Photonic Quantum Information Processing
OPTI 647: Lecture 17

Christos Gagatsos
Saikat Guha

October 17, 2019
College of Optical Sciences
Meinel 501A

Recap (partial) of previous lectures

1. Gaussian states (pure and mixed) and Gaussian transformations in phase space and in Heisenberg picture.
2. Homodyne/Heterodyne detection.
3. CV teleportation.

Outline of Lecture 15

THE UNIVERSITY OF ARIZONA

Introduction to non-Gaussianity and photon subtraction.

1. What non-Gaussian states and transformations are.

2. Why they are useful.

3. A way to produce non-Gaussian states: photon subtraction.

$$\hbar = 1: \;\; \hat{a} = \frac{\hat{q}_a + i\hat{p}_a}{\sqrt{2}}, \;\; \hat{a}^\dagger = \frac{\hat{q}_a - i\hat{p}_a}{\sqrt{2}}$$

Non-Gaussian v Gaussian states

THE UNIVERSITY OF ARIZONA

Non-Gaussian state is any state that… is not a Gaussian state, which is a trivial definition which however underlines how vast the set of non Gaussian state is.

$\hat{\rho}_G \rightarrow$ described by a $2N \times 2N$ positive definite matrix (CM) and $2N$ vector (1st moments). N is the number of modes, which is a finite number even if the dimension of the Hilbert space is infinite. ⇒ Elegant description.

Pure Gaussian states are generated by **quadratic (in $\hat{a}^\dagger, \hat{a}$ or in $\hat{x}, \hat{p}$) Hamiltonians**, whose corresponding $\hat{U}$ acts on $|0\rangle$. Equivalently, pure Gaussian states are ground states of quadratic Hamiltonians. Mixed Gaussian states are the outcome of tracing out a part of pure Gaussian state.

Problem 69: prove that pure Gaussian states are ground states of quadratic Hamiltonians.

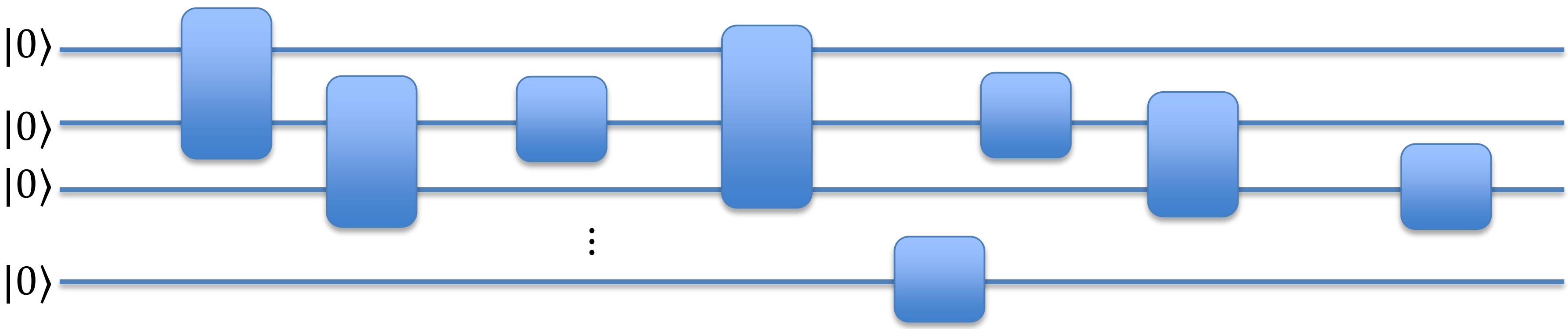
Examples of non-Gaussian pure states:

- Fock states $|n\rangle$. There is no way to go from $|0\rangle$ to $|n\rangle$ with a Gaussian operator (we will go to $|\alpha, \xi\rangle$).
- Superposition of Gaussian states, e.g., cat states: $|\mathrm{c}\rangle = \frac{1}{\sqrt{N_\pm}}(|\alpha\rangle \pm |-\alpha\rangle)$.
- Anything of the form $\hat{\rho} = \exp(-\frac{1}{2}\sum_{ij}\underbrace{\hat{r}_i\, G_{ij}\, \hat{r}_j}_{\text{Quadratic term}} + \sum_{ijk}\underbrace{\hat{r}_i\hat{r}_j F_{ijk}\hat{r}_k + \hat{r}_k F_{ijk}\hat{r}_j\hat{r}_i + \cdots}_{\text{Non quadratic terms}})$

From Gaussian to Non-Gaussian transformations

THE UNIVERSITY OF ARIZONA

It is apparent, that if we want to access **any** transformation we must include non-quadratic Hamiltonians. But how much non-Gausianity is necessary?



