Photonic Quantum Information Processing
OPTI 647: Lecture 27

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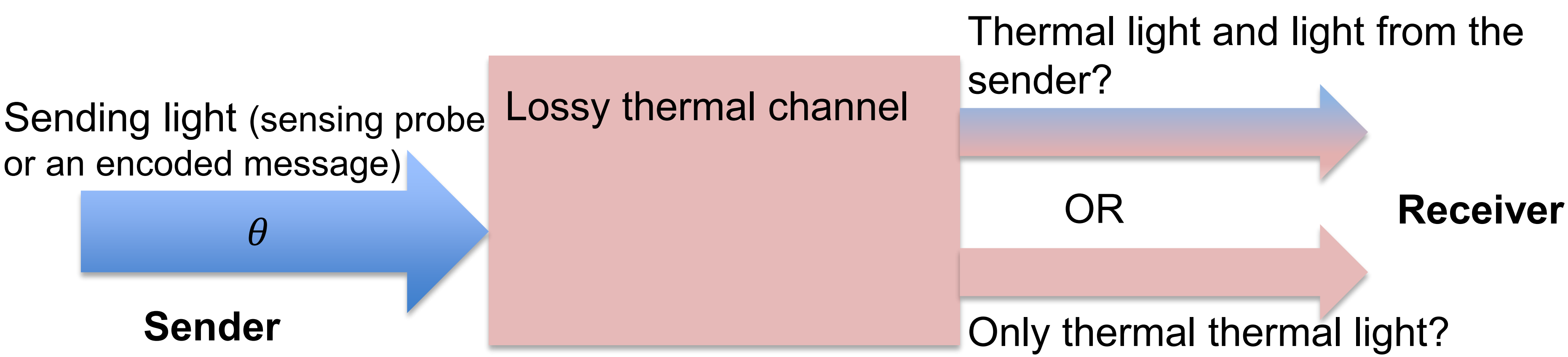
Plan for today

- The notion of *covertness*
- To get an idea of how the math on covert protocols work.
- Covert sensing:
  1. Brief overview of quantum sensing
  2. Covert quantum sensing
  3. Covert quantum sensing: Floodlight illumination

The idea of *covert*

Covert protocols are those where the task is to prevent an adversary (Willie) from detecting the protocols activity.

- Covert communications. Where the task is to transmit useful information while the adversary cannot detect the message.
- Covert sensing. Where the task is to sense a set of unknown parameters, while an adversary (who can be the target), is unaware of the sensing attempt.



Crucial ingredient: Thermal environment, as this will help to hide our probe light.

Quantum relative entropy (QRE)

QRE is one more measure of “closeness” of two density matrices.

$$D(\hat{\rho}|\hat{\sigma}) = \mathrm{tr}(\hat{\rho}\ln\hat{\rho} - \hat{\rho}\ln\hat{\sigma})$$

(Some) properties:

1. $D(\hat{\rho}|\hat{\sigma}) \geq 0.$
2. It can go infinity.
3. Not symmetric: $D(\hat{\rho}|\hat{\sigma}) \neq D(\hat{\sigma}|\hat{\rho})$, i.e., not a metric.
4. Jointly convex: $D(\lambda\hat{\rho}_1 + (1-\lambda)\hat{\rho}_2|\lambda\hat{\sigma}_1 + (1-\lambda)\hat{\sigma}_2) \leq \lambda D(\hat{\rho}_1|\hat{\sigma}_1) + (1-\lambda)D(\hat{\rho}_2|\hat{\sigma}_2)$
5. Decreasing under CPTP maps $\mathcal{E}$: $S(\mathcal{E}(\hat{\rho})|\mathcal{E}(\hat{\sigma})) \leq S(\hat{\rho}|\hat{\sigma})$
6. Additive: $D(\hat{\rho}|\hat{\sigma})$: $D\left(\hat{\rho}^{\otimes n}\middle|\hat{\sigma}^{\otimes n}\right) = n\,D(\hat{\rho}|\hat{\sigma})$.
7. Pinsker’s inequality: $\sqrt{2D(\hat{\rho}|\hat{\sigma})} \geq |\hat{\rho} - \hat{\sigma}|_1$ (lower bound on trace distance)

