OPTI 415/515 Final Exam May 8, 2013

Undergraduates do any two problems. Graduate students do all three problems

1. The spot pattern from a Shack Hartmann wavefront sensor expands and contracts with varying levels of defocus. If the wavefront impinging on the lenslet array with focal length f and pitch p is given by $W(x,y) = W_{20} (x^2 - y^2)$, where $W_{20} > 0$. Do the following:

(a) What does the wavefront look like along the x-axis? What does the wavefront look like along the y-axis?

Since W20 is positive, the wavefront is a converging spherical (parabolic) wavefront in the x-direction and a diverging spherical wavefront in the y-direction.

(b) Write an expression for the displacement of the spots relative to their location when there is no wavefront error.

The spot displacements are given by

$$\Delta x = -f\frac{\partial W(x,y)}{\partial x} \quad \text{and} \quad \Delta y = -f\frac{\partial W(x,y)}{\partial y}.$$

Plugging in the derivatives gives

$$\Delta x = -2fW_{20}x \quad \text{and} \quad \Delta y = 2fW_{20}y.$$

(c) If the lenslet pitch is 100 microns and our sensor can accurately separate spots that are spaced by 10 microns. What is the required focal length of the lenslets, if we wish to be able to measure the wavefront along the x-direction with a minimum radius of curvature of 100 mm?

For a wavefront with a radius of curvature of 100 mm, the coefficient $W_{20} = 0.005\ mm^{-1}$. For the lenslet on axis, $\Delta x = 0$. For the at x = 0.1 mm, we want $\Delta x = -0.09$ mm to leave a gap of 0.01 mm between this spot and the on axis spot. Plugging into our results from part (b) gives

$$f = -\frac{-0.09}{2(0.005)(0.1)} = 90mm$$

(d) For the pitch and focal length above, we wish to be able to also measure the wavefront along the y-direction with a minimum radius of curvature of 100 mm. How big is the spot pattern if there are nine spots across the diameter of the pattern.

For nine spots, the uppermost lenslet will be located at four times the lenslet pitch or $y = 0.4$ mm. The displacement of the spot formed by this lenslet is

$$\Delta y = 2fW_{20}y = 2(90)(0.005)(0.4) = 0.36mm$$

So the spot is formed at a height of $y = 0.76$ mm and the size of the pattern will be double this value or 1.52 mm.

2. Suppose we measure a wavefront error $W(x, y) = W_{20}(x^2+y^2) + W_{40}(x^4+2x^2y^2+ y^4)$ with a Twyman-Green interferometer with $\lambda = 0.6328$ μm. Both the reference and test arms have the same irradiance, $I_1 = I_2$.

(a) Write an expression for the interference pattern for this wavefront error. You can assume the relative phase difference $\phi = 0$.

$$I(x,y) = I_1 + I_2 + 2\sqrt{I_1 I_2}\, cos\left[\frac{2\pi}{\lambda}\left(W_{20}\left(x^2+y^2\right) + W_{40}\left(x^4 + 2x^2y^2 + y^4\right)\right)\right]$$

(b) What is the visibility of the fringe pattern?

Since $I_1 = I_2$, the visibility is 1.

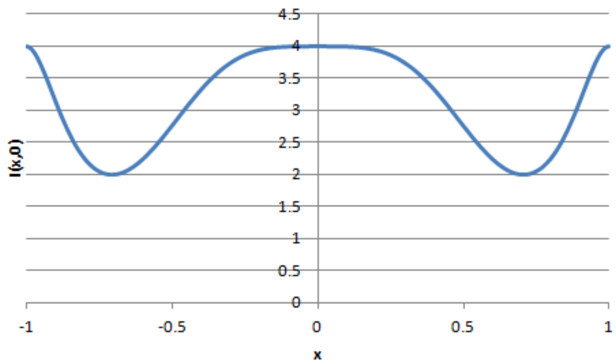
(c) For $y = 0$, $W_{20} = -0.6328$ μm, $W_{40} = 0.6328$ μm, $I_1 = I_2 = 1$, write and expression for the location of the bright fringes along the x-axis.

The interference pattern in this case is

$$I(x,0) = 2 + 2\,cos\left[2\pi\left(-x^2 + x^4\right)\right]$$

Bright fringes occur when $2\pi\left(-x^2+x^4\right) = 2m\pi$ for m = integer. For m = 0, x = -1, 0, or 1.

(d) Given the values from part (c), plot the irradiance profile of the interference fringes along the x-axis for $-1 \le x \le 1$.



Note, the first dark fringe doesn't occur until outside the specified range of x values.

3. Write the wavefront $W(\rho,\theta)=\rho^4 \sin 2\theta$ as a linear combination of Zernike polynomials. A table of Zernike polynomials is included on the next page.

Since the maximum power of ρ is four, we only need to consider Zernike terms with $n \le 4$. Furthermore, the $\sin 2\theta$ term suggests that only terms with $m = -2$ need to be considered. These requirements suggest

$$W(\rho,\theta)=\rho^4 \sin 2\theta = a_{4,-2}Z_4^{-2}(\rho,\theta)+a_{2,-2}Z_2^{-2}(\rho,\theta)$$

From the table of Zernike polynomial definitions

$$\rho^4 \sin 2\theta = a_{4,-2}\sqrt{10}\left(4\rho^4-3\rho^2\right)\sin 2\theta + a_{2,-2}\sqrt{6}\rho^2 \sin 2\theta$$

Rearranging gives

$$\rho^4 = 4\sqrt{10}a_{4,-2}\rho^4 + \left\lfloor a_{2,-2}\sqrt{6}-3\sqrt{10}a_{4,-2}\right\rfloor \rho^2$$

Equating like powers gives two equations for the two unknowns

$$1=4\sqrt{10}a_{4,-2} \Rightarrow a_{4,-2}=\frac{1}{4\sqrt{10}} \quad \textit{and} \quad a_{2,-2}\sqrt{6}-3\sqrt{10}a_{4,-2}=0 \Rightarrow a_{2,-2}=\frac{3}{4\sqrt{6}}$$

So,

$$W(\rho,\theta)=\rho^4 \sin 2\theta = \frac{1}{4\sqrt{10}}Z_4^{-2}(\rho,\theta)+\frac{3}{4\sqrt{6}}Z_2^{-2}(\rho,\theta).$$