When we change position to the operators, we basically have to commute (BCH relation) their generating Hamiltonians.

$\left[\hat{q}_i, \hat{p}_j\right] = i\delta_{ij}$ quadratures of the e/m field

$\hat{q}_i \rightarrow$ Generator of momentum displacement

$\hat{p}_i \rightarrow$ Generator of position displacement

$\widehat{\Phi}_i = \hat{q}_i^2 + \hat{p}_i^2 \rightarrow$ Phase generator

$\hat{S}_i = \frac{1}{2}(\hat{q}_i\hat{p}_i + \hat{p}_i\hat{q}_i) \rightarrow$ Squeezing generator

} Hamiltonians of single mode generators

From Gaussian to Non-Gaussian transformations

By commuting the single mode Gaussian (quadratic) Hamiltonians $\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i$, we can produce any other single mode Hamiltonian, *but nothing else.*

To include any number of modes $N > 1$, we just need $\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i$ and a beam splitter $\hat{B}_{ij} = \hat{p}_i\hat{q}_j - \hat{q}_i\hat{p}_j$. In that way we can construct any multimode Gaussian Hamiltonian, which will be given by commutating the operators $\{\hat{q}_i, \hat{p}_i, \hat{\Phi}_i, \hat{S}_i, \hat{B}_{ij}\}$. **Recall: Reck decomposition.**

Reck decomposition of U(N)

N-mode passive transformation U(N).
We need **N(N-1)/2** beam splitters and phase shifters.
[Reck et al. 1994]

$$U(N)T_{N,N-1}\dots T_{2,1} = \begin{pmatrix} e^{i\phi_1} & & \\ & \ddots & \\ & & e^{i\phi_N} \end{pmatrix}$$

Where the matrix $T_{n,m}$ is a **unit matrix** with the matrix elements $t_{nn}, t_{mm}, t_{nm}, t_{mn}$ are replaced by the four matrix elements of some 2x2 unitary matrix.

$$U(N) = \begin{pmatrix} e^{i\phi_1} & & \\ & \ddots & \\ & & e^{i\phi_N} \end{pmatrix} T^\dagger_{2,1}\dots T^\dagger_{N,N-2}T^\dagger_{N,N-1}$$

Advanced Problem 5: Study Phys. Rev. Lett. **73**, 58 (1994), present it briefly and apply the Reck decomposition to the Hadamard gate $H = H_1 \otimes \ldots \otimes H_1$

$$H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U(N)T_{N,N-1}T_{N,N-2}\dots T_{N-1,1} = \begin{pmatrix} U(N-1) & \\ & e^{i\phi_N} \end{pmatrix}$$

$$U(N-1)T_{N-1,N-2}T_{N-1,N-3}\dots T_{N-2,1} = \begin{pmatrix} U(N-2) & & \\ & e^{i\phi_{N-1}} & \\ & & e^{i\phi_N} \end{pmatrix}$$

$$\vdots$$

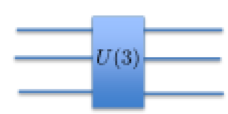
Example on Reck decomposition

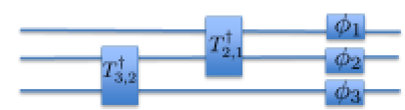
$$U(3) = \begin{pmatrix} e^{i\phi 1}\cos(\theta 2) & e^{i\phi 1}\cos(\theta 1)\sin(\theta 2) & e^{i\phi 1}\sin(\theta 1)\sin(\theta 2) \\ -e^{i\phi 2}\sin(\theta 2) & e^{i\phi 2}\cos(\theta 1)\cos(\theta 2) & e^{i\phi 2}\cos(\theta 2)\sin(\theta 1) \\ 0 & -e^{i\phi 3}\sin(\theta 1) & e^{i\phi 3}\cos(\theta 1) \end{pmatrix}$$