Pinsker’s inequality is useful to calculate the average probability of error P_e when discriminating between two density matrices:

$$P_e = \frac{1}{2}\left(1 - \frac{1}{2}|\hat{\rho} - \hat{\sigma}|_1\right) \geq \frac{1}{2}\left(1 - \frac{1}{2}\sqrt{2D(\hat{\rho}|\hat{\sigma})}\right)$$

Quantum sensing (single parameter)

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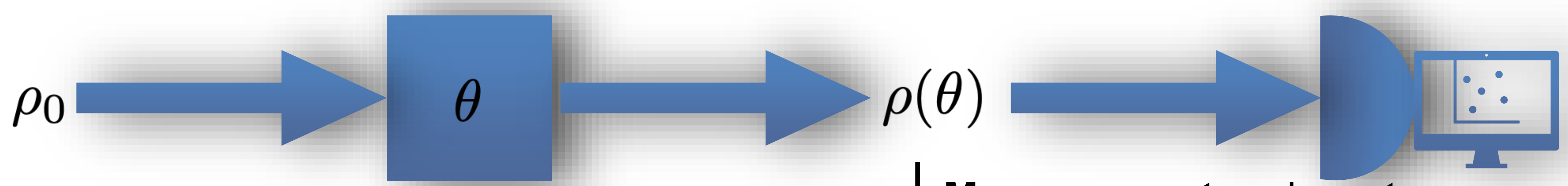

An **input probe state** picks up an **unknown parameter** (e.g. single or many phases, loss rate, etc.) when is injected into a **quantum channel**

Fidelity: $\mathcal{F}\left(\rho(\theta), \rho(\theta+\epsilon)\right) = \mathrm{tr}\left[\left(\rho(\theta)^{1/2}\rho(\theta+\epsilon)\rho(\theta)^{1/2}\right)^{1/2}\right]$

Quantum Fisher Information (QFI):

$$F = -4\frac{d^2\mathcal{F}\left(\rho(\theta), \rho(\theta+\epsilon)\right)}{d\epsilon^2}\Big|_{\epsilon=0}$$

Captures the distance between two states.

Measurement and **post processing** are applied on the **final state**

Estimator $\hat{\theta}_n$ consists of two things: a measurement and data processing.

From these, the covariance of the estimator is constructed: $\langle(\theta-\hat{\theta})^2\rangle$

Unbiased estimator: $\langle\hat{\theta}\rangle = \theta$

Cramér-Rao bound: $\langle(\theta-\hat{\theta}_n)^2\rangle \geq 1/(nF)$

Gives the best performance of any unbiased estimator.

$\hat{\theta}_n$:unbiased estimator, repeated n times.

Quantum sensing (single parameter)

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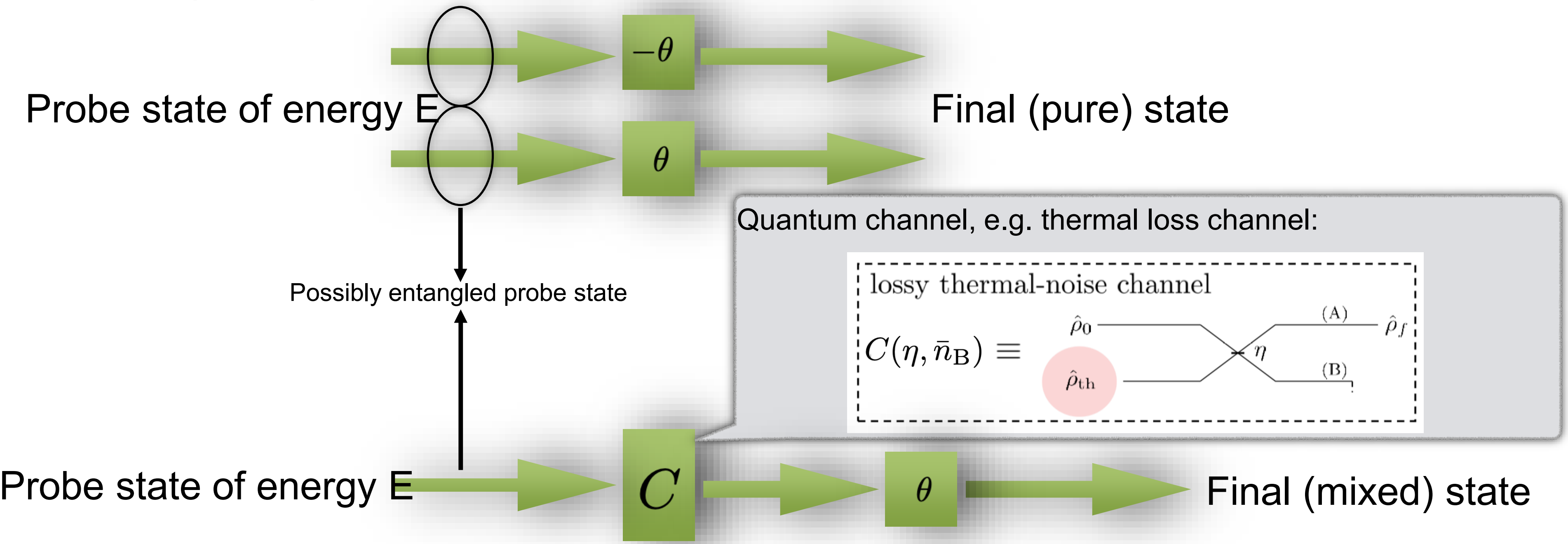
The task is to find the maximum possible QFI and an estimator which achieves that.

