Undergraduate students choose any two problems. Graduate Students do all three problems.

1. Suppose we have a wavefront error given by $\mathrm{W}(\mathrm{x}, \mathrm{y})=\mathrm{W}_{22}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)$.
(a) For a Shack Hartmann sensor with lenslets of focal length $=\mathrm{f}$ and pitch $=\mathrm{p}$, write an expression for the displacement of the spots formed by the lenslets.

The displacements of the spots are given by

$$
\Delta \mathrm{x}=-\mathrm{f} \frac{\partial \mathrm{~W}(\mathrm{x}, \mathrm{y})}{\partial \mathrm{x}}=-2 \mathrm{~W}_{22} \mathrm{fx} \text { and } \Delta \mathrm{y}=-\mathrm{f} \frac{\partial \mathrm{~W}(\mathrm{x}, \mathrm{y})}{\partial \mathrm{y}}=2 \mathrm{~W}_{22} \mathrm{fy} .
$$

(b) The lenslet focal length $\mathrm{f}=10 \mathrm{~mm}$, the pitch $\mathrm{p}=1 \mathrm{~mm}$ and $\mathrm{W}_{22}=0.0125 \mathrm{~mm}^{-1}$. The figure below shows the spot pattern for a system with a 4 mm diameter pupil. Show how each spot moves for the aberrated wavefront $\mathrm{W}(\mathrm{x}, \mathrm{y})$.


(c) If we use a $2 / 3^{\prime \prime}$ CCD array (Dimensions $8.8 \times 6.6 \mathrm{~mm}$ ) to record the spot pattern. What is the largest value of $\mathrm{W}_{22}$ that will keep all of the spots on the sensor?

Rearranging the spot displacement in the vertical direction gives

$$
\Delta \mathrm{y}=2 \mathrm{~W}_{22} \mathrm{fy} \Rightarrow \mathrm{~W}_{22}=\frac{\Delta \mathrm{y}}{2 \mathrm{fy}}
$$

The uppermost spot will be on the edge of the sensor when $2+\Delta y=3.3$, so $\Delta y=1.3 \mathrm{~mm}$. This means the largest value of $W_{22}$ is given by

$$
\mathrm{W}_{22}=\frac{1.3}{2 \cdot 10 \cdot 2}=0.0325 \mathrm{~mm}^{-1}
$$

2. Suppose we measure the same wavefront error $W(x, y)=W_{22}\left(x^{2}-y^{2}\right)$ with a Twyman-Green interferometer with $\lambda=0.6328 \mu \mathrm{~m}$. The reference beam has an irradiance of 1 unit, while the test beam has an irradiance of 0.04 units.
(a) Write an expression for the interference pattern for this wavefront error. You can assume the relative phase difference $\phi=0$.
The interferogram is given by

$$
\mathrm{I}(\mathrm{x}, \mathrm{y})=\mathrm{I}_{1}+\mathrm{I}_{2}+2 \sqrt{\mathrm{I}_{1} \mathrm{I}_{2}} \cos \left[\frac{2 \pi}{\lambda} \mathrm{~W}_{22}\left(\mathrm{x}^{2}-\mathrm{y}^{2}\right)\right]
$$

(b) What is the visibility of the fringe pattern?
$I_{1}=1$ and $I_{2}=0.04$. The visibility is given by

$$
\text { Visibility }=\frac{2 \sqrt{0.04}}{1.04}=0.385
$$

(c) For $\mathrm{y}=0$, what is the separation between bright fringes?

Bright fringes occur when

$$
\frac{2 \pi}{\lambda} \mathrm{~W}_{22} \mathrm{x}^{2}=2 \mathrm{~m} \pi \Rightarrow \mathrm{x}_{\mathrm{m}}=\sqrt{\frac{\mathrm{m} \lambda}{\mathrm{~W}_{22}}}
$$

The separation between two adjacent fringes is

$$
\mathrm{x}_{\mathrm{m}}-\mathrm{x}_{\mathrm{m}-1}=\sqrt{\frac{\mathrm{m} \lambda}{\mathrm{~W}_{22}}}-\sqrt{\frac{(\mathrm{m}-1) \lambda}{\mathrm{W}_{22}}}
$$

(d) We use a CCD array with $10 \mu \mathrm{~m}$ square pixels to capture an image of the interferogram.

The interferogram has a diameter of 4 mm on the sensor. What is maximum value of $\mathrm{W}_{22}$ that keeps the spacing between the bright fringes to at least 2 pixels? HINT: The narrowest fringe spacing occurs along the x -axis at the edge of the interferogram.

For $\mathrm{x}_{\mathrm{m}}=2 \mathrm{~mm}=\sqrt{\frac{\mathrm{m} \lambda}{\mathrm{W}_{22}}} \Rightarrow \mathrm{~m}=\frac{4 \mathrm{~W}_{22}}{\lambda}$. The fringe spacing at this point is
$\mathrm{x}_{\mathrm{m}}-\mathrm{x}_{\mathrm{m}-1}=\sqrt{\frac{4 \mathrm{~W}_{22} \lambda}{\lambda \mathrm{~W}_{22}}}-\sqrt{\frac{\left(4 \mathrm{~W}_{22} / \lambda-1\right) \lambda}{\mathrm{W}_{22}}}=0.02$
$2-\sqrt{\frac{\left(4 \mathrm{~W}_{22}-\lambda\right)}{\mathrm{W}_{22}}}=0.02$
$W 22=0.00795 \mathrm{~mm}^{-1}$.
4. Describe the meaning of all of the various features and codes in the attached ISO 10110 drawing.


