   - What is the magnification?

     The magnification is $m=\frac{L'}{L}=-0.111$

   - What is the F-Number f/#?

     $$f/\#=\frac{f}{D_E}=4.0$$

   - What is the Working F-Number $f/\#_w$?

     $$f/\#_w=(1-m)\frac{f}{D_E}=4.444$$

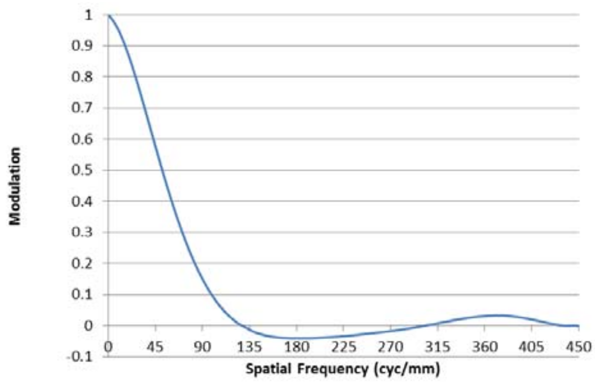
   - Based on the Rayleigh Criterion, what is the resolution limit of the system?

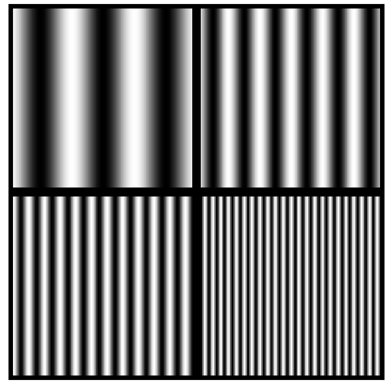
     $\text{Resolution Limit}=1.22\lambda f/\#_w=2.71\,\mu\text{m}$.

   - What is the cutoff frequency f_c of the Optical Transfer Function (OTF) for this system?

     $$f_c=\frac{1}{\lambda f/\#_w}=450.45\text{ cyc/mm}$$

2. The OTF for the optical system in Question 1 is shown below. The object (also shown below) consists of four sinusoidal patterns with 100% contrast and spatial frequencies of 10, 20, 40 and 80 cyc/mm, respectively (You can ignore edge effects and just treat these as pure sinusoids). Using your results from Question 1, answer the following questions:





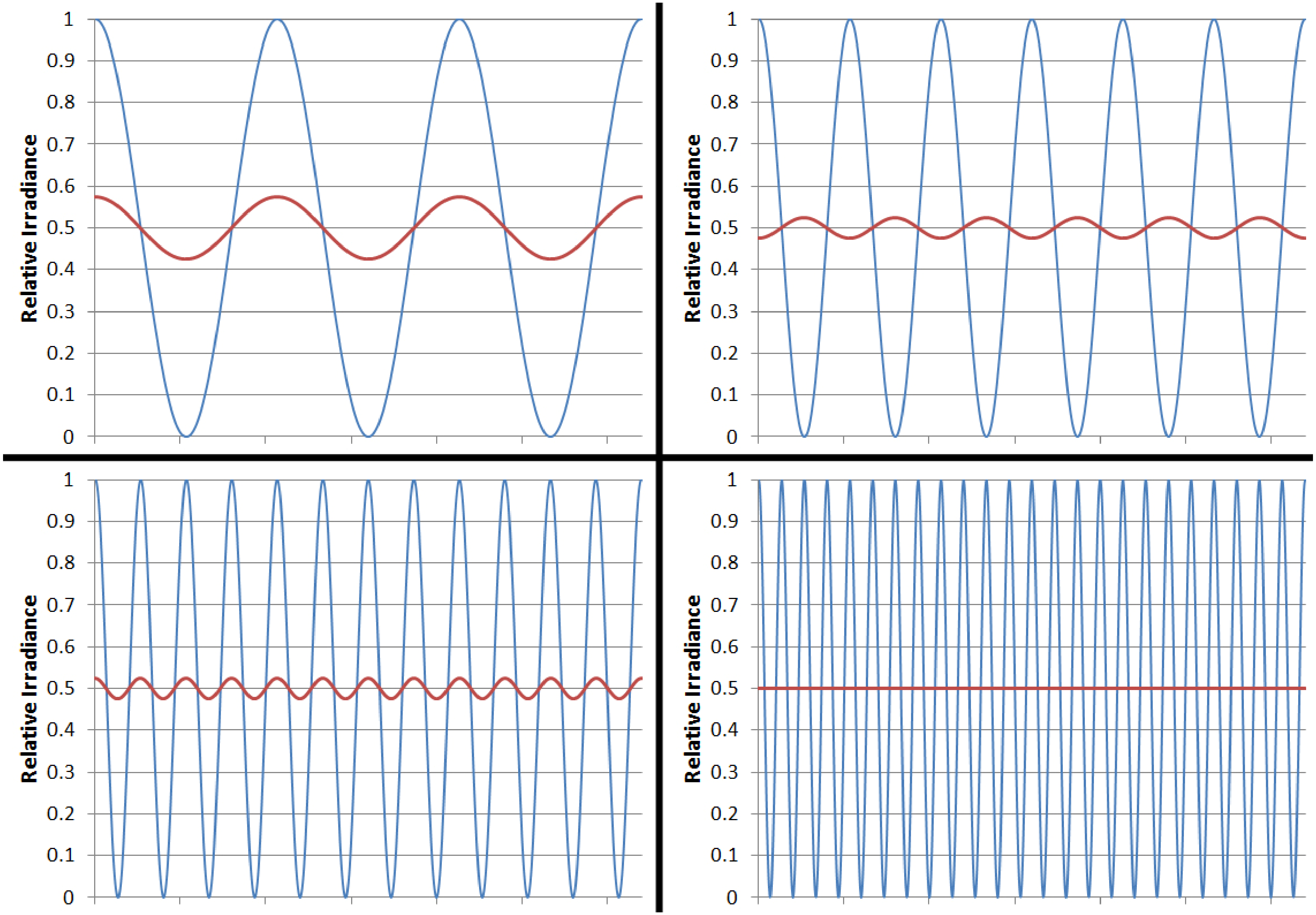
- What are the four spatial frequencies found in the image of the object described above?