For example, in phase estimation: $\hat{U}(\theta) = \exp(i\theta\hat{n})$



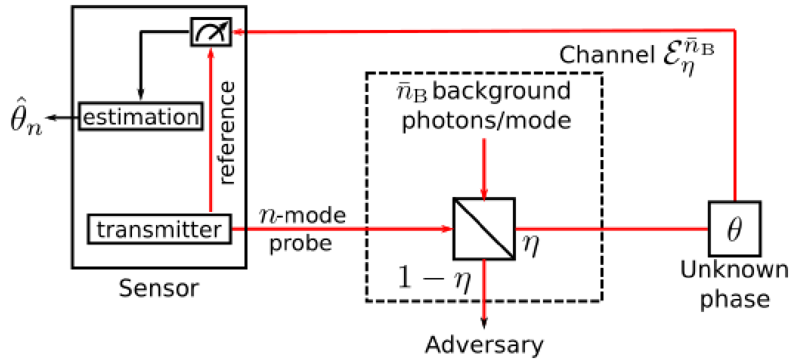
Depending on the probe state, different results have been obtained in the extensive literature and in class, on how to obtain the optimal precision and what estimator achieves it.

$$\langle(\theta - \hat{\theta}_n)^2\rangle \geq 1/(nF)$$

QFI $F \propto E^2$ (Heisenberg limit: better precision)

QFI $F \propto E$ (Classical limit)

Covert quantum sensing (CQS)



CQS: We ask if it is possible to optically probe and estimate an unknown phase while a fully quantum-equipped adversary is not able to credibly decide if such a sensing attempt took place or not.

The adversary performs a **binary test:** detection (0)/ no detection (1).

Photon number detected by the adversary

Threshold chosen By the adversary

If $X_{tot} \geq S$ the adversary declares that the sensor Interrogates.

Performance of the hypothesis test: detection error probability: false alarm (FA) and missed detection (MD).

$$P_e^{\text{det}} = \frac{P_{\text{FA}} + P_{\text{MD}}}{2}$$

Covertness condition:

$$P_e^{\text{det}} \geq \frac{1}{2} - \varepsilon, \;\; \varepsilon > 0$$

Arbitrarily close to random choice.

CQS conditions

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Adversary chooses a detected photons threshold S.

$P_{\mathrm{FA}} = P(\text{d.c.} + \text{th.ph.} \geq S)$

$P_{\mathrm{MD}} = P(\text{probe} + \text{d.c.} + \text{th.ph.} < S)$

Adversary chooses a detected photons threshold S.

High detection probability if

$$\langle N_{\mathrm{S}} \rangle = \omega(\sqrt{n})$$

Mean (total) photon number lower bounded by the sqrt of the number of modes (or uses).