$$U(3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta 1) & -\sin(\theta 1) \\ 0 & \sin(\theta 1) & \cos(\theta 1) \end{pmatrix} = \begin{pmatrix} e^{i\phi 1}\cos(\theta 2) & e^{i\phi 1}\sin(\theta 2) & 0 \\ -e^{i\phi 2}\sin(\theta 2) & e^{i\phi 2}\cos(\theta 2) & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix}$$

$$U(3)\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta 1) & -\sin(\theta 1) \\ 0 & \sin(\theta 1) & \cos(\theta 1) \end{pmatrix}\begin{pmatrix} \cos(\theta 2) & -\sin(\theta 2) & 0 \\ \sin(\theta 2) & \cos(\theta 2) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix}$$

$$U(3)T_{3,2}T_{2,1} = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix} \Rightarrow U(3) = \begin{pmatrix} e^{i\phi 1} & 0 & 0 \\ 0 & e^{i\phi 2} & 0 \\ 0 & 0 & e^{i\phi 3} \end{pmatrix} T^\dagger_{2,1}T^\dagger_{3,2}$$





From Gaussian to Non-Gaussian transformations

THE UNIVERSITY OF ARIZONA

[Lloyd&Braunstein Quantum Computation over Continuous Variables, Vol. 82, Num. 8, p. 1784 (1999)]

If we include just one, single mode, non-quadratic Hamiltonian $\widehat{K}_i$, it is enough to construct any non-quadratic Hamiltonian by commutation relations of $\{\hat{q}_i, \hat{p}_i, \widehat{\Phi}_i, \hat{S}_i, \widehat{B}_{ij}, \widehat{K}_i\}$. For example Kerr non-linearity $\widehat{K}_i = \left(\hat{q}_i^2 + \hat{p}_i^2\right)^2$. Any other non-quadratic Hamiltonian $\widehat{K}_i$ would do the job.

Intuition/proof: for $\widehat{K}_i = \left(\hat{q}_i^2 + \hat{p}_i^2\right)^2$, when trying to commute $\widehat{K}_i$ with the Gaussian set $\{\hat{q}_i, \hat{p}_i, \widehat{\Phi}_i, \hat{S}_i, \widehat{B}_{ij}\}$, you'll need commutations of the form:

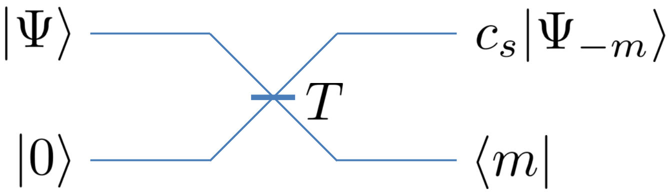
$$[\hat{q}_i^3, \hat{p}_i^m\, \hat{q}_i^n] = i\boxed{\hat{p}_i^{m+2}}\hat{q}_i^{n-1} + \text{lower order terms}$$

The exponent is increasing

$$[\hat{p}_i^3, \hat{p}_i^m\, \hat{q}_i^n] = i\hat{p}_i^{m-1}\boxed{\hat{q}_i^{n+2}} + \text{lower order terms}$$

Photon subtraction

THE UNIVERSITY OF ARIZONA



Subtraction of m photons from some mode of the multimode state $|\Psi\rangle$.

$$\hat{P}_{-m}|\Psi\rangle = c_s|\Psi_{-m}\rangle$$

For coherent input state (single mode) $|\Psi\rangle = |\alpha\rangle$:

$$\hat{P}_{-m}|\alpha\rangle = c_s|\alpha_{-m}\rangle = \frac{\left(-\sqrt{1-T}\right)^m}{\sqrt{m!}}\alpha^m e^{-(1-T)\frac{|\alpha|^2}{2}}|\alpha\sqrt{T}\rangle$$

$$p_s = |c_s|^2 = \frac{(1-T)^m}{m!}|\alpha|^{2m}e^{-(1-T)|\alpha|^2}$$ Probability of success

$|\alpha\sqrt{T}\rangle$ Resulting state. Coherent state with decreased amplitude (phase doesn't change)

For coherent input state (single mode) $|\Psi\rangle = |\alpha\rangle$:

$$\hat{P}_{-m} = \frac{\left(-\sqrt{1-T}\right)^m}{\sqrt{m!}}T^{\frac{\hat{n}}{2}}\hat{a}^m$$

Problem 70: Consider that $|\Psi\rangle = |\alpha\rangle$, and that the PNR detector returns $m = 0$. Find the resulting state. What is your explanation why the final state is not just $|\alpha\rangle$?

