OPTI 415/515 Midterm 2 April 7, 2011

Undergraduates do any three problems. Graduate students do all four problems.

1. Given a Scheimpflug system with a 15D thin lens located at z = 0 in the x-y plane. Two objects points are located at (0, 10 mm, -105 mm) and (0, -5 mm, -95 mm).
   (a) Where are the image points formed?
   From the Gaussian imaging equation

   $\frac{1}{z_1'}+\frac{1}{105}=0.015 \Rightarrow z_1'=182.609\,\text{mm}$; $\frac{1}{z_2'}+\frac{1}{95}=0.015 \Rightarrow z_2'=223.529\,\text{mm}$

   $m_1=\frac{182.609}{-105}=-1.739$; $m_2=\frac{223.529}{-95}=-2.353$

   $y_1'=-17.391\,\text{mm}$; $y_2'=11.765\,\text{mm}$

   The image points are formed at (0, -17.391, 182.609) and (0, 11.765, 223.529).

   (b) What is the equation of the line that passes through the two object points?
   The two point form of a line is

   $$y-10=\frac{(-15)}{(10)}(z+105)\Rightarrow y=-1.5z-147.5$$

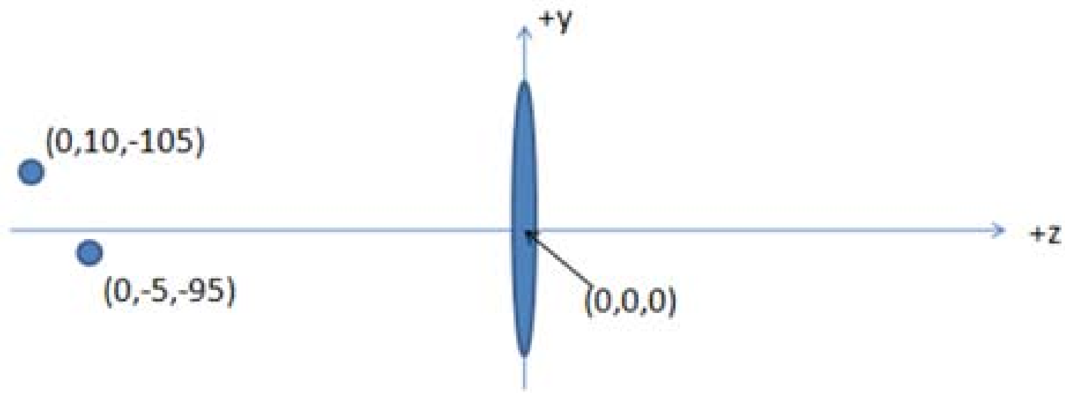
   (c) What is the equation of the line that passes though the two image points?

   $$y+17.391=\frac{(11.765+17.391)}{(223.529-182.609)}(z-182.609)\Rightarrow y=0.712512z-147.5$$

   (d) Where do these two lines intersect?
   The lines intersect when

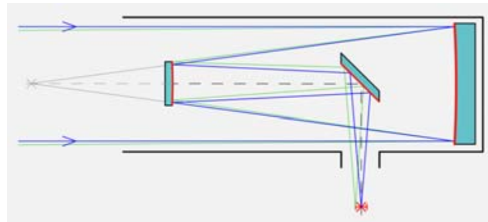
   $$0.712512z-147.5=-1.5z-147.5\Rightarrow z=0$$

   (e) How does the magnification change from the top to bottom of the image?
   The magnification changes linearly (i.e. keystone distortion) from -1.739 to -2.353.



2. In 1970, a disgruntled employee fired shots into the primary mirror of the Harlan J. Smith telescope. The bullet holes were bored out and the telescope is still in use today.

Photo Credit: Bill Keel



The mirror is 170 inches in diameter and does not have a hole in its middle. Suppose we can model the three bullet holes as one inch diameter cylinders, each of which is 0.25 inches deep. Furthermore, suppose we measure the wavefront error over the whole surface and it is zero everywhere except at the three bullet holes where is takes on a value of -0.5 inches.

(a) What is the peak-to-valley wavefront error?

The Peak-to-Valley wavefront error is the max minus the min wavefront error or 0.5 inches.

(b) What is the wavefront variance?