The sensor doesn't interrogate

$$P_{\mathrm{FA}} = P(X_{\mathrm{tot}}^{(0)} \geq S) \leq \frac{\langle \Delta N_{\mathrm{N}}^2 \rangle}{(S - n\bar{n}_{\mathrm{N}})^2}$$

$$X_{\mathrm{tot}}^{(0)} = X_{\mathrm{D}} + X_{\mathrm{T}}$$

Mean thermal photons

Dark counts (Poisson statistics with rate λ)

Total noise photons per mode and variance: $\bar{n}_{\mathrm{N}} = \lambda + \eta \bar{n}_{\mathrm{B}}$

$$\langle \Delta N_{\mathrm{N}}^2 \rangle = n\lambda + n\eta^2(\bar{n}_{\mathrm{B}} + \bar{n}_{\mathrm{B}}^2)$$

The adversary sets threshold: $S = n\bar{n}_{\mathrm{N}} + \sqrt{\langle \Delta N_{\mathrm{N}}^2 \rangle / P_{\mathrm{FA}}^*}$

to obtain the desired P_{FA}^*

The sensor interrogates

$$P_{\mathrm{MD}} = P(X_{\mathrm{tot}}^{(1)} < S) \leq \frac{\langle \Delta N_{\mathrm{N}}^2 \rangle + (1-\eta)^2 \langle \Delta N_{\mathrm{S}}^2 \rangle}{((1-\eta)\langle N_{\mathrm{S}} \rangle - \sqrt{\langle \Delta N_{\mathrm{N}}^2 \rangle / P_{\mathrm{FA}}^*})^2}$$

$$X_{\mathrm{tot}}^{(1)} = X_{\mathrm{D}} + X_{\mathrm{T}} + X_u$$

Count from probe

$\langle \Delta N_{\mathrm{N}}^2 \rangle = \mathcal{O}(n)$ and assuming: $\langle \Delta N_{\mathrm{S}}^2 \rangle = \mathcal{O}(n)$

If $\langle N_{\mathrm{S}} \rangle = \omega(\sqrt{n}) \longrightarrow \lim_{n \to \infty} P_{\mathrm{MD}} = 0$

Thus, given large enough n, the adversary can detect the probes that have mean photon number $\langle N_{\mathrm{S}} \rangle = \omega(\sqrt{n})$ with arbitrary low error of detection probability.

Covertness conditions

$\langle N_{\mathrm{S}} \rangle = O(\sqrt{n})$ (for total photons) $\bar{n}_{\mathrm{S}} = O(1/\sqrt{n})$ (for photons per mode or per use)

CQS converse

1. The sensor must use an n-mode probe with photons $\langle N_{\mathrm{S}} \rangle = O(\sqrt{n})$ to avoid detection.
2. The Cramér-Rao bound and a further lower bound for the thermal loss channel:

$$\langle (\theta - \hat{\theta})^2 \rangle \geq F_{\mathrm{Q}}^{-1} \geq C_{\mathrm{Q}}^{-1} \qquad C_{\mathrm{Q}}^{-1} = O(\tfrac{1}{\sqrt{n}})$$

[CNG, B. Bash, S. Guha, A. Datta Phys. Rev. A **96**, 062306 (2017)]

Suppose the target is interrogated using an n-mode probe with a total of photons $\langle N_{\mathrm{S}} \rangle = n\bar{n}_{\mathrm{S}}$ and that the total photon number variance of the probe is $\langle \Delta N_{\mathrm{S}}^2 \rangle = \mathcal{O}(n)$

Then, the sensing attempt is either detected by the adversary with arbitrarily low detection error probability, or the estimator has mean squared error:

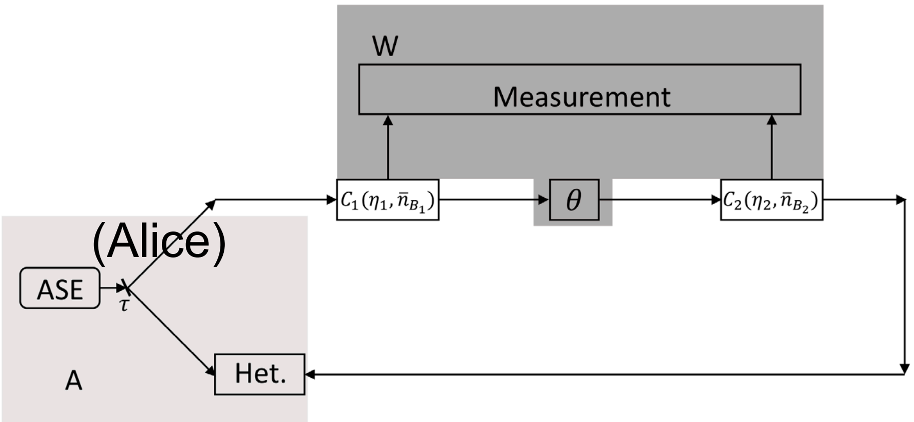
$$\langle (\theta - \hat{\theta}_n)^2 \rangle = \Omega\left(\frac{1}{\sqrt{n}}\right) \quad \text{(tight lower bound)}$$