Cat states

An important class of non-Gaussianity are the CV cat states.

$$|c_+\rangle = \frac{1}{\sqrt{N_+}}(|\alpha\rangle + |-\alpha\rangle)$$

$$|c_-\rangle = \frac{1}{\sqrt{N_-}}(|\alpha\rangle - |-\alpha\rangle)$$

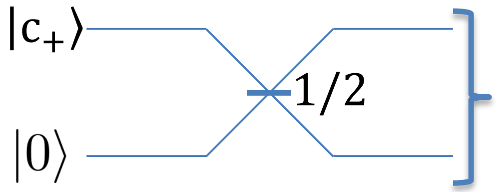
Qubit basis.

$$|c_+\rangle = \frac{1}{\sqrt{N_+}} e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}}(1 + (-1)^n)|n\rangle$$

Even number of photons

$$|c_-\rangle = \frac{1}{\sqrt{N_-}} e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^n}{\sqrt{n!}}(1 - (-1)^n)|n\rangle$$

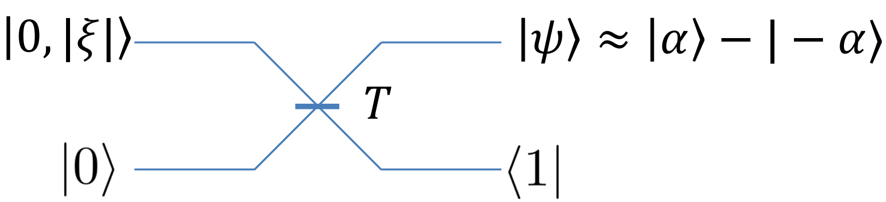
Odd number of photons



$$\frac{1}{\sqrt{N_+}}\left(|\frac{\alpha}{\sqrt{2}}, -\frac{\alpha}{\sqrt{2}}\rangle + |-\frac{\alpha}{\sqrt{2}}, \frac{\alpha}{\sqrt{2}}\rangle\right)$$

In this way, we can produce multimode cats.

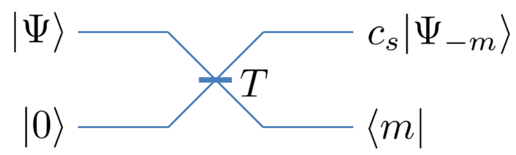
Non-Gaussian states can be also produced by Gaussian operators and post-selection



Problem 71: prove that the resulting state $|\psi\rangle$ looks like $|c_-\rangle$ in Fock space.

Multimode photon subtraction (MPS)

THE UNIVERSITY OF ARIZONA



Subtraction of m photons from some mode of the multimode state $|\Psi\rangle$.

$$\hat{P}_{-m}|\Psi\rangle = c_s|\Psi_{-m}\rangle$$

.

For coherent input state (single mode) $|\Psi\rangle = |\alpha\rangle$:

$$\hat{P}_{-m} = \frac{\left(-\sqrt{1-T}\right)^m}{\sqrt{m!}} T^{\frac{\hat{n}}{2}} \hat{a}^m$$

$$c_s|\alpha_{-m}\rangle = \hat{P}_{-m}|\alpha\rangle = \frac{\left(-\sqrt{1-T}\right)^m}{\sqrt{m!}} \alpha^m e^{-(1-T)\frac{|\alpha|^2}{2}} |\alpha\sqrt{T}\rangle$$

$$p_s = |c_s|^2 = \frac{(1-T)^m}{m!}|\alpha|^{2m} e^{-(1-T)|\alpha|^2}$$ Probability of success

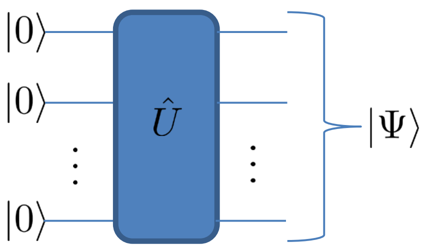
$|\alpha\sqrt{T}\rangle$ Resulting state. Coherent state with decreased amplitude (phase doesn't change)

In general the PS is proportional to the destruction operator $\hat{a}_i^{m_i}$ (m_i being the number of subtracted photons from the i-th mode). To work it out in:

1. Fock space, forget about it.
2. P Glauber-Sudarshan representation: it can be functional or doesn't even exist. Note that it doesn't exist for the cases of interest such as (multimode) squeezed states.
3. Q Husimi representation. The photon subtracted Q representation would look demand to find $\langle\alpha\left|\hat{a}^m\hat{\rho}\hat{a}^{\dagger m}\right|\alpha\rangle$, which is difficult and weird because of $\hat{a}^{\dagger m}|\alpha\rangle$ for all m.