The wavefront variance is given by

$$\sigma_w^2 = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 [W(\rho,\theta)]^2 \rho d\rho d\theta - \left[\frac{1}{\pi}\int_0^{2\pi}\int_0^1 W(\rho,\theta)\rho d\rho d\theta\right]^2$$

These integrals are zero everywhere except over the bullet holes where the wavefront error is -0.5 inches. Thus we can rewrite the variance as

$$\sigma_w^2 = \frac{3}{\pi}\int_0^{2\pi}\int_0^{r_0} [-0.5]^2 \rho d\rho d\theta - \left[\frac{3}{\pi}\int_0^{2\pi}\int_0^{r_0} (-0.5)\rho d\rho d\theta\right]^2$$

where r_0 is the relative radius of the bullet hole. If the semi-aperture of the mirror is 170 / 2 =85 inches and the semi-aperture of a bullet hole is 0.5 inches, then r_0 = 0.00588. Evaluating these gives $\sigma_w^2 = 0.0000259$ *in^2.*

(c) What is the rms wavefront error?

The rms wavefront error is given by $\sqrt{\sigma_w^2} = 0.00509$ *inches.*

(d) Which of these metrics poorly describes the performance of the mirror in this case?

The peak-to-valley metric is a poor metric in this case because the wavefront error is extremely large, but only over a tiny region of the mirror.

(e) If the bullet holes are painted black, what fraction of light is collected by the mirror relative to its performance before it was damaged?

The relative area of the bullet holes compared to the whole mirror is given by

$\frac{3\pi(0.5)^2}{\pi(85)^2} = 0.000104$, *so we would expect one minus this value as the collecting area of the mirror after the damage, or 99.9896% of the undamaged mirror.*

3. Suppose we design a plano-convex singlet from N-BK7. The attached data sheet gives the properties of the glass. At 20° C, the lens is flat on one side and has a convex radius of curvature of 14 mm on the other side. At this temperature, the center thickness of the lens is 4.5 mm, the edge thickness of the lens is 3.149111 mm and the diameter of the lens is12 mm. The design wavelength is 0.555 μm.

   (a) What is the refractive index at 20° C for the design wavelength to six decimal places?
   Evaluating the Sellmeier Formula for the coefficients below gives $n_{555} = 1.518274$.

B1	1.039612		0.555
B2	0.231792		1.518274025
B3	1.010469		
C1	0.006001		
C2	0.020018		
C3	103.5607		

   (b) What is the refractive index at 40° C for the design wavelength to six decimal places?
   Here we use the Constants of Dispersion dn/dT and a ΔT *of 20°to calculate* Δn*. The new index is the index from part (a) plus* Δn *or 1.518306.*

D0	1.86E-06	dt	20	
D1	1.31E-08	dn	3.19E-05	1.59733E-06
D2	-1.37E-11			
E0	4.34E-07			
E1	6.27E-10			
λTK	1.70E-01			
			1.518305972	1.000017775

   (c) At 40° C, calculate the new center thickness, edge thickness and diameter.
   The value $\alpha_{-30/+70\,^\circ C} = 7.1\text{x}10^{-6}$ *describes the linear expansion of the material. Each of the linear dimensions needs to be scaled by* $1 + \alpha\Delta T = 1.000142$.

7.10E-06	20	40
r	6	6.000852
ET	3.149111	3.149558
CT	4.5	4.500639

   (d) What is the new radius of curvature of the rear surface? HINT: Invert the sag equation for a sphere and use your results from part (c).
   If we invert the sag of a sphere equation, the radius R is given by

$$z = R - \sqrt{R^2 - r^2} \Rightarrow R = \frac{r^2 + z^2}{2z}$$

   When r = 6.000852, the sag z of the surface is the edge thickness minus the center thickness or 4.500639 - 3.149558 = 1.351081. This gives R = 14.002 mm.

   (e) How does the effective focal length change from 20° C to 40° C.
   The second surface has all the power in this lens, so

$$f_{20} = \frac{-14}{1 - 1.518274} = 27.013\,\text{mm}\,; \quad f_{40} = \frac{-14.002}{1 - 1.518306} = 27.015\,\text{mm}$$

4. A Hartmann screen with five equi-spaced apertures in the y direction is placed in the pupil of an optical system. Two patterns at planes separated by 10 mm are recorded and the spot positions are summarized in the table below. Plane 1 is located 20 mm behind the optical system and plane two is located 30 mm behind the optical system. The spots merge in between the two planes and become unreadable. From the recorded spot pattern, estimate the location of the best focus of the system.

Plane 1 (z = 20 mm)		**Plane 2 (z = 30 mm)**	
x (mm)	y (mm)	x (mm)	y (mm)
0.000	0.644	0.000	0.516
0.000	0.373	0.000	0.188
0.000	0.000	0.000	0.000
0.000	-0.373	0.000	-0.188
0.000	-0.644	0.000	-0.516

If we connect the spots assuming the focus is between the two planes, then the best focus is around 25.9 mm.