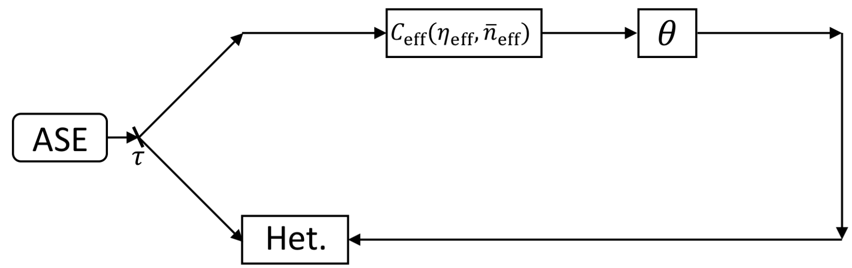
Fundamental limit: $\langle (\theta - \hat{\theta}_n)^2 \rangle = O\left(\frac{1}{\sqrt{n}}\right)$

The converse holds for any input state which satisfies this set of restrictions.

CQS achievability: Floodlight illumination

(Willie)





Equivalent to left scheme for sensor (Alice).

Floodlight illumination (~THz ASE source):
Thermal correlated light with zero mean and covariance matrix,

$$V_{\mathrm{ASE}} = \begin{pmatrix} A_{\mathrm{ASE}} & 0_{2\times 2} \\ 0_{2\times 2} & A_{\mathrm{ASE}} \end{pmatrix} \quad A_{\mathrm{ASE}} = \begin{pmatrix} \bar{n}_{\mathrm{S}} + \frac{1}{2} & \sqrt{\bar{n}_{\mathrm{S}}\bar{n}_{\mathrm{LO}}} \\ \sqrt{\bar{n}_{\mathrm{S}}\bar{n}_{\mathrm{LO}}} & \bar{n}_{\mathrm{LO}} + \frac{1}{2} \end{pmatrix}$$

CQS achievability: Floodlight illumination (continued)

Willie's task is to discriminate between two states:

$\hat{\rho}_1$ (Alice is sensing)

$\hat{\rho}_0$ (Alice is not sensing).

$$P_e^{\text{det}} \geq \frac{1}{2} - \sqrt{\frac{1}{8} D(\hat{\rho}_0^{\otimes n} \| \hat{\rho}_1^{\otimes n})}$$

$$D(\hat{\rho}_0 \| \hat{\rho}_1) = \operatorname{tr}(\hat{\rho}_0 \ln \hat{\rho}_0) - \operatorname{tr}(\hat{\rho}_0 \ln \hat{\rho}_1)$$

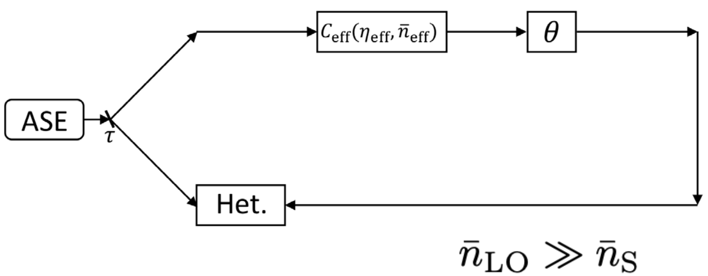
Quantum relative entropy (which is additive)