Pure Gaussian states: Gaussian cluster states

The University of Arizona



$$\hat{U} = \exp\left(-i\mathbf{r}\cdot\hat{H}\right)$$

We will deal with quadratic (in $\hat{a}_i$ and $\hat{a}_i^\dagger$) Hamiltonians → $\hat{U}$ is Gaussian unitary, i.e., transforms Gaussian states to Gaussian states preserving the purity.

1. $|\Psi\rangle$ is Gaussian state → it is described by a covariance matrix (CM) V and first moment vector $\boldsymbol{d}$.
2. We will not consider first moments here since we'll talk about clusters. However I have developed the subsequent formulae to work even if there's displacement.

$$Q(\vec{\alpha}) = \frac{1}{\pi^N}\langle\vec{\alpha}|\hat{\rho}|\vec{\alpha}\rangle \to (\hbar = 1) \to Q(\vec{q}_\alpha, \vec{p}_\alpha) = \frac{1}{(2\pi)^N}\langle\vec{\alpha}|\hat{\rho}|\vec{\alpha}\rangle$$

N-mode Gaussian state

$$Q(\vec{q}_\alpha, \vec{p}_\alpha) = \frac{1}{(2\pi)^N\sqrt{\det\Gamma}}\exp\left[-\frac{1}{2}\vec{x}^T\Gamma^{-1}\vec{x}\right]$$

$$\vec{x}^T = (\vec{q_\alpha}^T, \vec{p_\alpha}^T) \qquad \Gamma = \frac{1}{2}I + V$$

Coherent basis representation

$$\frac{1}{\pi^N}\int d^{2N}\vec{\alpha}\ |\vec{\alpha}\rangle\langle\vec{\alpha}| = \frac{1}{(2\pi)^N}\int d^N\vec{q}_\alpha d^N\vec{p}_\alpha\ |\vec{\alpha}\rangle\langle\vec{\alpha}| = I$$

$$|\Psi\rangle = \frac{1}{(2\pi)^N}\int d^N\vec{q}_\alpha d^N\vec{p}_\alpha\ \langle\vec{\alpha}|\Psi\rangle|\vec{\alpha}\rangle = \int d^N\vec{q}_\alpha d^N\vec{p}_\alpha\ K(\vec{q}_\alpha,\vec{p}_\alpha)|\vec{\alpha}\rangle$$

Anything we do now (PS, Fock state projection, etc) will hit a coherent state: easy to deal with, but some more math are required…

$$K(\vec{q}_\alpha,\vec{p}_\alpha) = \frac{1}{(2\pi)^N}\langle\vec{\alpha}|\Psi\rangle$$

$$Q(\vec{q}_\alpha,\vec{p}_\alpha) = \frac{1}{(2\pi)^N}\langle\vec{\alpha}|\Psi\rangle\langle\Psi|\vec{\alpha}\rangle$$

$$\frac{1}{(2\pi)^N}Q(\vec{q}_\alpha,\vec{p}_\alpha) = |K(\vec{q}_\alpha,\vec{p}_\alpha)|^2 \Rightarrow K(\vec{q}_\alpha,\vec{p}_\alpha) = \frac{1}{(2\pi)^{N/2}}Q_{1/2}(\vec{q}_\alpha,\vec{p}_\alpha)$$

To find K, we have to separate the Q representation into a product of two conjugate parts, such that their product is the Q representation. If we're able to do that, we will find K as a function of the CM (and first moments, if any).