  The spatial frequencies are scaled by the system magnification. Given that the magnification m = -0.111, the image is smaller than the object, so the spatial frequencies are higher in the image. The spatial frequencies are given by

  10 / |m| = 90 cyc/mm, 20 / |m| = 180 cyc/mm, 40 / |m| = 360 cyc/mm and

  80 / |m| = 720 cyc/mm, respectively.

- The plots on the following page show the irradiance profile through each of the four sinusoids in the image plane. Each plot assumes a "perfect" imaging system in which the 100% contrast in each sinusoid is preserved. However, the OTF tells us that the true image is degraded. Sketch on each plot, the true irradiance profile through each sinusoidal pattern. Be sure to label important features such as period, average irradiance, maximum and minimum irradiance, etc.


Relative Irradiance
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

3. The International Glass Code for an optical material is 487704. Model this glass with a two term Cauchy-type dispersion formula such that

$$n(\lambda) = A + \frac{B}{\lambda^2}$$

Answer the following:

- What are the index n_d and the Abbe number ν_d for the d-wavelength?

  The first three numbers of the glass code encode the index such that n_d = 1.487.

  The last three numbers of the glass code encode the Abbe number such that ν_d = 70.4.

- Write an expression for the Abbe number ν_d in terms of the Cauchy coefficients A and B and the wavelengths λ_d, λ_F, and λ_C, where the subscripts represent the d, F and C wavelengths respectively.

$$\nu_d = \frac{n(\lambda_d) - 1}{n(\lambda_F) - n(\lambda_C)} = \frac{A + \dfrac{B}{\lambda_d^2} - 1}{\dfrac{B}{\lambda_F^2} - \dfrac{B}{\lambda_C^2}}$$

- Solve for the coefficients A and B for the optical material specified above

  Rearranging the results from the previous part, we have

$$A + B\left(\frac{1}{\lambda_d^2} - \left(\frac{1}{\lambda_F^2} - \frac{1}{\lambda_C^2}\right)\nu_d\right) = 1$$

  Also, we know that

$$A + \frac{B}{\lambda_d^2} = n_d$$

  Solving these two equations for A and B gives

$$B = \frac{(n_d - 1)}{\nu_d} \frac{\lambda_C^2 \lambda_F^2}{(\lambda_C^2 - \lambda_F^2)} \quad and \;\; A = n_d - \frac{(n_d - 1)}{\lambda_d^2 \nu_d} \frac{\lambda_C^2 \lambda_F^2}{(\lambda_C^2 - \lambda_F^2)}$$

- Based on this model of the glass dispersion, what are n_F and n_C?

  Plugging in the results gives n_F = 1.49184 and n_C = 1.48492.