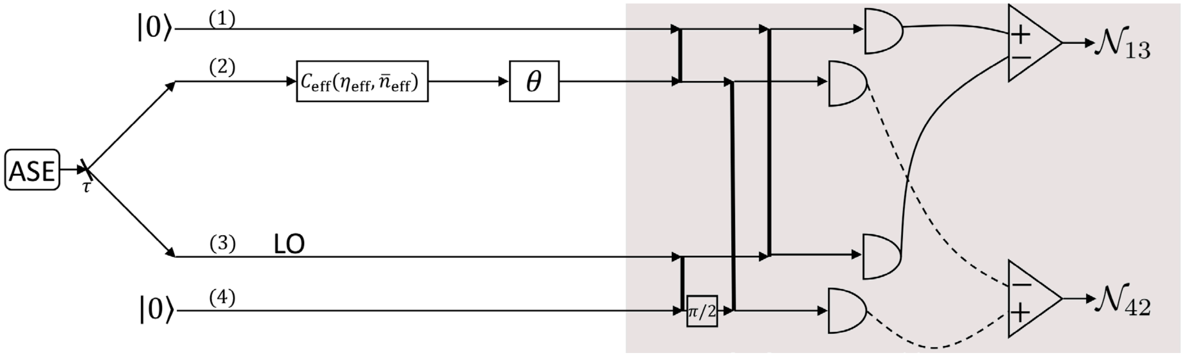
$$P_e^{\text{det}} \geq \frac{1}{2} - \frac{\sqrt{c_2}}{4} \sqrt{n}\, \bar{n}_S \qquad c_2 = f(\eta_1, \eta_2, \bar{n}_{B_1}, \bar{n}_{B_2}) > 0$$

Alice sets $\bar{n}_S = \frac{4\varepsilon}{\sqrt{c_2}} \frac{1}{\sqrt{n}}$ to ensure covertness:

$$P_e^{\text{det}} \geq \frac{1}{2} - \varepsilon, \; \varepsilon > 0$$

CQS achievability: Floodlight illumination





Can we attain the covert quantum limit?

Hetorodyne measurement (dual homodyne):

$$\langle(\theta - \hat{\theta}_{\mathrm{het},n})^2\rangle \approx \frac{\tilde{c}_{\mathrm{het}}}{\varepsilon\sqrt{n}} = \mathcal{O}\left(\frac{1}{\sqrt{n}}\right)$$

$$\tilde{c}_{\mathrm{het}} = [1 + \bar{n}_{\mathrm{B_{eff}}}(1 - \eta_{\mathrm{eff}})]\sqrt{c_2}/(8\eta_{\mathrm{eff}})$$

QFI calculation for the ultimate quantum bound:

$$\langle(\theta - \hat{\theta}_n)^2\rangle \geq \frac{c_{\mathrm{ASE}}}{\varepsilon\sqrt{n}}$$

$$c_{\mathrm{ASE}} = [1 + 2\bar{n}_{\mathrm{B_{eff}}}(1 - \eta_{\mathrm{eff}})]\sqrt{c_2}/(16\eta_{\mathrm{eff}})$$

Heterodyne detection yields an MSE that is at most twice compared to what is attainable by the optimal quantum receiver (but it scales the same).

Comparison of sources

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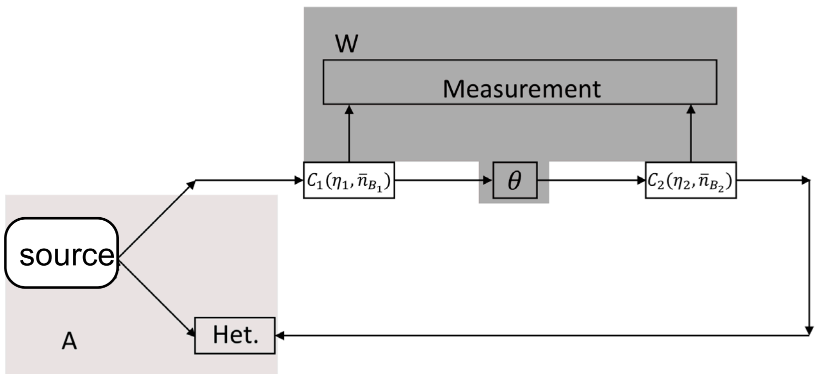
Quantum limits: LASER (coherent state sources) v ASE

We compare the QFI's which correspond to the two sources.

$\mu = F_{\mathrm{A}}^{-1}/F_{\mathrm{coh}}^{-1}$ If $\mu < 1$, the ASE source outperforms the coherent probe.