The “square root” of a Q function

[Gagatsos & Guha: Phys. Rev. A **99**, 053816]

$$K(\vec{q}_\alpha, \vec{p}_\alpha) = \frac{1}{(2\pi)^{N/2}} Q_{1/2}(\vec{q}_\alpha, \vec{p}_\alpha)$$

$$Q(\vec{q}_\alpha, \vec{p}_\alpha) = \frac{1}{(2\pi)^N \sqrt{\det \Gamma}} \exp\left[-\frac{1}{2}\vec{x}^T \Gamma^{-1} \vec{x}\right]$$

$\vec{x}^T = (\vec{q}_\alpha^{\,T}, \vec{p}_\alpha^{\,T})$

$\Gamma = \frac{1}{2} I + V$

Covariance matrix

$$Q(\vec{q}_\alpha, \vec{p}_\alpha) = Q(\vec{q}_\alpha, \vec{p}_\alpha)_{1/2} Q(\vec{q}_\alpha, \vec{p}_\alpha)^*_{1/2}$$

The task is to break the Q function into two conjugate parts.

V is symmetric → Γ symmetric → Γ^{-1} symmetric $\quad \Gamma^{-1} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix} \quad A^T = A,\ B^T = B$

Change of basis: $(\vec{q}_\alpha\ \vec{p}_\alpha) \to (\vec{\alpha}, \vec{\alpha}^*)$ or in the compact notation $\vec{x} = R\,\vec{z},\ \vec{x}^T = \vec{z}^\dagger R^\dagger$, where the coordinates transformation matrix is:

$$R = \frac{1}{\sqrt{2}} \begin{pmatrix} I & I \\ -iI & iI \end{pmatrix}$$

Reminder: $\vec{\alpha} = \frac{1}{\sqrt{2}}(\vec{q}_\alpha + \mathrm{i}\,\vec{p}_\alpha),\ \ \vec{\alpha}^* = \frac{1}{\sqrt{2}}(\vec{q}_\alpha - \mathrm{i}\,\vec{p}_\alpha)$

$$\vec{x}^T \Gamma^{-1} \vec{x} = \vec{z}^\dagger R^\dagger \Gamma^{-1} R \vec{z} = \vec{z}^\dagger \tilde{\Gamma}^{-1} \vec{z}, \quad \boxed{\tilde{\Gamma}^{-1} = R^\dagger \Gamma^{-1} R}$$

$$\Gamma^{-1} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix} \to \tilde{\Gamma}^{-1} = \frac{1}{2} \begin{pmatrix} A + B - i(C - C^T) & A - B + i(C + C^T) \\ A - B - i(C + C^T) & A + B + i(C - C^T) \end{pmatrix} \Rightarrow \boxed{\tilde{\Gamma}^{-1} = \begin{pmatrix} \tilde{A} & \tilde{C} \\ \tilde{C}^* & \tilde{A}^* \end{pmatrix}}$$

The "square root" of a Q function

THE UNIVERSITY OF ARIZONA

$$\tilde{\Gamma}^{-1} = \begin{pmatrix} \tilde{A} & \tilde{C} \\ \tilde{C}^* & \tilde{A}^* \end{pmatrix}$$

$$\vec{z}^\dagger \tilde{\Gamma}^{-1} \vec{z} = \begin{pmatrix} \vec{\alpha}^{*T} & \vec{\alpha}^T \end{pmatrix} \begin{pmatrix} \tilde{A} & \tilde{C} \\ \tilde{C}^* & \tilde{A}^* \end{pmatrix} \begin{pmatrix} \vec{\alpha} \\ \vec{\alpha}^* \end{pmatrix} = \vec{\alpha}^{*T} \tilde{A} \vec{\alpha} + \vec{\alpha}^{*T} \tilde{C} \vec{\alpha}^* + \vec{\alpha}^T \tilde{C}^* \vec{a} + \vec{\alpha}^T \tilde{A}^* \vec{\alpha}^* = \vec{z}^\dagger \tilde{\mathcal{B}} \vec{z} + \vec{z}^\dagger \tilde{\mathcal{B}}^\dagger \vec{z}$$

$$\Gamma^{-1} = R \tilde{\Gamma}^{-1} R^\dagger = R \tilde{\mathcal{B}} R^\dagger + R \tilde{\mathcal{B}}^\dagger R^\dagger = \mathcal{B} + \mathcal{B}^\dagger$$

