1. An optical system has a focal length $f = 100$ mm and an entrance pupil diameter $D_E = 25$ mm. The object distance $L = -1000$ mm and the wavelength $\lambda = 0.5 \, \mu m$. Answer the following:

- Where is the image formed?

  The Gaussian imaging equation gives

  \[
  \frac{1}{L'} + \frac{1}{1000} = \frac{1}{100} \Rightarrow L' = 111.11 \, mm
  \]

- What is the magnification?

  The magnification is

  \[
  m = \frac{L'}{L} = -0.111
  \]

- What is the F-Number $f/#$?

  \[
  f/\# = \frac{f}{D_E} = 4.0
  \]

- What is the Working F-Number $f/#_w$?

  \[
  f/\#_w = (1 - m) \frac{f}{D_E} = 4.444
  \]

- Based on the Rayleigh Criterion, what is the resolution limit of the system?

  Resolution Limit = $1.22 \lambda f/\#_w = 2.71 \, \mu m$.

- What is the cutoff frequency $f_c$ of the Optical Transfer Function (OTF) for this system?

  \[
  f_c = \frac{1}{\lambda f/\#_w} = 450.45 \, \text{cyc/mm}
  \]
2. The OTF for the optical system in Question 1 is shown below. The object (also shown below) consists of four sinusoidal patterns with 100% contrast and spatial frequencies of 10, 20, 40 and 80 cyc/mm, respectively (You can ignore edge effects and just treat these as pure sinusoids). Using your results from Question 1, answer the following questions:

• What are the four spatial frequencies found in the image of the object described above?

   The spatial frequencies are scaled by the system magnification. Given that the magnification \( m = -0.111 \), the image is smaller than the object, so the spatial frequencies are higher in the image. The spatial frequencies are given by
   
   \[
   \frac{10}{|m|} = 90 \text{ cyc/mm}, \quad \frac{20}{|m|} = 180 \text{ cyc/mm}, \quad \frac{40}{|m|} = 360 \text{ cyc/mm} \quad \text{and} \quad \frac{80}{|m|} = 720 \text{ cyc/mm},
   \]

• The plots on the following page show the irradiance profile through each of the four sinusoids in the image plane. Each plot assumes a “perfect” imaging system in which the 100% contrast in each sinusoid is preserved. However, the OTF tells us that the true image is degraded. Sketch on each plot, the true irradiance profile through each sinusoidal pattern. Be sure to label important features such as period, average irradiance, maximum and minimum irradiance, etc.
3. The International Glass Code for an optical material is 487704. Model this glass with a
two term Cauchy-type dispersion formula such that
\[ n(\lambda) = A + \frac{B}{\lambda^2} \]

Answer the following:

- What are the index \( n_d \) and the Abbe number \( \nu_d \) for the d-wavelength?
  
  *The first three numbers of the glass code encode the index such that \( n_d = 1.487.\) 
  *The last three numbers of the glass code encode the Abbe number such that \( \nu_d = 70.4.\)

- Write an expression for the Abbe number \( \nu_d \) in terms of the Cauchy coefficients A
  and B and the wavelengths \( \lambda_d, \lambda_F, \) and \( \lambda_C, \) where the subscripts represent the d, F
  and C wavelengths respectively.

  \[ \nu_d = \frac{n(\lambda_d) - 1}{n(\lambda_F) - n(\lambda_C)} = \frac{A + \frac{B}{\lambda_d^2} - 1}{\frac{B}{\lambda_F^2} - \frac{B}{\lambda_C^2}} \]

- Solve for the coefficients A and B for the optical material specified above

  *Rearranging the results from the previous part, we have

  \[ A + B \left( \frac{1}{\lambda_d^2} - \frac{1}{\lambda_F^2} - \frac{1}{\lambda_C^2} \right) \nu_d = 1 \]

  *Also, we know that

  \[ A + \frac{B}{\lambda_d^2} = n_d \]

  *Solving these two equations for A and B gives
$$B = \frac{(n_d - 1) \lambda_C^2 \lambda_F^2}{\nu_d (\lambda_C^2 - \lambda_F^2)} \quad \text{and} \quad A = n_d - \frac{(n_d - 1) \lambda_C^2 \lambda_F^2}{\lambda_d^2 \nu_d (\lambda_C^2 - \lambda_F^2)}$$

- Based on this model of the glass dispersion, what are $n_F$ and $n_C$?

*Plugging in the results gives $n_F = 1.49184$ and $n_C = 1.48492$. 