For thermal equilibrium between the two stations

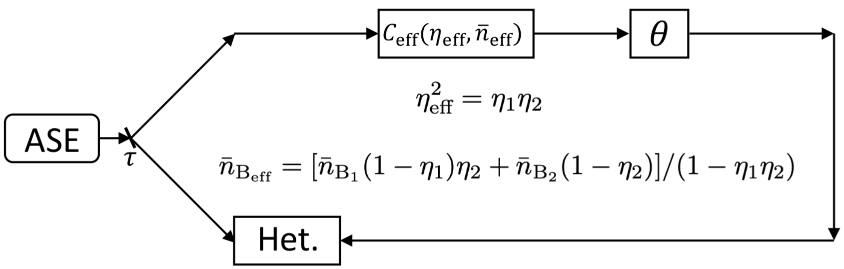
$\bar{n}_{\mathrm{B}_1} = \bar{n}_{\mathrm{B}_2}$

$n =$(optical bandwidth) × (fixed interrogation time)

$\mu = \sqrt{W_{\mathrm{coh}}/W_{\mathrm{ASE}}} < 1$ W: optical bandwidth

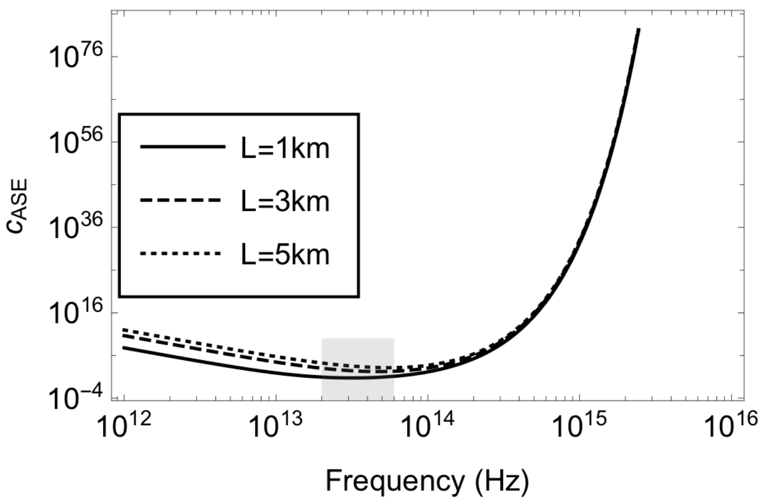
The ASE source would outperform a coherent state probe because of its larger bandwidth

Towards a realistic model



$$\langle(\theta - \hat{\theta}_n)^2\rangle \geq \frac{c_{\text{ASE}}}{\varepsilon\sqrt{n}}$$

$$c_{\text{ASE}} = [1 + 2\bar{n}_{\text{B}_{\text{eff}}}(1-\eta_{\text{eff}})]\sqrt{c_2}/(16\eta_{\text{eff}})$$



$$r_t = 4\text{cm} \; r_T = 10\text{cm}, \; T = 1s, \; \varepsilon = 10^{-3}$$

$$\eta_1 = \eta_2 = \eta = [1 + 2D - \sqrt{1+4D}]/(2D)$$

(e.g. monostatic sensor)

$$D = (A_t A_T)/(\lambda L)^2$$

A_t: Alice's transmitter apparatus exit area.

A_T: Effective area of target's cross section.

λ : Central wavelength.

L : Distance to target.

$$\bar{n}_{B_1} = \bar{n}_{B_2} = \bar{n}_B = [\exp(hc/(\lambda k_B T_0)) - 1]^{-1}$$

(thermal equilibrium) $T_0 = 300K$

Smaller frequencies have more thermal noise to hide the probe, which tends to make c_{ASE} smaller. On the other hand, smaller frequencies have more diffraction limited loss, giving more photons to Willie, which tends to make c_{ASE} higher.

Minimum in the mid to long wave infrared (ML- IR) region.

Conclusions

- Covert sensing *is* possible.
- Covertness condition: $\bar{n}_{\mathrm{S}} = O(1/\sqrt{n})$
- Fundamental limit: $\langle(\theta - \hat{\theta}_n)^2\rangle = O\left(\frac{1}{\sqrt{n}}\right)$
- Attempt to go beyond the fundamental limit leads to detection.
- ASE source and heterodyne detection appear to be efficient.

CNG, B. Bash, A. Datta, Z. Zhang, S. Guha, Phys. Rev. A **99**, 062321 (2019)

B. Bash, CNG, A. Datta, S. Guha, 2017 IEEE (ISIT 2017), 10.1109/ISIT.2017.8007122

Upcoming topics

- Covert sensing (continued)
- Covert communications
- Other topics…