Therefore, we broke $\tilde{\Gamma}^{-1}$ into two conjugate parts, namely

$$\tilde{\Gamma}^{-1} = \tilde{\mathcal{B}} + \tilde{\mathcal{B}}^\dagger, \quad \tilde{\mathcal{B}} = \frac{1}{2} \begin{pmatrix} \tilde{A} & \tilde{C} \\ 0 & \tilde{A}^* \end{pmatrix}, \quad \tilde{\mathcal{B}}^\dagger = \frac{1}{2} \begin{pmatrix} \tilde{A} & 0 \\ \tilde{C}^* & \tilde{A}^* \end{pmatrix}$$

Going back to Cartesian coordinates $\vec{x} = (\vec{q}_\alpha \;\; \vec{p}_\alpha)$:

$$V \Rightarrow \Gamma = \frac{1}{2} I + V \Rightarrow \Gamma^{-1} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

Proper half of the Q representation correlation matrix

$$\mathcal{B} = \frac{1}{2} \begin{pmatrix} A + \frac{i}{2}\left(C + C^T\right) & C - \frac{i}{2}(A - B) \\ C^T - \frac{i}{2}(A - B) & B - \frac{i}{2}\left(C + C^T\right) \end{pmatrix}$$

$$K(\vec{x}) = \frac{1}{(2\pi)^N} \frac{1}{(\det \Gamma)^{1/4}} \exp\left[-\frac{1}{2} \vec{x}^T \mathcal{B} \vec{x}\right]$$

$$|\Psi\rangle = \int d^N \vec{q}_\alpha d^N \vec{p}_\alpha \;\; K(\vec{q}_\alpha, \vec{p}_\alpha) |\vec{\alpha}\rangle$$

Multimode squeezed state

THE UNIVERSITY OF ARIZONA

$$\hat{H} = -\frac{i}{2}\sum_{m,n}^{N} G_{mn}\left(\hat{a}_m^\dagger \hat{a}_n^\dagger - \hat{a}_m \hat{a}_n\right) \longrightarrow \hat{U}_r = \exp\left(-ir\hat{H}\right) \longrightarrow S_r = \begin{pmatrix} e^{rG} & 0 \\ 0 & e^{-rG} \end{pmatrix}$$

The symplectic transformation will act on the N-mode vacuum CM, the N-mode squeezed state will be:

$$V = \frac{1}{2}S_r S_r^T = \frac{1}{2}\begin{pmatrix} e^{2rG} & 0 \\ 0 & e^{-2rG} \end{pmatrix} \qquad \Gamma = \frac{1}{2}I + V$$

$$K(\vec{x}) = \frac{1}{(2\pi)^N}\frac{1}{(\det\Gamma)^{1/4}}\exp\left[-\frac{1}{2}\vec{x}^T\mathcal{B}\vec{x}\right] \qquad \mathcal{B} = \frac{1}{2}I + \frac{1}{2}\begin{pmatrix} -\tanh Gr & i\tanh Gr \\ i\tanh Gr & \tanh Gr \end{pmatrix}$$

$$|\Psi\rangle = \int d^N\vec{q}_\alpha d^N\vec{p}_\alpha \; K(\vec{q}_\alpha, \vec{p}_\alpha)|\vec{\alpha}\rangle$$

Just a tad more compact →

$$|\Psi\rangle = \int d^{2N}\vec{x} K(\vec{x})|\vec{\alpha}\rangle \qquad \vec{x} = (\vec{q}_\alpha \; \vec{p}_\alpha)$$

TMSV: $G = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

Advanced Problem 10

Take a single mode squeezed state $|0, |\xi|\rangle$.

i. Subtract 1 photon and find the probability of success, and the fidelity with $|c_+\rangle$ and $|c_-\rangle$.
ii. Subtract 2 photon and find the probability of success, and the fidelity with$|c_+\rangle$ and $|c_-\rangle$.

Analyze your results as a function of $|\xi|$, T (beam splitter's transmissivity).
Can you create small or big cat states (in terms of $|\alpha|^2$)?
You can work with Fock basis or coherent basis representation presented in this lecture (or any other way you want).

Upcoming topics

THE UNIVERSITY OF ARIZONA

1. Probabilistic, noiseless amplification.
2. Introduction to quantum channels and their capacity.
3. Discrete variables teleportation and application of fidelity.
4. More optical circuits other than teleportation (e.g. entanglement swapping).
5. Introduction to metrology/sensing (using the fidelity as starting point to introduce the quantum Fisher information metric).